

A-E-I-O-U (Algebraic Enriching Opportunities for U) NCTM Conference Boston 2015

Numerous Files can be downloaded from <http://wsfcs.k12.nc.us/Page/51682> and from the Conference Planner Handout
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Links:

Box Whiskers/Standard Deviation/Median: http://math.cowpi.com/geogebra/histogram_mean_median.html

Regression Line of Best Fit: <http://www.shodor.org/interactivate/activities/Regression/>
<http://mathbits.com/MathBits/PPT/EstimateAge.htm>

Demonstration of Exponential Growth of Walmart and Target:

<http://projects.flowingdata.com/walmart/> <http://projects.flowingdata.com/target/>

Demonstrations of Simple Movement on Graphs: <http://www.graphingstories.com>

Introduction to Functions (Input-Output): <https://www.youtube.com/watch?v=VUTXsPFx-qQ>

Distance Formula Applet: http://math.cowpi.com/geogebra/distance_formula.html

Worksheet Builder (answers and how to solve): www.easyworksheet.com

Go to <http://www.mrbartonmaths.com/jigsaw.htm> to download Tarsia (Puzzle maker) and for pre-made puzzle

Songs: Distance/Midpoint/Slope: <https://www.youtube.com/watch?v=Z1BjPATzFXA>

Distance Formula: https://www.youtube.com/watch?v=YeNqkeYwX_U
https://www.youtube.com/watch?v=m_0yBtstjzs

Slope Song (Slope Rida) <https://www.youtube.com/watch?v=HUATvvVxwj0>
<https://www.youtube.com/watch?v=AV76i4kJHmU>

Factor: <https://www.youtube.com/watch?v=OFSrINhfNsQ>

Systems of Equations: <https://www.youtube.com/watch?v=1qHTmxlaZWQ>

Exponential Growth: <https://www.youtube.com/watch?v=aDkRHY16Py4>

Exponents: <https://www.youtube.com/watch?v=QIZTruxt2rQ>

Graphing Lines: <https://www.youtube.com/watch?v=TTYKcHJyLN4>
<https://www.youtube.com/watch?v=2BHzXltkByU>

Solving Equations (more for Algebra 2)

<https://www.youtube.com/watch?v=OsEd7X5XuCU&list=PLO7ZzJGcOQfDFVsNIZgXeYS-wf5SaMXbb>

Trig (Algebra 2) <https://www.youtube.com/watch?v=t2uPYLH4Zo>

Exponent Song (Sung to "Flintstones")

Exponents, meet the exponents.

They're a common Algebra Family

When you multiply them, you add the exponents

When you divide them, you subtract the exponents

When you raise one to a power, you multiply the exponents

When you have a fraction one, the denominator is a root

When you have a negative one, you switch the location

Let's see then when the exponent is zero,

Then you always make the base one.

Exponents, use them correctly...

Use them correctly and you'll get an "A."

Log Song (Sung to "Jingle Bells") (For algebra 2)

Adding logs, Adding logs, multiply them

A number in front of the log becomes the exponent.

Minus logs, minus logs, divide them

Log of 1 is zero and the domain is positive

Factoring Binomials (Sung to "If you are happy and know it")

$(+ +) = (+)(+)$ $(- +) = (-)(-)$

If the second is a plus, two of the first.

If the second is a plus, two of the first.

If the second is a plus, then you add to get the middle

If the second is a plus, two of the first

$(+ -) = (+)(-)$

If the second is a minus, one of each

If the second is a minus, one of each

If the second is a minus, then you subtract to get the middle

If the second is a minus, one of each.

Math Aerobics

Students act out the "chants" with their bodies and do each one twice

$Y=3$ $x=3$

Positive Negative Zero Undefined

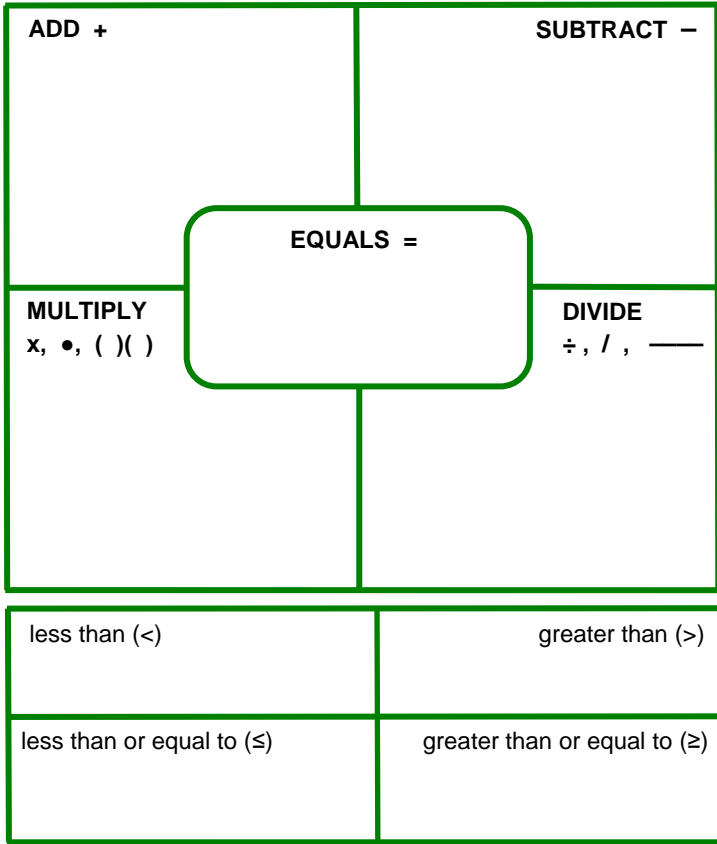
Parallel same, perpendicular negative flip

Rise over Run

$Y - y$ over $x - x$

$Y = \text{Slope } x + B$

TRANSLATION TERMS SORT ACTIVITY



- | | |
|--|---|
| is
is the same as
is equivalent to
is equal to
sum of ... and ..
plus
more than
increased by
combine
deposit
total
difference of ... and ...
minus
less than
decreased by
less
withdrawal
take away
At least
No less than | product of ... and ...
times
twice
double
triple
half
one-third
squared
cubed
quotient of ... and ...
per
into
over
separate into equal groups
Fewer than
At most
No more than
A maximum of
More than
A minimum of |
|--|---|

SOLVING INEQUALITIES

1. Begin by exploring the effects of multiplying both sides of an inequality by a negative number.

- a. Consider the following true statements. $3 < 7$ $-2 < 1$ $-8 < -4$

For each statement multiply the number on each side by -1. Then indicate the relationship between the resulting numbers using < or >.

- b. Based on your observations in Part a, complete the statement: *If $a < b$, then $(-1)a$ ___ $(-1)b$.*
 c. Next, consider relations of the form $c > d$ and multiplication by -1. Test several examples and make a conjecture: *If $c > d$, then $(-1)c$ ___ $(-1)d$.*

2. Pairs of numbers are listed below. For each pair, describe how it can be obtained *from the pair above it*. Then indicate whether the direction of the inequality stays the same or reverses. The first two examples have been done for you.

	<u>Inequality Operation</u>	<u>Inequality Direction</u>
$9 > 4$		
$12 > 7$	add 3 to both sides	stays the same
$24 > 14$	multiply both sides by 2	stays the same
a. 20 ___ 10	_____	_____
b. -4 ___ -2	_____	_____
c. -2 ___ -1	_____	_____
d. 8 ___ 4	_____	_____
e. 6 ___ 2	_____	_____
f. -18 ___ -6	_____	_____
g. 3 ___ 1	_____	_____
h. 21 ___ 7	_____	_____

3. Look back at Problem 2 and identify cases where operations reversed the inequality.

- a. What operations seem to cause this reversal of inequality relationships?
 b. See if you can explain why it makes sense for those operations to reverse inequality relationships.

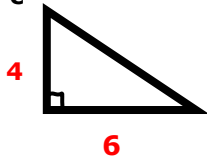
The Pythagorean Theorem/Distance Formula Connection

Pythagorean Theorem $c^2 = a^2 + b^2$

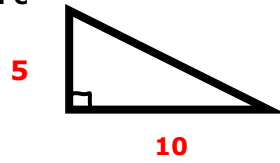
Find the missing side. Show your work.

Round your answer to the nearest tenth, if necessary.

1. Find c



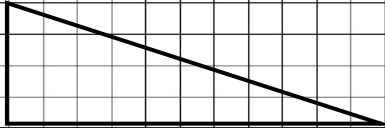
2. Find c



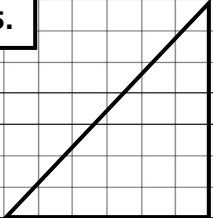
3. $a = 9$, $b = 6$, $c = ?$

Find the length of the hypotenuse for the triangle shown.

4.

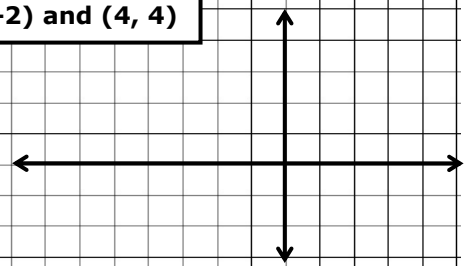


5.



What's the length of the line segment connecting the two points given?

8. $(-6, -2)$ and $(4, 4)$

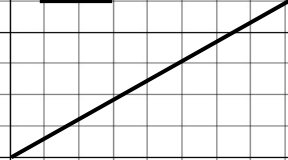


What is the length of the line segment? Assume it is the hypotenuse of a triangle and draw in the missing sides to help you determine the answer.

6.



7.



What's the distance between the two points given?

9. $(4, 10)$ and $(6, 18)$

find the "slope numbers" (these are a and b)
square each number and add these together
find the square root

Directions: Find the mistake(s) if any in working out the problems. Work the problem correctly on the right side.

Problem 1

- a. $2 + 3(x + 4) = 8$ _____
- b. $2 + 3x + 4 = 8$ _____
- c. $6 + 3x = 8$ _____
- d. $3x = 2$ _____
- e. $x = 2/3$ _____

Problem 2

- a. $5 - (x + 9) > 7$ _____
- b. $5 - x - 9 > 7$ _____
- c. $4 - x > 7$ _____
- d. $-x > 3$ _____
- e. $x < -1$ _____

Problem 3

- a. $3(x + 2) - 5x < 8$ _____
- b. $3x + 6 - 5x < 8$ _____
- c. $-2x + 6 < 8$ _____
- d. $-2x < 2$ _____
- e. $x < -1$ _____

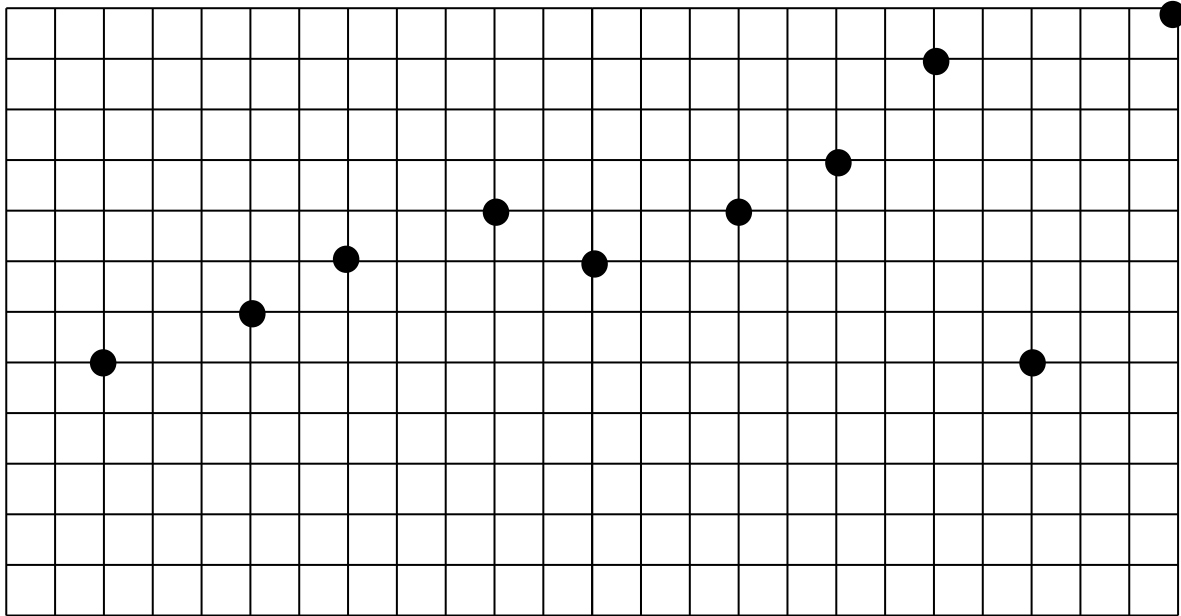
Problem 4

- a. $4(5x + 1) - 8x > -20$ _____
- b. $20x + 4 - 8x > -20$ _____
- c. $12x + 4 > -20$ _____
- d. $12x < -24$ _____
- e. $x < -2$ _____

Lesson Plan on Residuals

- 1) Find the current ages of 6-10 famous people (include your principal etc...)
- 2) Have the students guess the ages of the people.
- 3) Then have them calculate the residual of $|\text{Actual} - \text{Guessed}|$ and sum the total. Teacher can decide if they want the “squared difference” or just the difference.
- 4) Talk about which famous person had the highest residual etc...
- 5) Then have the students complete the following and then talk about the residuals. Teacher can decide if they want the “squared distance” or the regular distance.

Draw what you consider the line of best fit that has the least amount of “net distance”. Calculate the vertical distance from the line and then add them.



Teachers can further expound on the subject by going to: <http://mathbits.com/MathBits/PPT/EstimateAge.htm>

Systems of Equations Around the World, also called a Scavenger Hunt.

Enlarge and place these cards around the room. Students start at different places, solve the problem at the bottom and then look for the answer on top of another card. They then look for their answer etc.. until they have gone around the room.

A (15,-7)

$$4x + 6y = -12$$

$$3x - 5y = 29$$

B (50,30)

$$5x - 3y = 4$$

$$2x + 3y = 52$$

C (3,-4)

$$y = 10x + 60$$

$$y = 8x + 52$$

D (2,6)

$$x + y = 80$$

$$3x + 2y = 210$$

E (-4,20)

$$3y = 12x - 66$$

$$8x - 3y = 26$$

F (12,20)

$$x + y = 8$$

$$x - y = 22$$

G (10,18)

$$4x + 7y = 50$$

$$y = 5x - 4$$

H (8,12)

$$.25x + .05y = 4$$

$$x + y = 32$$

Use the table below to answer questions 1-3.

Actual	Predicted	Residual (Predicted – Actual) Keep it positive
4	4.5	$4.5 - 4 = 0.5$
5	5.2	$5.2 - 5 =$
6	6.7	
7	6.8	
8	8.3	

- 1) How many residuals were above .5?
- 2) What percentage of residuals were above .5?
- 3) What percentage of residuals were above .2?

Use the table below to answer questions 4-6.

Actual	Predicted	Residual (Predicted – Actual)
11.2	11.5	$11.5 - 11.2 = 0.3$
12.4	12.4	$12.4 - 12.4 =$
13.5	13.8	
14.8	14.2	
15.2	15.9	

- 4) How many residuals were at least 0.3?
- 5) What percentage of the residuals were less than 0.1?
- 6) What percentage of the residuals were at least 0.6?

Use the table below to answer questions 7-8.

Actual	Equation $Y = 1.2x - 1$	Residual (Predicted – Actual)
4	3.8	$3.8 - 4 = -0.2 = 0.2$
5		
6		
7		
8.3		

- 7) What percentage of the residuals were above 0.3?
- 8) Which value had the highest residual?

Use the table below for question 9-12.

Day	3	4	5	6	7	8
Height of flower (inches)	5	5.3	5.7	6.1	6.3	6.6
Equation (find by linear regression)						
Residuals						

9) What is the coefficient of correlation?

10) How many data points had a residual greater than 0.1?

11) What percentage had residuals less than 0.2?

12) Which data point had the highest residual?

The following table shows the population of Smithville.

Year	1980	1990	1995	2005	2008
Population	52,000	55,432	57,145	60,580	62,123

13) Based on the line-of-best fit, find the percentage of residuals that were greater than 400?

The following shows the amount of wages that Sally took home based on the number of hours she worked in a restaurant.

Hours	1	2	3	4	5
Wages	12	20	30	42	54

14) Write the linear equation of best fit

15) What is coefficient of correlation?

16) What is the slope and interpret the slope

17) What is the y-intercept and interpret the y-intercept

18) Predict how much Sally would make if she worked 8 hours

19) Predict how much Sally needs to work to make \$83

20) What percentage of data points had residuals higher than 1.5?

Exponential Growth of Stores

<http://projects.flowingdata.com/walmart/>

<http://projects.flowingdata.com/target/>

<http://projects.flowingdata.com/ross/>

The growth of Walmart and Sam's Club in the United States can be modeled by the equation:

$$W(x) = 1(1.1867)^x \text{ where } x \text{ is the number of stores in 1961.}$$

The growth of Target can be modeled by the equation:

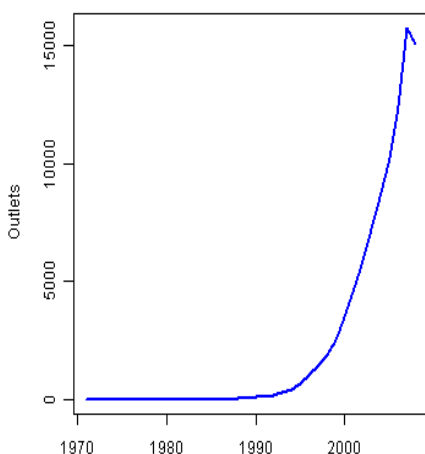
$$T(x) = 1(1.1712)^x \text{ where } x \text{ is the number of stores in 1961.}$$

The growth of Ross Stores can be modeled by the equation:

$$R(x) = 1(1.2588)^x \text{ where } x \text{ is the number of stores in 1984.}$$

1. How many stores did Walmart have in 1961?
2. How many stores did Target have in 1961?
3. Which company grew at the fastest rate?
4. By what growth did Walmart have between 1961 and 2010?
5. By what growth did Target have between 1961 and 2008?
6. How much greater of a rate did Walmart grow faster than Target?
7. Based on the equation, predict the number of stores in 2010 for Walmart.
8. Based on the equation, predict the number of stores in 2008 for Target.
9. Based on the equation, predict the number of stores in 2008 for Ross.
10. Even though Ross grew at a faster rate, why were there less Ross stores in 2008?
11. There were 1240 Target stores by the end of 2004. Find the residual.
12. Can Target and Walmart sustain the same rate of growth?
13. Based on the video, why are there so few Walmarts in Nevada?

Number of Starbucks in the World



$$S(x) = 1(1.2718)^x \text{ where } x \text{ is the number of years since 1970}$$

14. What is the rate of yearly growth of Starbucks?
15. Using the equation, predict the number of Starbucks in the US in 2000. How does it compare to the graph?
16. Why do you think the number of Starbucks decreased after 2010?

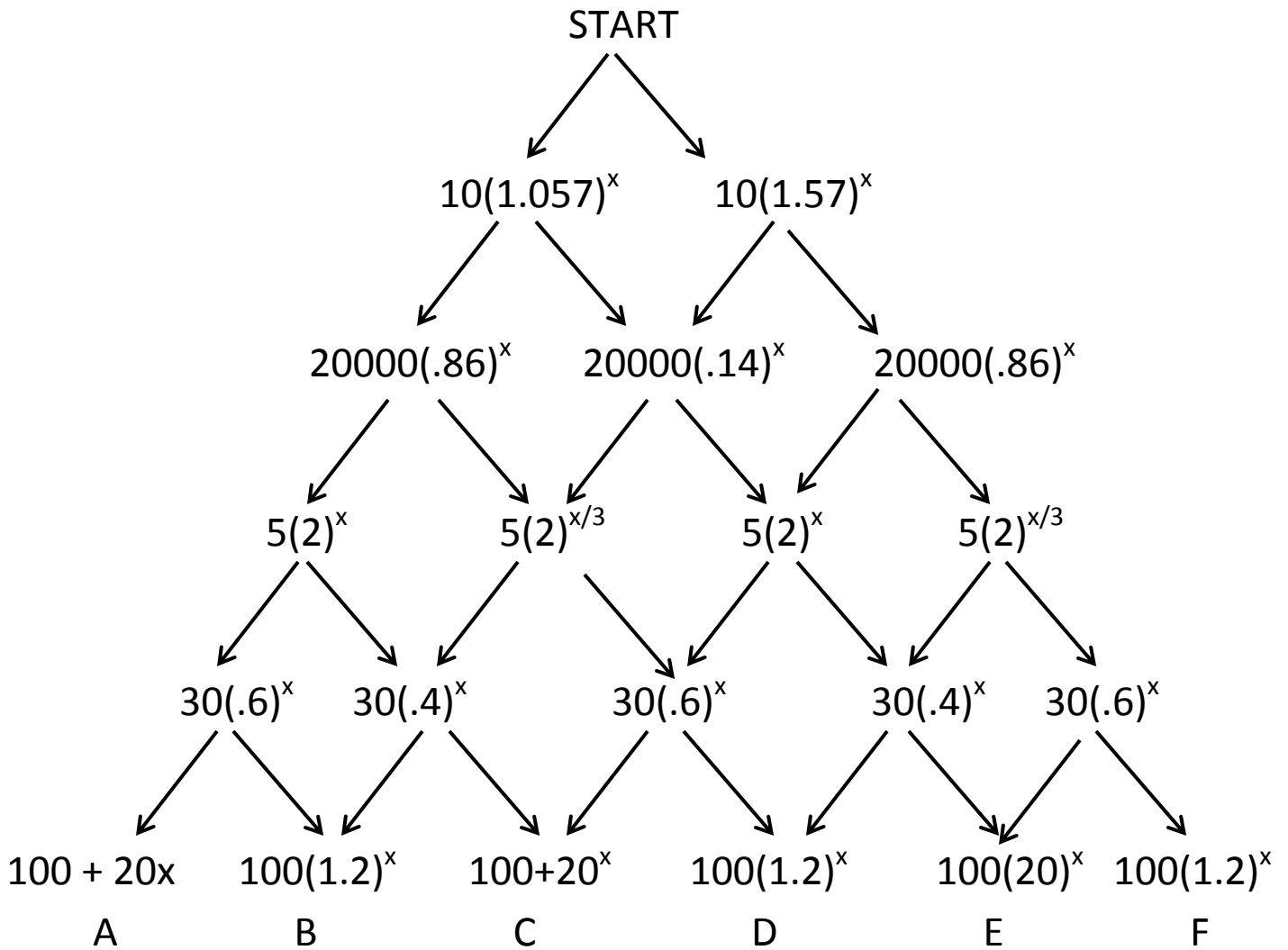
Teacher notes: Target increased a lot in California in the 80's and in other places because it bought out other retailers. It focused on larger cities. Walmart started with small towns in Arkansas and slowly expanded.

Slap Jack

Directions: Teacher gives a board (see below) to every group of 3-6 students who compete against people in their own group. Each group has a score keeper. Teacher displays question (orally or shown) and everyone tries to “touch” the correct square. The first person gets 2 points, other correct people get 1 point and any incorrect response gets -1 points.

- Is the following growth, decay, or neither? $Y = 5(.6)^x$ Decay K
 Is the following growth, decay, or neither? $Y = 2x^3$ Neither B
 $Y = 7(2)^x$. What is the initial value? 7 E
 $Y = 56(2)^x$. What is the rate of growth? 2 C
 $Y = 56(7)^x$. What is the y-intercept? 56 G
 $Y = 2(1.05)^x$. What is the rate? 5% increase D
 $Y = 56(1.37)^x$. What is the rate? 37% increase N
 $Y = 7(.7)^x$. What is the rate? 30% decrease L
 6 butterflies increase exponentially by 4% a year. Write the equation. $6(1.04)^x$ J
 \$6 baseball card depreciates 4% a year. Write the equation. $6(.96)^x$ O
 200 people decrease by 8% yearly. How many people in 5 years? $200(.92)^x \rightarrow 132$ M

$Y = (1.056)^x$	Neither	2	5% increase	7	$Y = 6(1.4)^x$
A	B	C	D	E	F
56	50% increase	Growth	$6(1.04)^x$	Decay	30% decrease
G	H	I	J	K	L
132	37% increase	$6(.96)^x$	3% decrease	3.7% increase	$Y = (1.56)^x$
M	N	O	P	Q	R



Direction: This “Tree” is displayed (or copied) for the students to see. The teacher does an example by saying start and then saying just one of the ones directly below it. She then continues going down until she gets to the bottom. The students try to follow her and land at the same spot she ended up at. Teacher does another example. Then students are paired up. One becomes the reader and the other becomes the listener. They try to get to the same spot.

$2x^{\frac{5}{2}}y^{\frac{1}{2}}$ C $\sqrt[3]{27x^{12}y^5}$	$3x^4y^{\frac{5}{3}}$ G $\sqrt{64x^2y^3}$	_____ L $\sqrt{9x^6y^4}$
$3x^3y^2$ F $(16x^5y)^{\frac{1}{2}}$	$4x^{\frac{5}{2}}y^{\frac{1}{2}}$ M $(8x^7y^2)^{\frac{1}{3}}$	$2x^{\frac{7}{3}}y^{\frac{2}{3}}$ B $\sqrt[3]{64x^2y^3}$
$4x^{\frac{2}{3}}y$ I $(9x^5y)^{\frac{1}{2}}$	$3x^{\frac{5}{2}}y$ A $\sqrt[3]{8x^2y^7}$	$2x^{-\frac{7}{3}}y^{\frac{2}{3}}$ D $(9x^3y^7)^{\frac{1}{2}}$
$3x^{\frac{3}{2}}y^{\frac{7}{2}}$ E $(64x^5y)^{\frac{1}{3}}$	$4x^{\frac{5}{3}}y^{\frac{1}{3}}$ K $(27x^6y^9)^{\frac{1}{3}}$	$3x^2y^3$ H $\sqrt{4x^5y}$

EXPONENT DOMINOES

The problem is on the right side, with simplified "answers" on the left side. Start with any tile. Tile H leads to tile C. The dominoes are cut up to each group and they try to place them together. They also fill in the blanks. Use the blanks to create your own!

Teacher Says (Similar to Simon Says)

Students stand up. Have the students make their chin their “origin.” The teacher then instructs the students to make graphs such as “ $Y=x$,” “ $x=2$,” “ $y=5$,” “ $y=x-3$,” or to show on their fingers the answer to easy questions such as “What is the y -intercept of $y=5x+3$?” or “ X -intercept of $2x-y=8$.” If the teacher begins the instructions with “Teacher Says” then the students perform the task. If the teacher doesn’t say “Teacher Says” then students don’t move. Students who either show an incorrect answer or move when they shouldn’t are asked to sit down. Play continues until there is a winner. (It’s best for the teacher to display the instructions.)

Partner Team Work

The class is split into pairs which each person designated as a “left” or a “right” Teacher displays a set of problems simultaneously for the partners to do. When each pair is done, they raise their hand and the teacher verifies if it is correct or not. Teacher can give “prizes” to the fastest pairs.. Here are some examples:

Left person: Solve for x : $x+2=7$

Right Person: Solve for y : $2x-y=8$

(x is what you get from your partner)

Left person: Solve for x : $3x+4=-11$

Right Person: Solve for y : $2x-y=25$

(x is what you get from your partner)

Right person: Solve for x : $-3x+4=-20$

Left Person: Solve for y : $2x-3y=25$

(x is what you get from your partner)

For Algebra 2: $L(x) = 3x - 2$ $R(x) = 2x^2 - 5x - 1$. Find $LoR(3)$; $R \circ L(x)$; etc..

Partner Worksheet:

Partner A does the left side and Partner B does the right side. After both partners have completed the first four problems, compare your answers. Each partner should have the same 4 answers (but in a different order.)

A. $(5n^3)(4n^2)$ _____

B. $\frac{30n^{10}}{2n}$ _____

C. $\frac{4n^4}{0.25n^{-2}}$ _____

D. $(3n^4)^2$ _____

E. $\frac{10r^3t^5}{40r^7t^3}$ _____

F. $\left(\frac{2r}{3t^3}\right)^2$ _____

G. $\frac{6r^0 \cdot 9t^9}{t}$ _____

H. $(4r^3)^2(3rt^2)$ _____

1. $\frac{18n^6}{2n^{-2}}$ _____

2. $\frac{40n^8}{2n^3}$ _____

3. $(4n^3)^2$ _____

4. $(5n^8)(3n)$ _____

5. $(3t^3)^2 \cdot 6t^2$ _____

6. $\left(\frac{t}{2r^2}\right)^2$ _____

7. $\frac{8r^4t}{18r^2t^7}$ _____

8. $\frac{16r^0 3r^7 t^3}{t}$ _____

Vocabulary Recall

1. Split the following 10 cards to people in your group
2. Select a scorekeeper.
3. One person goes first and says his card, and then says another card. That person then says his card and then someone else's card. Play continues until someone makes a mistake by not responding quickly enough, or not saying another card.
4. The person making a mistake gets a point. (Lowest points wins.)
5. The person making a mistake then begins the next round by saying his card and then another card.

3% Increase $(1.03)^x$	30% Increase $(1.3)^x$	3% Decrease $(.97)^x$	30% decrease $(.7)^x$
5.3% Increase $(1.053)^x$	5.3% Decrease $(.947)^x$	15% Tip $(1.15)^x$	15% Discount $(.85)^x$
7% Tax $(1.07)^x$	7% Discount $(.93)^x$		

Slope Activity Matching (SOLUTIONS)

Slope	Pair #1	Pair #2	Pair #3
5	(1, 6) and (2, 11)	(-2, -3) and (0, 7)	(4, 8) and (7, 23)
2/3	(-1, -8) and (5, -4)	(5, 6) and (8, 8)	(-4, 1) and (-13, -5)
-1/7	(0, 3) and (14, 1)	(3, -2) and (-11, 0)	(2, 4) and (9, 3)
0	(8, 12) and (4, 12)	(5, -2) and (-3, -2)	(-1, 5) and (10, 5)
Undefined	(3, 8) and (3, 0)	(-2, 6) and (-2, -2)	(0, 7) and (0, 2)
9/5	(3, 6) and (13, 24)	(-3, -8) and (2, 1)	(-7, 8) and (-2, 17)
-6	(2, -8) and (-1, 10)	(-3, -15) and (-5, -3)	(4, 9) and (6, -3)
-7/6	(5, 12) and (11, 5)	(-3, 8) and (3, 1)	(-7, -7) and (5, -21)

EXPRESSION BINGO

B I N G O

Answers for BINGO cards:

- A.** $2y^2$ **B.** $\frac{y}{3}$ **C.** $6y$ **D.** $3 + y$ **E.** FREE **F.** $-y - 3$ **G.** $2y - 4$
H. $y^2 + 4$ **I.** $2y + 5$ **J.** $\frac{y}{4}$ **K.** $3y$ **L.** $y + 2$ **M.** $-6y$ **N.** $3y + 2$
O. $y - 3$ **P.** $y - 5$ **Q.** $2y + 2$ **R.** $\frac{y}{-3}$ **S.** $2y + 3$ **T.** $2y$ **U.** y^2
V. $2y + 4$ **W.** y^3 **X.** $4y - 3$ **Y.** $6 - y$

Expressions:

- | | | |
|---------------------------|-----------------------------------|-----------------------|
| 2 times y squared | the difference of $-y$ and 3 | twice y |
| the product of 6 and y | y cubed | 3 more than 2 times y |
| y squared plus 4 | 3 less than y | 3 less than 4 times y |
| the sum of 3 and y | 2 times y increased by 5 | 4 more than twice y |
| the quotient of y and 3 | y divided by 4 | -6 times y |
| the difference of 6 and y | y divided by -3 | y decreased by 5 |
| the sum of y and 2 | the product of 3 and y | 3 times y plus 2 |
| the sum of $2y$ and -4 | the product of 2 and y, plus -2 | y squared |

SIMPLIFYING BINOMIAL MULTIPLICATION

Cut up the cards and distribute to the students – so they can practice the Distribution Property!

Students pair up with each other and work together to multiply the 2 binomials.

Each student records the problem and shows their work.

Students find another classmate and repeat the process.

Some different ways for students to pair up:

- ✓ **Same sign in the middle**
- ✓ **Different sign in the middle**
- ✓ **1 each: "a" coefficient = 1 and "a" coefficient ≠ 1**
- ✓ **Both constants are the same (either odd or even)**
- ✓ **1 odd and 1 even constant**

A $2x - 3$	B $3x + 8$	C $2x + 1$	D $4x - 6$
E $x + 5$	F $x - 5$	G $x + 4$	H $x - 2$
I $x + 10$	J $5x + 1$	K $4x - 1$	L $3x - 5$
M $2x - 9$	N $x + 6$	O $x - 5$	P $x + 8$
AA $x + 1$	BB $x - 1$	CC $x + 2$	DD $x - 6$
EE $2x - 5$	FF $4x + 1$	GG $2x + 3$	HH $4x - 3$
II $x + 4$	JJ $5x + 2$	KK $4x - 7$	LL $3x - 4$
MM $2x - 9$	NN $x + 9$	OO $x - 10$	PP $x + 7$

Quadratics Number Line

Place the following from least (left side) to largest (right side). (Teachers can cut these out or just give it as a worksheet)

A: Y intercept of $y = 3x^2 + 2x - 7$

B: x coordinate of vertex of $y = 2x^2 - 8x - 2$

C: y coordinate of vertex of $y = 2x^2 - 8x - 3$

D: The larger x-intercept of: $x^2 - 9x + 8 = 0$

E: The smaller x-intercept of: $x^2 - 9x + 8 = 0$

F: The smaller x-intercept of: $x^2 + 9x - 10 = 0$

G: The larger root of: $-x^2 + 10x - 24 = 0$

H: $f(4)$ of $y = 2x^2 - 3x - 8$

I: The rate of change of $y = x^2 - 7x + 10$ on the interval of $[1,5]$

J: The sum of the roots of: $y = -x^2 + 5x + 6$

Key:

A: -7 B: 2 C: -11 D: 8 E: 1 F: -10 G: 6 H: 12 I: -1 J: 5 So: C, F, A, I, E, B, J, G, D, H

QUADRATICS FUNCTIONS CONCEPT MAP

Identify the different characteristics for each of the quadratic functions below, using the Concept Map Graphic Organizer.

Show all of your work in each box.

Teacher's Notes:

1. Students can also make their own version of this concept map, either as a regular class assignment, or as creative project.

2. This organizer can also be used for vocabulary, or other "how to" notes.

1. $y = 2x^2 - 10x$

2. $y = -3x^2 + 24x$

3. $y = x^2 - 16$

4. $y = -x^2 + 25$

5. $y = x^2 - 8x + 12$

6. $y = x^2 - 5x - 14$

7. $y = -x^2 - 10x - 24$

8. $y = 2x^2 - 11x - 12$

9. $y = 4x^2 + 6x - 28$

10. $y = 4x^2 + 8x - 5$

11. $y = -5x^2 + 20x + 25$

12. $y = -16x^2 + 8x + 24$

EQUATION

**Axis of
Symmetry**

y-intercept:

Graph

Vertex

a= b= c=

x-intercept(s)

EQUATION

**Axis of
Symmetry**

y-intercept:

Graph

Vertex

a= b= c=

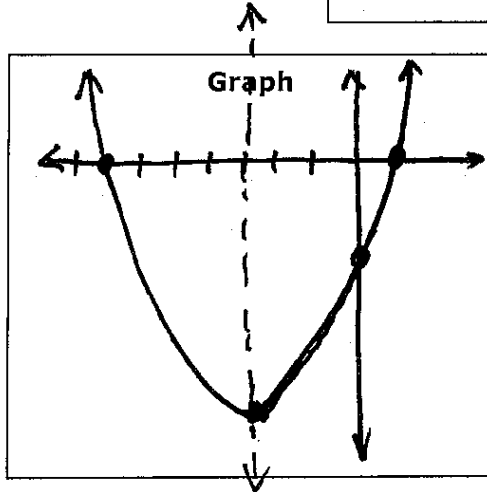
x-intercept(s)

EQUATION

$$y = x^2 + 6x - 7$$

Axis of Symmetry $x = \frac{-b}{2a} = \frac{-6}{2(1)}$
 $x = -3$

y-intercept: "c" -7



Vertex

$$y = x^2 + 6x - 7$$

$$y = (-3)^2 + 6(-3) - 7$$

$$y = 9 - 18 - 7$$

$$y = -16$$

$(-3, -16)$

$a = 1 \quad b = 6 \quad c = -7$

x-intercept(s) Factors: $-7 \rightarrow 7/-1$

$$x^2 + 6x - 7 = 0 \quad \text{Sum: } 6$$

$$x^2 + 7x - 1x - 7$$

$$x(x+7) - 1(x+7)$$

$$(x-1)(x+7) = 0$$

$$x-1 = 0 \quad x+7 = 0$$

$$x = 1 \quad x = -7$$

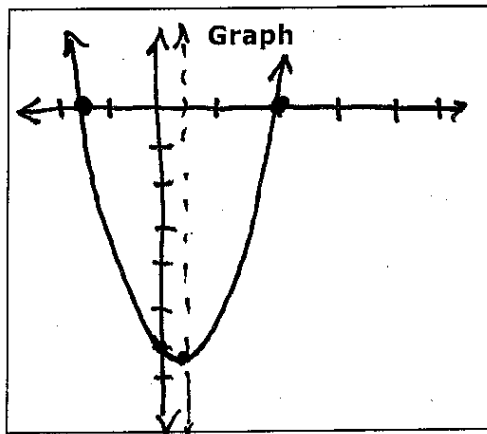
$\{-7, 1\}$

EQUATION

$$y = 2x^2 - x - 6$$

Axis of Symmetry $x = \frac{-b}{2a} = \frac{-(-1)}{2(2)}$
 $x = 1/4$

y-intercept: "c" -6



Vertex

$$y = \frac{b}{2}(x) + c$$

$$y = -\frac{1}{2}\left(\frac{1}{4}\right) - 6$$

$$= -6.125$$

$(.25, -6.125)$

$a = 2 \quad b = -1 \quad c = -6$

x-intercept(s) $\frac{x}{-12} \frac{+}{-1} \rightarrow -4/3$

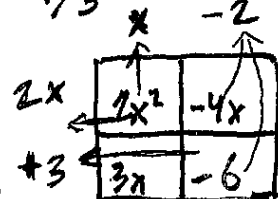
$$2x^2 - x - 6 = 0$$

$$(2x+3)(x-2) = 0$$

$$2x+3 = 0 \quad x-2 = 0$$

$$2x = -3 \quad x = 2$$

$$x = -3/2 \quad x = 2$$



$\{-3/2, 2\}$

Squaring a Binomial

Example 1: $32 * 24$ can be written as $(30 + 2)(20 + 4)$ which can be foiled to $30*20 + 30*4 + 2*20 + 2*4$
 $= 600 + 120 + 40 + 8$

Example 2: $28 * 53$ can be written as $(30-2)(50+3)$ which can be foiled to $30*50 + 30*3 - 2*50 - 2*3$
 $= 1500 + 90 - 100 - 6 = 1484$

1. Rewrite $84*53$ similar to example 1 and simplify.
2. Rewrite $98*23$ similar to example 2 and simplify.
3. **Jane thinks that $(3 + 4)^2 = 3^2 + 4^2$. Is she correct or incorrect?**
 (Prove/disprove your thought by simplifying each side of the '='.)
 If she is wrong, how much is she missing from the right side?
 Is what is missing equivalent to $2*3*4$?
 Is $(3+4)^2 = 3^2 + 2*3*4 + 4^2$?
 Is $(3+4)^2$ equivalent to $(3 + 4)(3 + 4)$? <show by "FOILING" that it works.>
4. **Jack thinks that $(1 + 5)^2 = 1^2 + 5^2$. Is he correct?**
 If not, what exact number does he need to add to the right to get it?
 The number that he is missing is equivalent to $___ * 1 * 5$.
 Therefore $(1 + 5)^2 = ______ + ______ + ______ <look at the part of #3>$
 Or $(1 + 5)^2 = (1 + 5)(1 + 5) = ______ + ______ + ______ + ______ < FOILING method>$
5. **Jack thinks that $(8-3)^2 = 8^2 - 3^2$. Is Jack correct?**
 Is $(8 - 3)^2 = 8^2 + 2*8*(-3) - 3^2$?
 Is $(8-3)^2 = 8^2 + 2*8*(-3) + (-3)^2$?
 $(8 - 3)^2$ is also $= (8 - 3)(8 - 3) = ______ + ______ + ______ + ______ < FOILING method>$
6. **Find $(9 - 2)^2$ two ways similar to #5**
7. **Is $(x + 3)^2 = x^2 + 3^2$?** (Verify by choosing a number for x and simplifying both sides)
 Therefore Similarly, $(x + 3)^2$ can be written as $x^2 + 2*x*3 + 3^2$ or _____
 Or $(x + 3)(x+3) = x^2 + 3x + 3x + 9 = ______$
8. **Is $(x - 5)^2 = x^2 - 5^2$?**
 How can $(x - 5)^2$ be found with two different methods?
9. **True or false:** <hint replace x with 1 and verify>
 $(4x + 3)^2 = 4x^2 + 9$
 $(4x + 3)^2 = 16x^2 + 9$?
 $(4x + 3)^2 = 4x^2 + 2*4x*3 + 9 = 4x^2 + 24x + 9$
 $(4x + 3)^2 = (4x)^2 + 2*4x*3 + (3)^2 = 16x^2 + 24x + 9$?
 $(4x + 3) = (4x + 3)(4x + 3) = 16x^2 + 12x + 12x + 9 = 16x^2 + 24x + 9$

10. **True or false** (use any method). If it is false, write the mistake

$$(5x - 2)^2 = 5x^2 - 4$$

$$(5x - 2)^2 = 5x^2 + 4$$

$$(5x - 2)^2 = (5x)^2 + 2(5x)(-2) + (-2)^2 = 25x^2 - 20x + 4$$

$$(5x - 2)^2 = (5x)^2 + 2(5x)(-2) + -2^2 = 25x^2 - 20x - 4$$

$$(5x - 2)^2 = 5x^2 + 2*(5x) (-2) + -2^2 = 25x^2 - 20x - 4$$

$$(5x - 2)^2 = (5x - 2)(5x - 2) = 25x^2 - 10x - 10x + 4 = 25x^2 - 20x + 4$$

11. Find $(5x + 2)^2$ two different ways

13. Find $(3x^2 - 5y)^2$ two different ways

12. Find $(3x - 2)^2$ two different ways

14. Find $(54)^2$ in a similar manner to example 1

15. Find $(126)^2$ in a similar manner to example 1 $(120 + 6)^2$

Summary: $(a + b)^2$ can be simplified to $(a)^2 + 2ab + (b)^2$ or $(a+b)(a+b) = a^2 + ab + ab + b^2 = (a)^2 + 2ab + (b)^2$
 $(a + b)^2$ is **NEVER** $(a)^2 + (b)^2$. It is equal to **First Squared + Last Squared + 2*First *Last**

Ex: $(3x + 5y)^2 = (3x)^2 + 2(3x)(5y) + (5y)^2 = 9x^2 + 30xy + 25y^2$

Or $(3x + 5y)(3x + 5y) = 9x^2 + 15xy + 15xy + 25y^2$ or $9x^2 + 30xy + 25y^2$

Ex: $(4x^3 - 2y)^2 = (4x^3)^2 + 2(4x^3)(-2y) + (-2y)^2 = 16x^6 - 16x^3y + 4y^2$

Or $(4x^3 - 2y)^2 = (4x^3 - 2y)(4x^3 - 2y) = 16x^6 - 8x^3y - 8x^3y + 4y^2$

16. Simplify $(5x^4 - 3y^2)^2$

17. Is $(2 + 3)^3 = 2^3 + 3^3$? Is it equal to $(2+3)(2+3)(2+3)$?

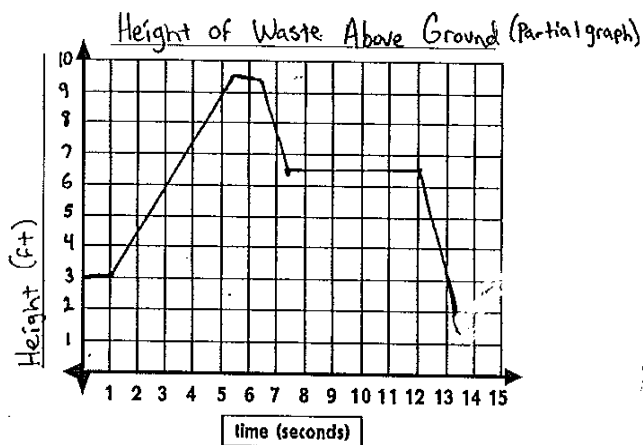
18. Will $(x - 2y)^3 = x^3 + (-2y)^3$?

$$(x - 2y)^3 = (x - 2y)(x - 2y)(x - 2y) = (x^2 - 2yx - 2yx + 4y^2)(x - 2y) = (x^2 - 4xy + 4y^2)(x - 2y)$$

$$= x^3 - 2x^2y - 4x^2y + 8xy^2 + 4xy^2 - 8y^3 = x^3 - 6x^2y + 12xy^2 - 8y^3$$

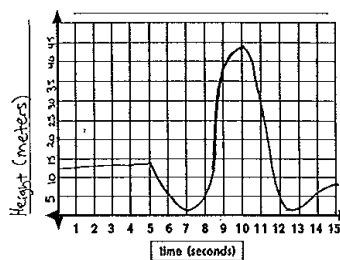
19. Simplify $(x - 4)^3$ by writing it out 3 times and "FOILING twice"

www.graphingstories.com Look at the "Height of Waste Above Ground" with the slide.



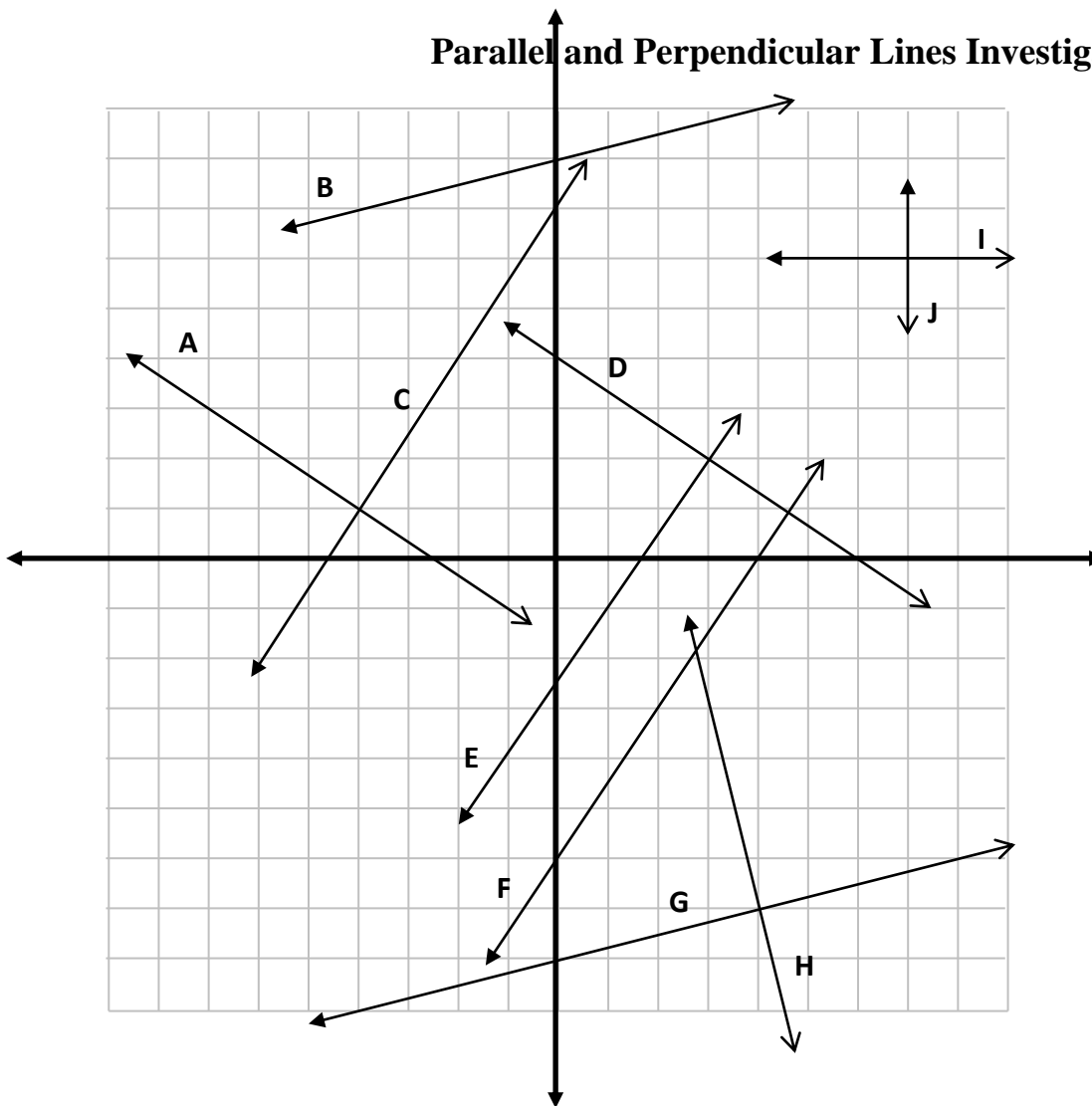
- 1) What is the domain and range? _____
- 2) Is this a function? Explain. _____
- 3) Is it a 1-1 function? Explain. _____
- 4) Find and interpret $f(0.5)$. _____
- 5) Find and interpret $f(4.5)$. _____
- 6) Find and interpret $f(8.5)$. _____
- 7) When is the graph increasing? _____
- 8) When is the graph decreasing? _____
- 9) What is the maximum height? _____

www.graphingstories.com . Look at the "Height" one with the swing.



1. What is the domain and range? _____
2. Is it a function? Is it 1-1? Explain. _____
3. Find and interpret $f(2)$ _____
4. Find and interpret $f(8)$ _____
5. Find and interpret $f(x) = 20$? _____
6. Find and interpret $f(x) = 46$? _____
7. How does your answer to #6 relate to your range? _____
8. Suppose that the swing started 10 feet higher. What would $f(3)$ become? _____
9. Explain how the answer to #8 is equivalent to $f(3) + 10$. _____

Parallel and Perpendicular Lines Investigation



1. Find the slopes of all 10 lines.

A:

B:

C:

D:

E:

F:

G:

H:

I:

J:

2. Which line appears to be parallel to Line A?

What do you notice about the slopes of these two lines?

3. What line appears to be parallel to Line B?

What do you notice the slopes of these two lines?

4. What lines appear to be parallel to E and what do you notice about the slopes of these three lines?

5. Complete this statement:

Two Lines are parallel if they have the _____ slope.

6. Do Lines A and C appear to be parallel or perpendicular?

What do you notice about the slopes of these two lines?

7. Do Lines G and H appear to be parallel or perpendicular?

What do you notice about the slopes of these two lines?

8. Complete this statement:

If two lines are perpendicular then one slope will be positive and the other will be _____.

They will be _____ of each other.

9. If a line has an undefined slope then what is the slope of any line that is parallel to it?

Perpendicular to it?

Distance and Midpoint Project

You are planning a 5-day trip across the United States.

Choose a place to start and continue in a "round-trip" throughout the country.

Use the map to determine how far you travel each day (distance formula), with a pit stop along the way (midpoint).

Each block on the map equals 50 miles.

Show your work for all questions (NEATLY please!) on a separate piece of paper.

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

	Start (ordered pair & State)	End (ordered pair & State)	Distance Traveled (in miles)	Pit Stop (ordered pair & State)
Day 1				
Day 2				
Day 3				
Day 4				
Day 5				

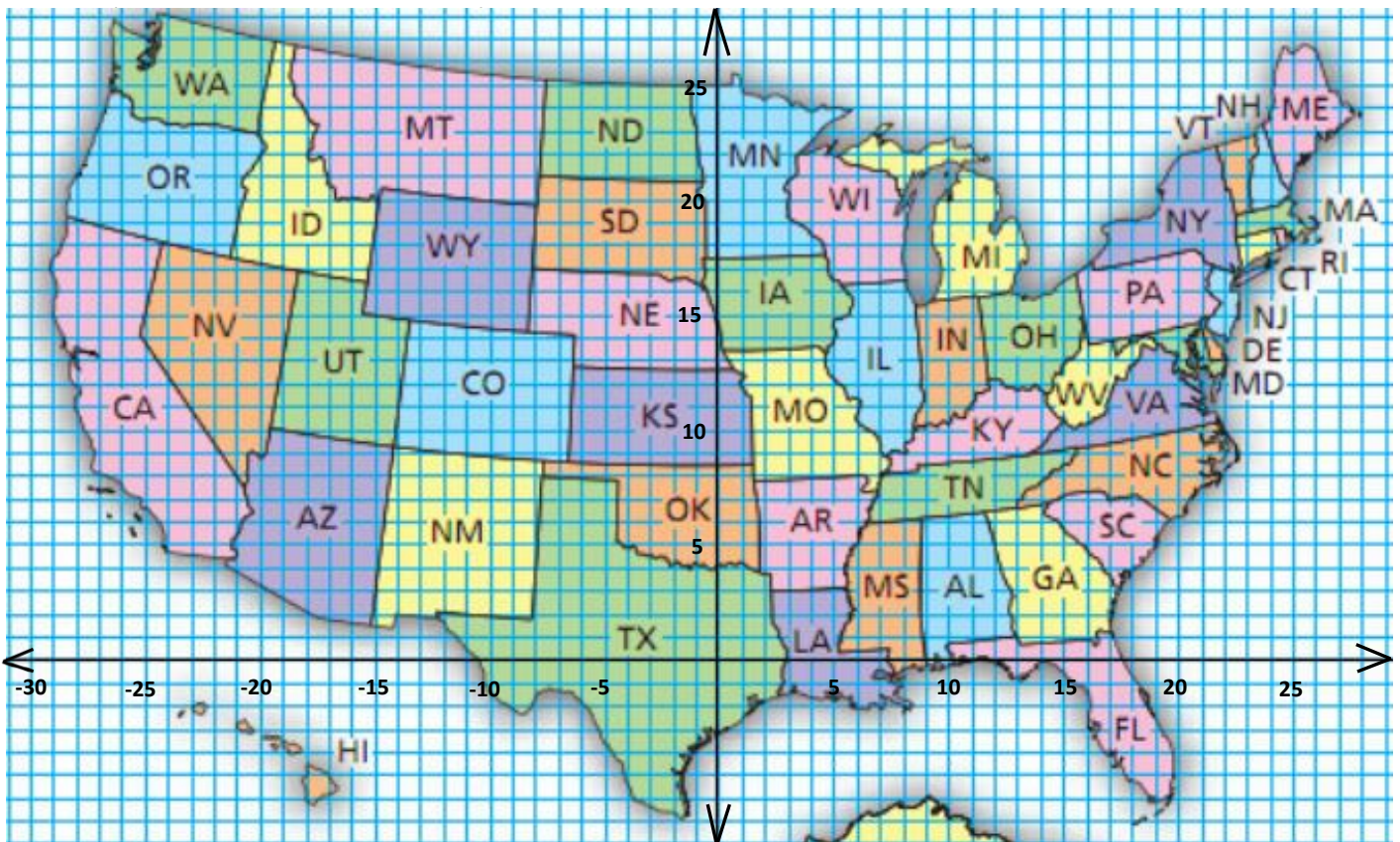
Total Distance of Trip (in miles): _____

TRIP TIME:

How long did it take? Assume your average speed was 60 mph. _____
 (time = distance/speed)

FUEL: How many gallons of gas did you use? Assume you averaged 25 miles per gallon (mpg). _____
 (gallons used = distance/mpg value used)

How much did the gas cost? Assume \$2.30 per gallon. _____



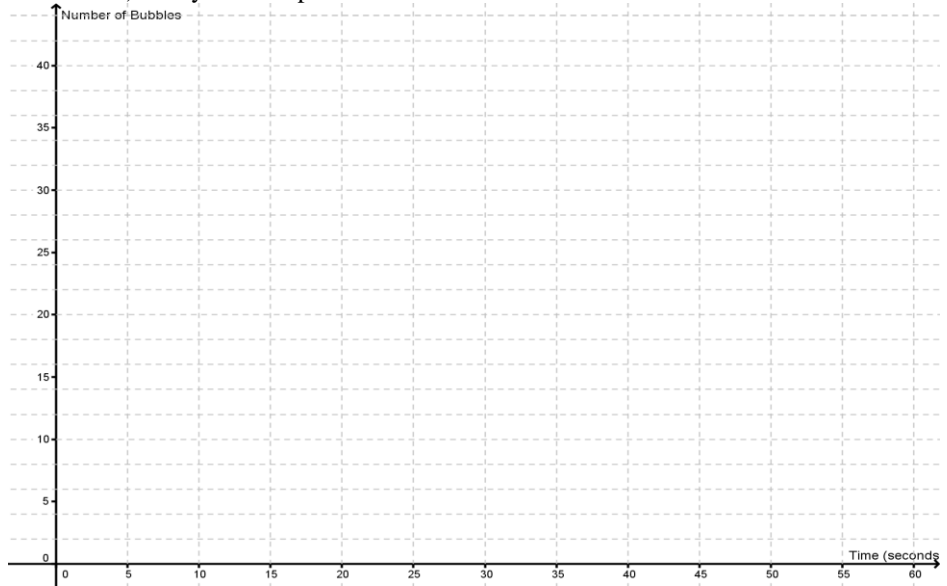
3D Rate of Change Investigation

Your teacher is going to give you bubble gum to chew. Count how many bubbles you blow during each 10 second increment. .

1) Fill in the chart based on your data

2) Plot your data points. Y is the total amount of Bubbles!

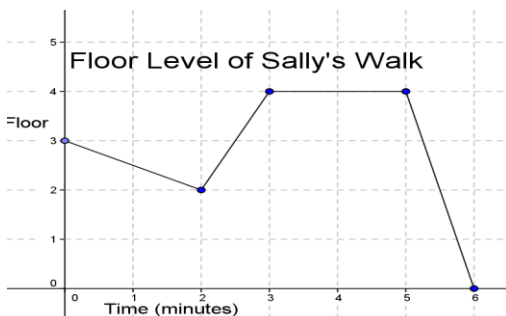
Time (s) (x)	Bubbles	Total Bubbles (y)
0		
10		
20		
30		
40		
50		
60		



- 4) Find and interpret the y-intercept from the table: _____
- 5) How could you find the y-intercept from the plot? _____
- 6 a) To find rate of change from 0-60 seconds, find out how many bubbles did you increase by from 0-60: _____ bubbles
 b) Then find out how much the time increased by from 0-60: Change in time: _____ seconds
 c) Then divide your answer from a) by b) Rate of change = _____ bubbles/seconds.
- 7 a) To find rate of change from 20-40 seconds, find out how many words did you increase by from 20-40: _____ bubbles
 b) Then find out how much the time increased by during that interval : _____ seconds
 c) Then divide your answer from a) by b) Rate of change = _____ bubbles/seconds.
- 8) Find the rate of change from 40-50 minutes. Show work
- 9) How could you have looked at your graph to find the answer from 0-60?

Day	Number of Flowers
0	2
3	5
6	7
8	10

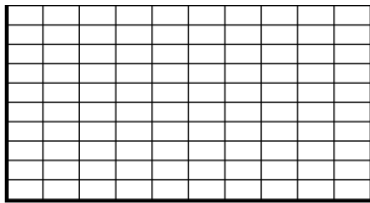
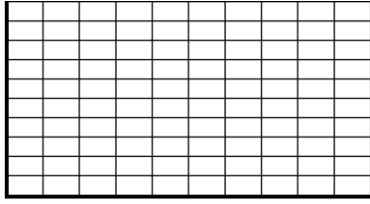
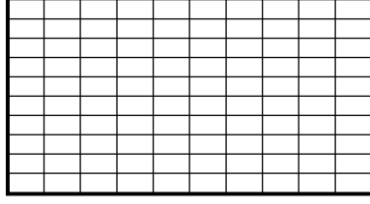
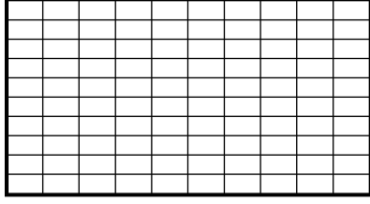
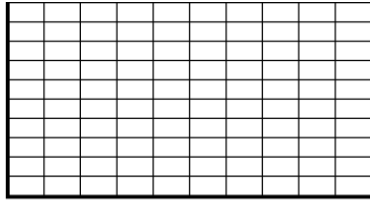
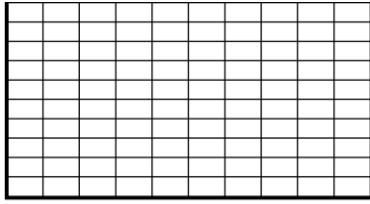
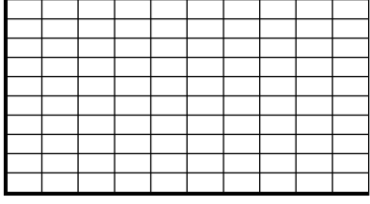
- 10) Find and interpret the y-intercept: _____
- 11) Find the rate of change from 3-8 days: a) Change in flowers: _____
 b) Change in days: _____
 c) Rate of change: _____ flowers/day



- 12) Find and interpret the y-intercept: _____
- 13) Will the rate of change always be positive? Explain.
- 14) Find the rate of change from 2-5 minutes a) Change in floors: _____
 b) Change in time: _____
 c) Rate of change: _____ floor/min
- 15) Find the rate of change from 5-6 minutes.

16) Explain in general terms how you can find the rate of change (use "x's" and "y's")

Roman opened a savings account with \$200 and saves \$10 per month.	$\text{slope} = 2$ $\text{y-intercept} = 200$	$\text{NEXT} = \text{NOW} + 10,$ starting at 200
Caleb's phone company charges a \$40 flat rate and \$0.10 per minute.	$\text{slope} = 0.10$ $\text{y-intercept} = 40$	$\text{NEXT} = \text{NOW} + 0.10,$ starting at 40
Ally's plant was 20 inches tall and grows 1 inch every 2 days.	$\text{slope} = \frac{1}{2}$ $\text{y-intercept} = 20$	$\text{NEXT} = \text{NOW} + 0.10,$ starting at 20
Allen weighs 200 pounds but loses 10 pounds per month on his diet.	$\text{slope} = -10$ $\text{y-intercept} = 200$	$\text{NEXT} = \text{NOW} - 10,$ starting at 200
Sabrina borrowed \$20 from her mom and pays her back \$2 each week.	$\text{slope} = -2$ $\text{y-intercept} = 20$	$\text{NEXT} = \text{NOW} - 2,$ starting at 20
Isaiah's job pays \$40 per day and \$3 for each sale that he makes.	$\text{slope} = 3$ $\text{y-intercept} = 40$	$\text{NEXT} = \text{NOW} + 3,$ starting at 40
Jamar began his hike at an elevation of 200 feet and climbed 2 ft per second.	$\text{slope} = 2$ $\text{y-intercept} = 200$	$\text{NEXT} = \text{NOW} + 2,$ starting at 200

$y = 200 + 10x$	<table border="1"> <tbody> <tr> <td>X</td> <td>0</td> <td>2</td> <td>4</td> <td>6</td> </tr> <tr> <td>Y</td> <td>200</td> <td></td> <td>240</td> <td>260</td> </tr> </tbody> </table>	X	0	2	4	6	Y	200		240	260	
X	0	2	4	6								
Y	200		240	260								
$y = 40 + 0.10x$	<table border="1"> <tbody> <tr> <td>X</td> <td>0</td> <td>10</td> <td>20</td> <td>40</td> </tr> <tr> <td>Y</td> <td>40</td> <td>41</td> <td>42</td> <td></td> </tr> </tbody> </table>	X	0	10	20	40	Y	40	41	42		
X	0	10	20	40								
Y	40	41	42									
$y = 20 + \frac{1}{2}x$	<table border="1"> <tbody> <tr> <td>X</td> <td>0</td> <td>2</td> <td>4</td> <td>6</td> </tr> <tr> <td>Y</td> <td>20</td> <td>21</td> <td></td> <td>23</td> </tr> </tbody> </table>	X	0	2	4	6	Y	20	21		23	
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X	0		6	10								
Y	200	160	140	100								
$y = 20 - 2x$	<table border="1"> <tbody> <tr> <td>X</td> <td>0</td> <td>3</td> <td>5</td> <td>8</td> </tr> <tr> <td>Y</td> <td></td> <td>14</td> <td>10</td> <td>4</td> </tr> </tbody> </table>	X	0	3	5	8	Y		14	10	4	
X	0	3	5	8								
Y		14	10	4								
$y = 40 + 3x$	<table border="1"> <tbody> <tr> <td>X</td> <td></td> <td>8</td> <td>10</td> <td>13</td> </tr> <tr> <td>Y</td> <td>55</td> <td>64</td> <td>70</td> <td>79</td> </tr> </tbody> </table>	X		8	10	13	Y	55	64	70	79	
X		8	10	13								
Y	55	64	70	79								
$y = 200 + 2x$	<table border="1"> <tbody> <tr> <td>X</td> <td>3</td> <td>7</td> <td></td> <td>11</td> </tr> <tr> <td>Y</td> <td>206</td> <td>214</td> <td>218</td> <td>222</td> </tr> </tbody> </table>	X	3	7		11	Y	206	214	218	222	
X	3	7		11								
Y	206	214	218	222								