



The Standards for Mathematical Practice of CCSSM in Assessment

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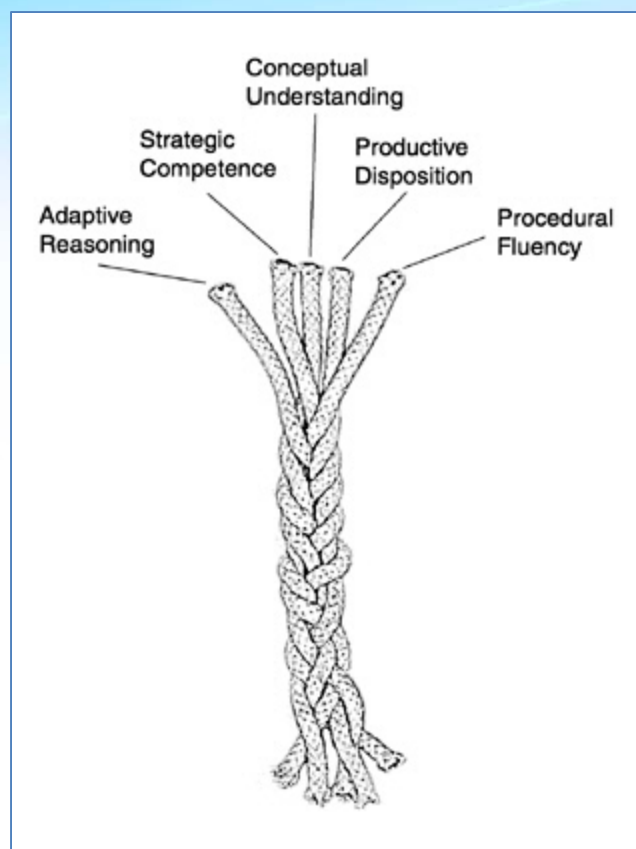


The Standards for Mathematical Practice of CCSSM in Assessment

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The Five Strands of Mathematical Proficiency



National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

NCTM Process Standards

- Problem Solving
- Representation
- Reasoning and Proof
- Connections
- Communication

Standards for Mathematical Practice of the CCSSM

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

Mathematical Practice 1

Make sense of problems and persevere in solving them.

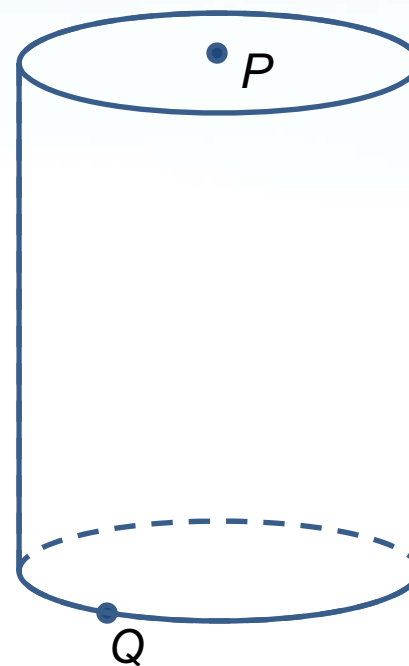
Mathematically proficient students:

- Explain the meaning of a problem
- Look for entry points to a solution
- Plan a pathway instead of simply “jumping in”
- Consider analogous problems and try special cases and simpler forms of the problem
- Monitor and evaluate their progress
- Change course if necessary

Mathematical Practice 1

Make sense of problems and persevere in solving them.

The figure shows a right circular cylinder. Point P is the center of one base and point Q lies on the circumference of the other base. The cylinder has a diameter of 10 centimeters and a height of 12 centimeters. What is the distance, in centimeters, from P to Q ?



Key = 13

Mathematical Practice 2

Reason abstractly and quantitatively.

Mathematically proficient students:

- Make sense of quantities in their relationships in problem situations
- Can *decontextualize*: abstract a situation, represent it symbolically, manipulate representing symbols
- Can *contextualize*: pause as needed to probe into the referents of the symbols involved
- Create a coherent representation of the problem at hand



Mathematical Practice 2

Reason abstractly and quantitatively.

$$T = \frac{1}{4}n + 10$$

The equation above can be used to estimate the temperature T , in degrees Fahrenheit, based on n , the number of times a cricket chirps in one minute.

What does the coefficient $\frac{1}{4}$ mean, in terms of the context?

- (A) The increase in chirps for each increase of 1 degree Fahrenheit
- (B) The increase in temperature, in degrees Fahrenheit, for each increase of 1 chirp per minute
- (C) The number of chirps per minute when the temperature is 0 degrees Fahrenheit
- (D) The temperature, in degrees Fahrenheit, when there are 0 chirps per minute

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- Use stated assumptions, definitions, and previously established results in constructing arguments
- Break situations into cases
- Recognize and use counterexamples
- Justify conclusions and respond to the arguments of others
- Distinguish correct from flawed logic
- Determine the domain of an argument

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Test questions may ask students to:

- Construct chains of reasoning that will justify or refute propositions or conjectures.
- Identify or describe errors in solutions to multi-step problems and present corrected solutions.
- Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any).
- Make inferences and justify conclusions from data.

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

If n is an integer, then 2^n is an integer.

Which of the following values of n can be used to show that the statement above is false?

- (A) -1
- (B) 0
- (C) 0.5
- (D) 1

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Martin makes the following claim.

“In the coordinate plane, if the graph of $y = f(x)$ is increasing and passes through the origin, the function f must represent a direct proportion.”

Is Martin’s claim correct?

- If Martin is correct, explain why.
- If Martin is not correct, provide the equation of a counterexample to show why not.

Martin is not correct, the graph of $y = x^3$ is increasing and passes through the origin, but is not a direct proportion.

Mathematical Practice 4

Model with mathematics.

Mathematically proficient students:

- Apply the mathematics they know to solve real-world problems
- Use geometry to solve a design problem
- Use a function to describe how one quantity of interest depends on another
- Make assumptions and estimations
- Simplify a complicated situation
- Apply their results in the original context

Mathematical Practice 4

Model with mathematics.

Test questions may ask students to:

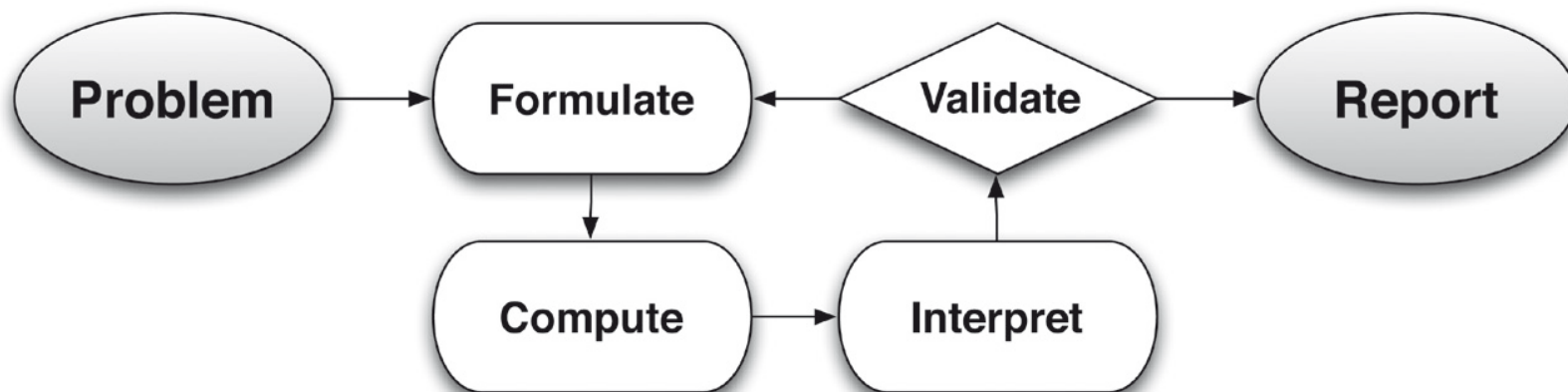
- Solve multi-step contextual word problems
- Apply a technique to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense
- Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity
- Identify relevant data in a data source, analyze it, and draw reasonable conclusions from it

Mathematical Practice 4

Model with mathematics.

Test questions may ask students to:

- Formulate a representation of a situation using variables, use the representation to obtain a result, interpret the result in terms of the original situation, validate the result by comparing it to the situation, and either improve the model or briefly report the conclusions





Mathematical Practice 4

Model with mathematics.

For a concert, a singer earns a \$500 fee plus 15% of the income from ticket sales. Tickets were sold for \$30 each.

Part A

Write an equation to represent the number of dollars the singer earns based on the number of tickets sold. Be sure to define the variables you use.

$y = 4.5x + 500$, where y represents the income, in dollars, and x represents the number of tickets sold.

Part B

The concert space has 500 seats. How much money, in dollars, will the singer earn if tickets are sold for all 500 seats?

\$2,750

Part C

How many tickets need to be sold in order for the singer to earn at least \$1,000?

112 tickets

Mathematical Practice 4

Model with mathematics.

A musician makes a song available to be downloaded from a Web site. The Web site counts the total number of times the song has been downloaded. One week after the song was made available, it had been downloaded a total 120 times. Five weeks after the song was made available, it had been downloaded a total of 760 times.

The musician assumes a linear relationship between the number of weeks since the song was made available and the total number of times the song has been downloaded. Write an equation for the linear relationship. Let x represent the number of weeks since the song was made available and let y represent the total number of times the song has been downloaded.

Does the linear relationship provide a realistic estimate of the number of times the song was downloaded at the time it was first made available? Explain why or why not.

$$y = 160x - 40, \text{ not realistic}$$

Mathematical Practice 5

Use appropriate tools strategically.

Mathematically proficient students:

- Consider the appropriate tools when solving a mathematical problem
- Use pencil and paper, ruler, calculator
- Make sound decisions about when each of these tools might be helpful
- Strategically use estimation and other mathematical knowledge
- Analyze graphs of functions with graphing calculators

Mathematical Practice 5

Use appropriate tools strategically.

- Tools include calculators and geometry software.
- Tools also include previously learned formulas, algorithms, and procedures.
- A coordinate plane or number line can be thought of as a mathematical tool.
- Use of tools may not be assessed directly, but students who are able to use tools effectively may have a advantage.

Mathematical Practice 5

Use appropriate tools strategically.

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t-t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		



Mathematical Practice 5

Use appropriate tools strategically.

If $x > 10^6$, what is the value of $\frac{x+1}{5x}$ to the nearest tenth?

Key = 0.2



Mathematical Practice 5

Use appropriate tools strategically.

A rectangle lies in the xy -plane so that its sides are not parallel to the coordinate axes. What is the product of the slopes of the four sides of the rectangle?

Key = 1

Mathematical Practice 5

Use appropriate tools strategically.

Indicate the location of the number 0.4 by clicking on the number line below.

To divide the number line into more parts, click “More Tick Marks.” To divide the number line into fewer parts, click “Fewer Tick Marks.”



More Tick Marks

Fewer Tick Marks

Mathematical Practice 5

Use appropriate tools strategically.

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More Tick Marks

Fewer Tick Marks

Mathematical Practice 6

Attend to precision.

Mathematically proficient students:

- Communicate precisely to others
- State the meaning of symbols they choose
- Carefully specify units of measure
- Label coordinate axes clearly
- Calculate accurately and efficiently
- Express numerical answers to a degree of precision appropriate to the problem context
- Avoid nonsense expressions such as
 $1 + 2 = 3 \times 3 = 9$



Mathematical Practice 6

Attend to precision.

The function f has the property that $f(3x) = 3f(x)$ for all x .

If $f(12) = 6$, what is the value of $f(4)$?

Key = 2



Mathematical Practice 6

Attend to precision.

The first term of a sequence is x , and each term after the first is 4 greater than $\frac{1}{4}$ of the preceding term. If $x \neq 0$, what is the ratio of the second term to the first term of the sequence, in terms of x ?

$$\frac{x+16}{4x}$$

Mathematical Practice 7

Look for and make use of structure.

Mathematically proficient students:

- Look closely to discern a pattern or structure
- Recognize the significance of an existing line in a geometric figure
- Step back for an overview and shift perspective
- See complicated things as single objects or as being composed of several objects



Mathematical Practice 7

Look for and make use of structure.

If $10x + 20x + 30x = 10 + 20 + 30$, what is the value of x ?

(A) -10

(B) -1

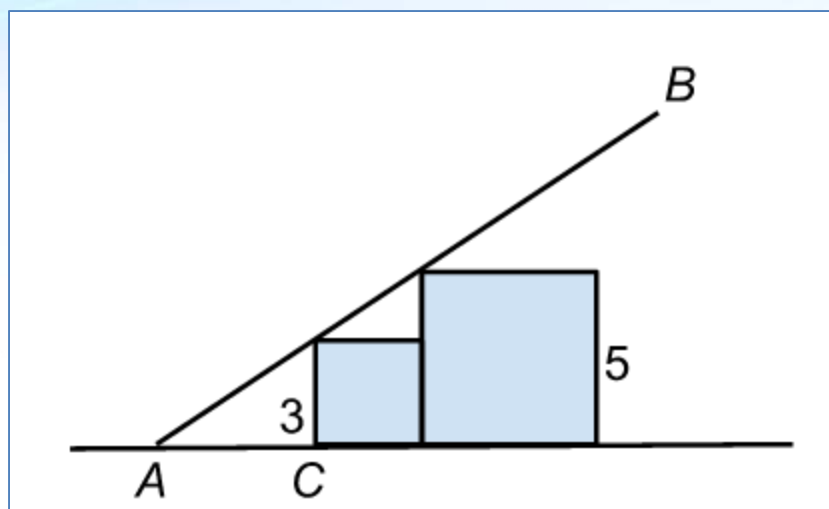
(C) 0

(D) 1

(E) 10

Mathematical Practice 7

Look for and make use of structure.



In the figure above, two squares have sides of length 3 and 5, respectively. Segment AB passes through a vertex of each of the squares. What is the length of segment AC ?

Key = 4.5

Mathematical Practice 8

Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- Notice if calculations are repeated and draw appropriate conclusions from this
- Look for general methods and for shortcuts
- Maintain oversight of the process as they work to solve a problem, while attending to the details
- Continually evaluate the reasonableness of their intermediate results



Mathematical Practice 8

Look for and express regularity in repeated reasoning.

How many positive three-digit integers have the hundreds digit equal to 5 and the tens digit not equal to 3?

Key = 90



Mathematical Practice 8

Look for and express regularity in repeated reasoning.

In a factory that manufactures electronic toys, Inspector *A* tests every 5th toy that rolls off the assembly line, beginning with the 5th. Inspector *B* tests every 7th toy that rolls off the assembly line, beginning with the 7th. If 200 toys roll off the assembly line, how many of the toys are tested by neither of the inspectors?

- 40 tested by A
- 28 tested by B
- 5 tested by both A and B
- 63 tested in total
- 137 tested by neither



Questions?

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For more information on interactive questions types, attend session #418 on Friday from 11-12.