

Session 346  
**Representations with Tape  
 Diagrams? What's That?**

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**Caution...**

Don't discount representations when giving formative assessments.... A picture is worth a ten-thousand words....?

And quite often shows you misconceptions better than words.

**What representations are helpful?**

- Base Ten
- Addition/Subtraction –
  - part/part/whole models
  - Comparison models – including double numberlines
  - Place value models, leading to algorithms
- Multiplication/Division
  - Equal quantities in each group models – set models
  - Equal groups models – numberlines
  - Area models, arrays
  - Place Value models, leading to algorithms
- Fractions/Decimals/Percent/Ratio
  - Bar Models (comparative and proportional models)

**Tape Diagrams...**

“Drawings that look like a segment of tape, used to illustrate number relationships. Also known as strip diagrams, bar models or graphs, fraction strips, or length models”

CCSS glossary

Also identified often : area models, arrays, numberlines (single and double)

**Which CCSS practices involve representations?...**

A great representation can support a student's explanation, IF the diagram:

- shows accurate relationships between parts and whole
  - Has labels that match the situation
  - Correctly identifies knowns vs unknowns
- Allows students to create abstract sentences that match their thinking in the diagram

**8 math practices for successful problem solving CCSS.**

1. Make sense of problems and persevere
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

1<sup>st</sup> graders at work



3rd grade  
Part/Whole Continuous and Comparisons

Janet and Austin together read 42 books in one month. Janet read 8 more books than Austin read. How many books did Austin read?

Guiding questions

1. How many groups or parts are in this situation? What is the question to be answered? (write an answer sentence with a blank)
2. Which group is larger, or is worth more? Least?
3. Can you represent the parts with diagrams that show the size relationship?
4. Label, label, label: nouns, numbers, question marks.
5. Number sentence(s). Write abstract number sentences to match the situation.
6. Solve and check for reasonableness.

Can you critique others?

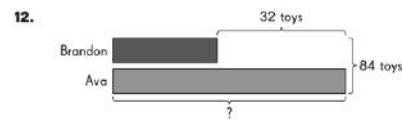
MP 3: Construct viable arguments and critique the reasoning of others

Can your students write a problem for a representation and then defend it as being a viable scenario for a representation? How about determining if someone else's situation matches?

Encourage student talk

3rd grade  
Parts/Whole continuous and Comparisons

Use the bar models to write a two-step word problem. Then solve the problem.



3rd grade  
Parts/Whole, continuous and Comparison

Soda pop \$1.25 Soda pop and Burger \$4.95  
Full meal deal (fries, drink and burger) \$7.20

Find the price of the a) burger, and b) fries

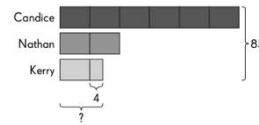
3<sup>rd</sup> grade  
The move from Partitioning phase  
(additive thinking) to Factoring Phase  
(multiplicative thinking)

*First Steps in Number*

Developmentally can happen over a 3 year span, thus makes it difficult in our "grade" system to meet all children's needs And highlights the importance to teach in the pictorial along with abstract and concrete.

3<sup>rd</sup> grade  
Multiply and Divide

Candice, Kerry, and Nathan sold a total of 85 tickets to the school's production.  
Candice sold three times as many tickets as Nathan.  
Kerry sold 4 more tickets than half of what Nathan sold.  
How many tickets did Kerry sell?



More 3<sup>rd</sup> grade tasks

- Nadia ordered 7 pizzas for dinner. Each pizza had 8 slices. She and her friends ate 49 slices, How many slices were left?
- Your school ordered 280 muffins for Mother's Day. They were evenly split between bran and cinnamon. They ran short, so they ordered another 66 bran muffins. How many bran muffins did they have altogether?

3<sup>rd</sup> grade tasks with numberlines

- Jack and Jill each had rain gauges side by side on the fence. Jack's gauge measured in fourths of an inch and Jill's measured in eighths of an inch. On Thursday, Jack looked and he had  $\frac{2}{4}$  inch of rain. How much rain did Jill's gauge have in it? Draw (a) numberline(s) to represent your answer.

Numberline progressions

- Single numberlines
  - Like denominators, counting up, value of the real number. Moving into abstract understanding
- Double numberlines
  - Equivalent fractions. Valuing start, end and whole numbers in between. Focus on unlike denominators

*Varied starts and stops on any given numberline*

4<sup>th</sup> grade  
Multiply and Divide

When a number is divided by 5, it has a quotient of 136 and a remainder of 3. What is the number?

Sometimes the simplest problems reveal major misunderstandings. And a representation clearly shows what they are thinking...

### 4th grade Multiply and Divide

How well are you able to connect multiplication models with division models?

*CCSS expects all 4<sup>th</sup> graders to use Area models, arrays and place value models to explain 3 digit by 1 digit and 2 digit by 2 digit multiplication problems.*

*All 5<sup>th</sup> graders will use the standard algorithm for multiplication problems, but must explain division using area models, arrays and place value models.*

### 4th grade Multiply and Divide

Charlie has 5 times as many stamps as Ryan. They have 1,608 stamps in all. How many more stamps does Charlie have than Ryan?

### 4th grade Multiply and Divide

Mr. Jackson sold 3 computer monitors in the first year. In the second year, he sold 4 times as many monitors as the first year. In the third year, he was able to sell 6 times as many monitors as the first year.

a. How many monitors did Mr. Jackson sell in three years?

\_\_\_\_\_

b. He sold the monitors for \$185 each. How much money did Mr. Jackson earn?

\_\_\_\_\_

### More 4th grade problems

• Julia has \$485. She has 5 times as much money as Charlene. How much money does Charlene have?

• Billy has 3 times as much money as Reagan. Madison has 4 times as much money as Reagan. If Madison has \$21 more than Reagan, how much money do they have altogether?

• At the flower shop there are 5 times as many roses as sunflowers. If there are 252 roses and sunflowers altogether, how many roses are there? How many sunflowers are there?

### 5th grade Multiply, Divide, Fractions

Klein read 30 pages of a book on Monday and  $\frac{1}{8}$  of the book on Tuesday. He completed the remaining  $\frac{1}{4}$  of the book on Wednesday. How many pages are there in the book?

Do 5<sup>th</sup> graders always begin with thinking? What is my whole? Oh, a book!  
Does it help to show the whole in a representation?

### 5th grade Multiply, Divide, Fractions

Lisa has some clips.  $\frac{1}{4}$  of the clips are pink,  $\frac{1}{3}$  of the remainder are blue, and the rest are yellow. What fraction of the clips are yellow?

### More 5<sup>th</sup> grade problems

- The party store had many balloons. Of all the balloons, 10% were red, 25% were orange, 15% were green, and 20% were yellow. The other 30 were blue. How many balloons did the store have in all?
- Hugo, the Labrador retriever, eats 4 times as much as Spencer, the Yorkie. Together they eat 120 ounces of food a week. How many ounces does Spencer eat in 1 week?
- Marco had \$16. Then he spent  $\frac{1}{4}$  of his money on lunch and  $\frac{1}{2}$  on a video game. How much money did he have left?

### 6th grade rates, ratios and proportions

A door is painted pink and blue. The area painted pink is 4 times that of the area painted blue.

- What is the ratio of the area that is painted blue to the area that is painted pink?
- The door has an area of 5 square meters. Find the area of the part of the door that is painted pink.

### 6th grade rates, ratios and proportions

The ratio of the ages of 3 children is 1 : 3 : 5. The oldest child is 12 years older than the youngest child. How old is the middle child?

### 6th grade rates, ratios and proportions

- In a sixth-grade class of 120 students, everybody played 1 sport. Of all the students, 25% played basketball, 15% played soccer, 20% were swimmers, 15% ran cross country, and the rest played baseball. How many students were either baseball players or swimmers?
- On Monday and Tuesday, Honor sent letters to her friends at sixth grade camp. She mailed 12 of the letters on Monday and 18 of them on Tuesday. What percentage of the letters did she mail on Monday?

### 6th grade rates, ratios and proportions

- Erica scored 40 points on a test. This was 80% of the total possible score. Find the total possible score.
- Superman flew for 3.5 hours and traveled 2,800 miles. At that rate, how far will he travel in 6 hours?

### Resources

- <http://sbac.portal.airast.org/practice-test/resources/>
- Utube – singapore math, bar models
- Utube – tape diagrams
- Google search tape diagrams
- Crystal Springs Books [www.SDE.com/crystalsprings](http://www.SDE.com/crystalsprings)  
ISBN:978-1-934026-(24-3)or(51-9)or(53-3)or(54-0)or(52-6)