

# Tasks Supporting English Learners in Mathematical Reasoning and Communication

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# Fundamental Role of Visual Representations

- Fostering Geometric Thinking field-test experience
- Singapore experience

I.

# Introduction: Confronting Myths

# Confronting Myths about Learning Mathematics

## *Myth 1*

*“English learners need to be competent in English before they can engage in learning mathematics, right?”*

# Confronting Myths about Learning Mathematics

## *Myth 1*

*“English learners need to be competent in English before they can engage in learning mathematics, right?”*

**NO**

# Confronting Myths about Learning Mathematics

## Myth 2

*“To solve a mathematics problem, you have to understand the wording in the problem as well as the context it describes. This cognitive load can overwhelm English learners, so it is unfair to subject them to the cognitive demand of problem solving.”*

# Confronting Myths about Learning Mathematics

## Myth 2

*“To solve a mathematics problem, you have to understand the wording in the problem as well as the context it describes. This **cognitive load** can overwhelm English learners, so it is unfair to subject them to the **cognitive demand** of problem solving.”*

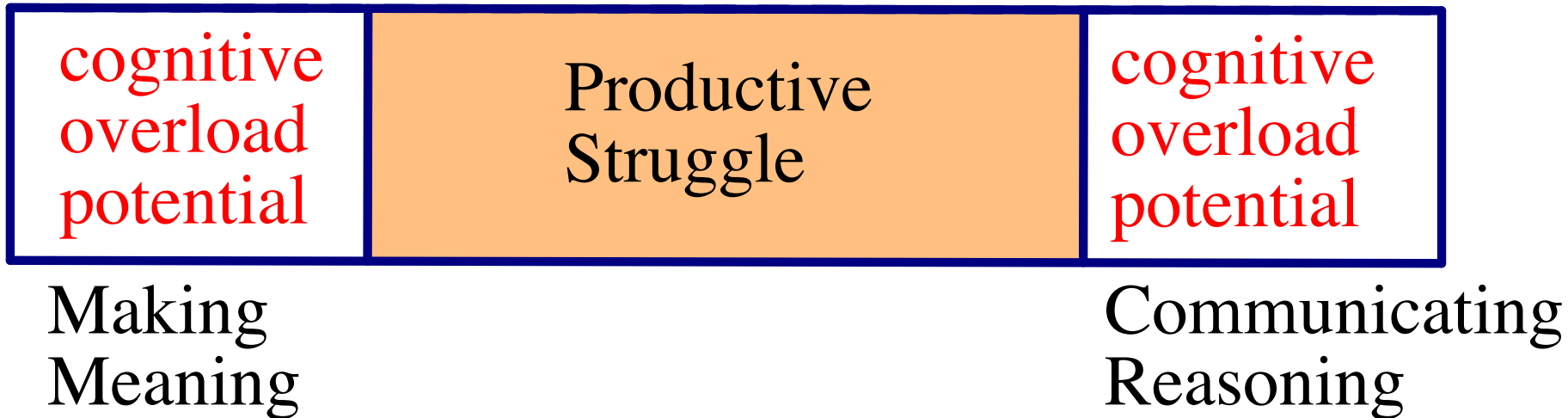
# Making a Distinction

***Cognitive Load:*** Refers to processing information in light of the limited capacity of working memory (e.g., 7 pieces of information at one time). Cognitive *overload* can happen pretty quickly—e.g., too much information to process too quickly.

***Cognitive Demand:*** Refers to how much a task invites or requires productive struggle through reasoning and problem solving---i.e., in our case, how much *mathematical thinking* is demanded.



# Stages of Working Through a Task



# Overarching Task Consideration

Increasing English learners' access to tasks and to the Mathematical Practices---- by helping students manage **cognitive load** without lowering the **cognitive demand**

# Another Consideration: Beware of Good Intentions Gone Awry

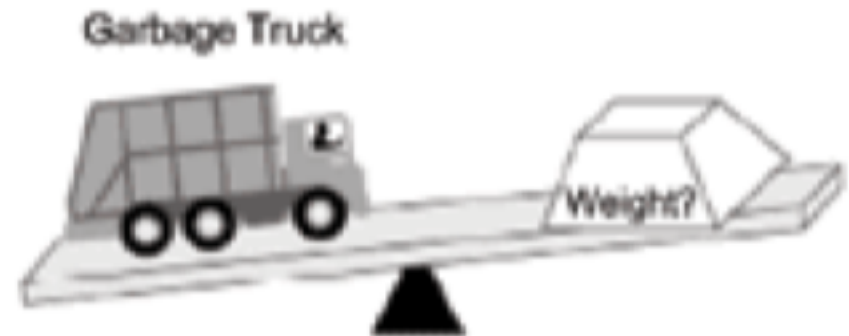
## Regular test item

Which is the best estimate for the weight of a garbage truck?

- 5 pounds
- 8 tons
- 14 ounces
- 1,400 ounces

## Modified for English-language learners

Which is the best estimate for the weight of a garbage truck?



- 5 pounds
- 8 tons
- 14 ounces
- 1,400 ounces

II.

Type 1 Tasks: Using  
Diagrams to Think

# Searching for Task Candidates, with ELL Students in Mind

- “Mathematical concepts, objects, and relationships arise through language, and within particular socio-cultural environments, in response to human thinking about quantity, relationships, and space” (Barton, *The Language of Mathematics*, 2008)
- Learning mathematics involves participation in the mathematical discourse practices in a classroom or other mathematical learning community (e.g., Moschkovich, 2002).

## Once You Have a Task Candidate:

- Is there potential in this task for eliciting student reasoning about quantities, relationships, or space?
- How might diagrams be tools for propelling student reasoning about the task?
- What supports can be provided for language access and production?
- What opportunities are there for student-student discussion?

# Candidate Task: Sharing Candies



Sara had a bag of candies.

She gave  $\frac{1}{3}$  of the candies to Raul.

Then Sara gave  $\frac{1}{4}$  of the candies she had left to Jasmine.

After giving candies to Raul and Jasmine, Sara had 24 candies left in her bag.

How many candies did Sara have at the beginning?



## Once You Have a Task Candidate:

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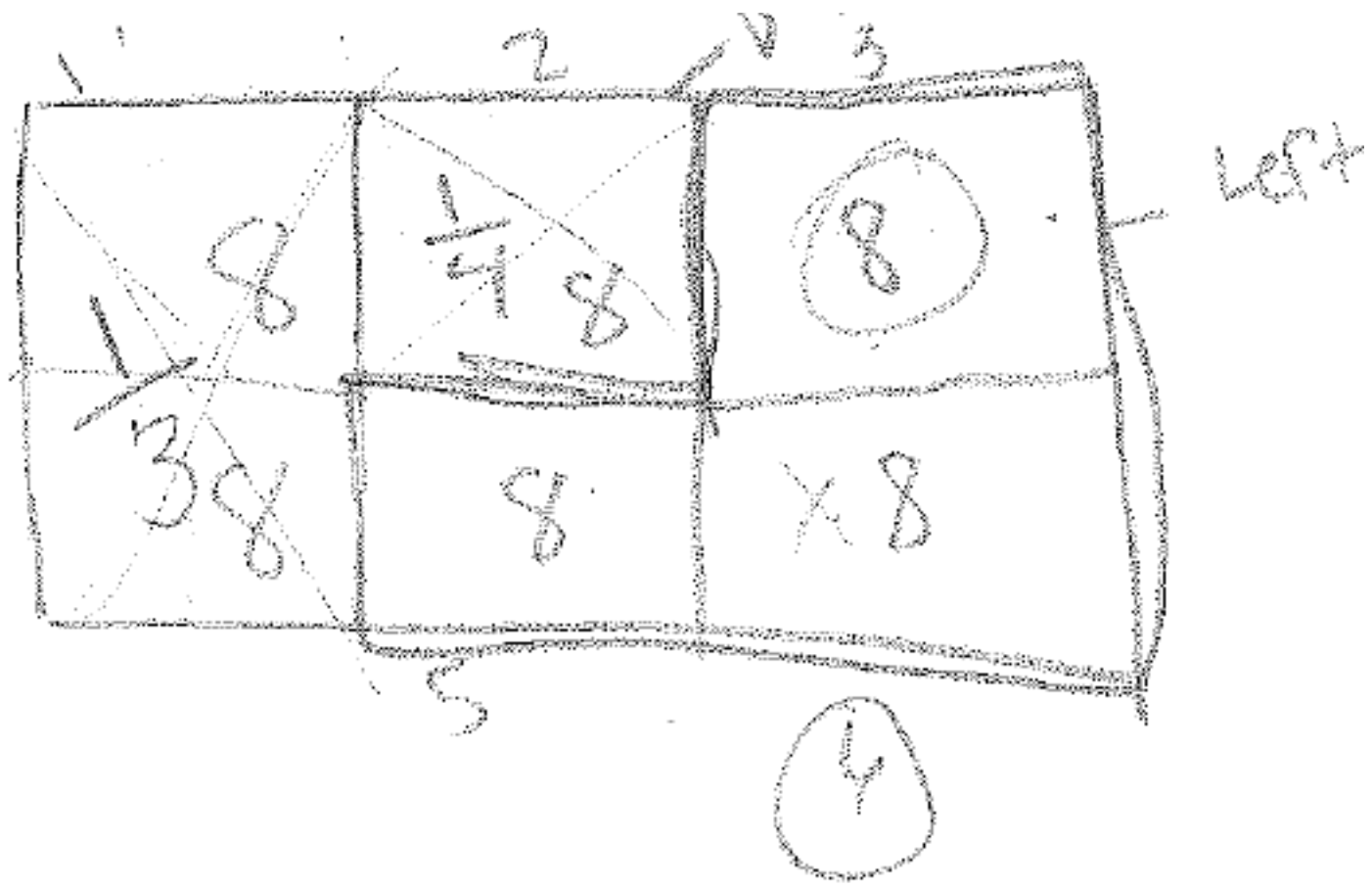
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# The Standards of Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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# 3-Reads Strategy

## Sharing Candies

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**Create a diagram that helps you to solve the problem. Show your work.**

<p><b>1<sup>st</sup> Read</b></p> 	<p><b>CONTEXT</b></p> <p>The problem is about _____.</p>
<p><b>2<sup>nd</sup> Read</b></p> 	<p><b>PURPOSE</b></p> <p>I need to _____.</p>
<p><b>3<sup>rd</sup> Read</b></p> 	<p><b>INFORMATION</b></p> <ul style="list-style-type: none"><li>•</li><li>•</li></ul>


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— = whole bag

 =  $\frac{1}{3}$  of candy  
gave away

 =  $\frac{1}{4}$  of candy  
gave away

The  
white  
space = leftover = 24  
candy

48 candy to begin with

Student A: “We noticed she kept the same amount of candy that she gave away.”

Student B (from across the room): “I just SAW that!”

# Recent Project: Mathematics Coaching Supporting English Learners (MCSEL)

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# Mathematics Coaching Supporting English Learners (MCSEL)

- Develop and study professional development (PD) materials to support mathematics teachers of English learners, grades 6-8:
  - materials for leading PD seminars
  - materials for Classroom Inquiry Cycles to follow each seminar

# Publication

Heinemann, Inc. will publish our MCSEL **professional development materials**, along with a **book** aimed primarily at a mathematics teacher audience, in late Summer/early Fall 2016.

A new website, born of our projects, which we hope will be helpful to you, and hope it will grow and grow in its helpfulness:

[mathandlanguage.edc.org](http://mathandlanguage.edc.org)

# Your Turn

- Form pairs
- Individually, take 3-4 minutes to think about how you might approach the presented problem with a visual representation
- Ideas for embellishing the task to support ELL language access and language production
- Discuss your ideas with your partner

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# Your Turn

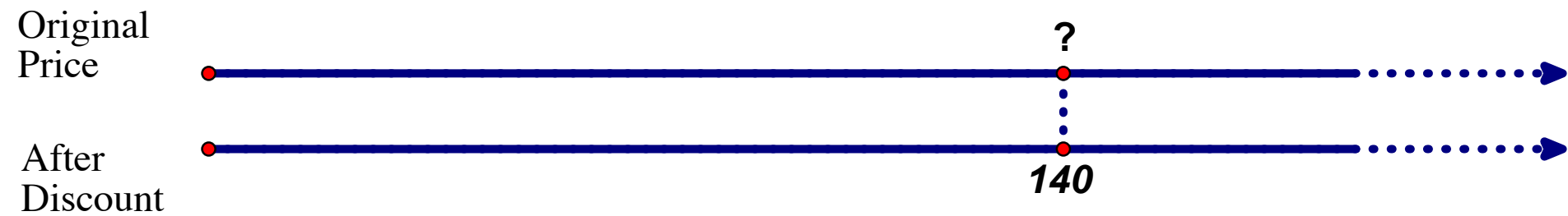
## Lee's New Jacket

*Lee purchased a new jacket and paid only 70% of its original price. He paid \$140 for the jacket. How much was the original price?*

# Your Turn

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- Individually, take 3-4 minutes to think about how you might approach the presented problem with a visual representation
- Ideas for embellishing the task to support ELL language access and language production
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# Two Possible Ways to Diagram the Task



Original  
Price

10

100

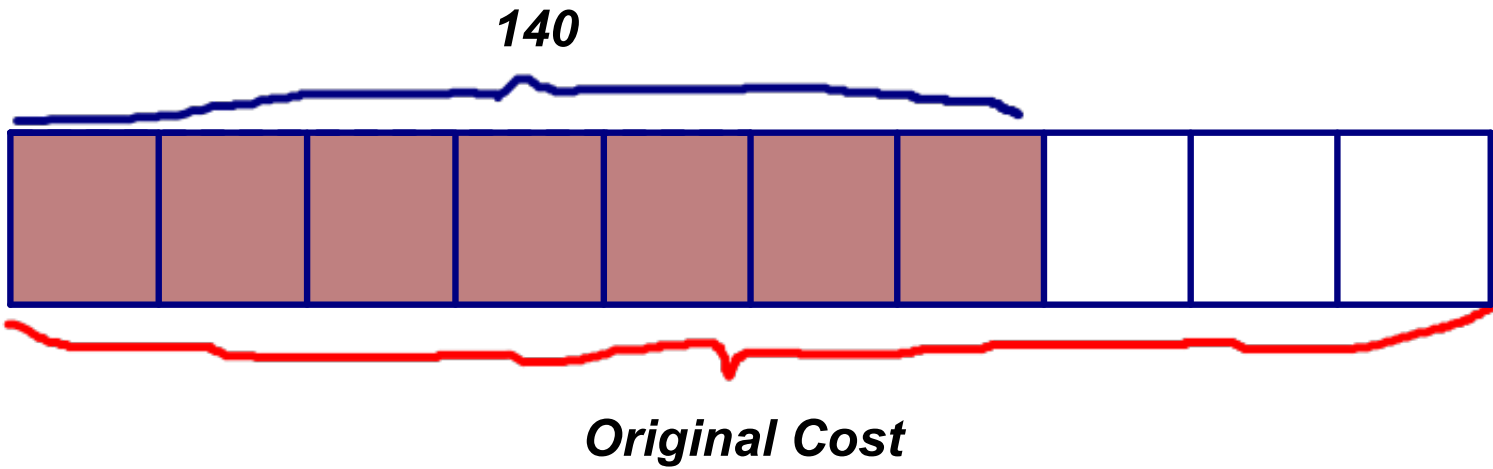
?

After  
Discount

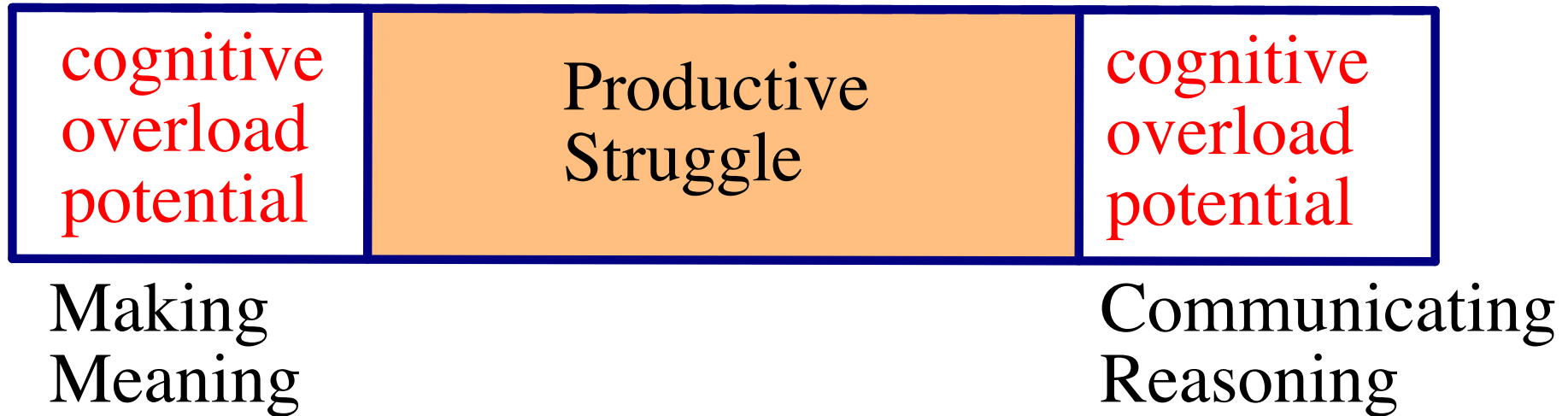
7

70

140



# Stages of Working Through a Task



III.

# Type 2 Tasks: Analyzing to Understand Diagrams



# “Draw a Diagram”

- Does ‘diagram’ = ‘picture’?

# “Draw a Diagram”

- Does ‘diagram’ = ‘picture’?
- Why not just use numbers and operations?

“Miss, I didn’t use a diagram. I did it with Math.”

# “Draw a Diagram”

- Does ‘diagram’ = ‘picture’?
- Why not just use numbers and operations?
- How neat do diagrams need to be?
- What kinds of mathematical diagrams are there?
- What diagram fits *this* problem?
- Where do I start? What do I draw first?

# Using Diagrams as Thinking Tools Needs To Be Learned

- So, a second kind of task can benefit English learners: one that has them *analyzing* rather than *creating* diagrams
- Two primary reasons: to **shed light on the meaning of ‘draw a diagram’**, and to allow **freer practice of the academic language** associated with diagrams.

# Worked Examples: An Instructional Device toward Multiple Strategies

“*Worked examples* typically include a problem statement and a procedure for solving the problem; together, these are meant to show how other similar problems might be solved. In a sense, they provide an expert's problem-solving model for the learner to study and emulate. Examples typically present solutions in a step-by-step fashion.”

Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K.R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide* (NCEE 2012-4055). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

## In our case...

- We wanted to help students to understand how diagramming can be a useful *thinking and problem solving tool*
- We decided to have them *analyze* Mario's use of diagrams to think about the task (other tasks have Estella and Janet), and
- We integrated *language tools* to increase access for ELL students
- During our pilot, we noted that the ELL students were *using the academic language* more---leading to conjecture that switching from problem solving to analyzing another's problem solving lowered cognitive load.

# Triangle Task: Mario's Thinking

An example of Analyzing Visual Representations (AVR) routine. (We also use Creating Visual Representations (CVR) routine.)

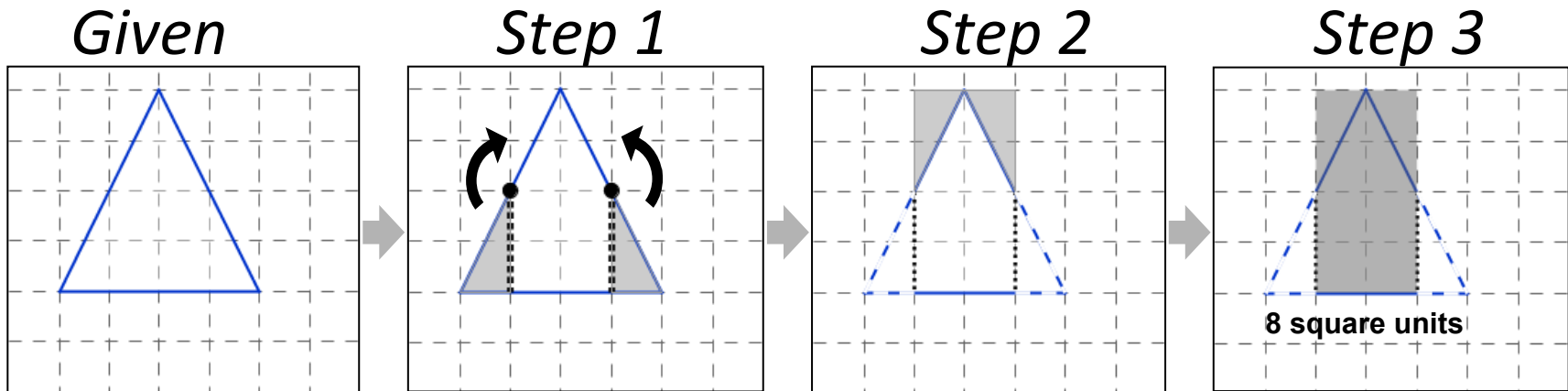




## GOALS

- ✓ Get better at making sense of someone else's math thinking.
- ✓ Learn how geometric drawings can be changed to reveal properties and spatial relationships (between those properties) that can help you solve a problem

# Transforming a Triangle: Mario

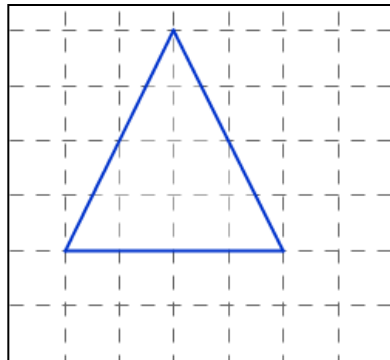


What do you notice?

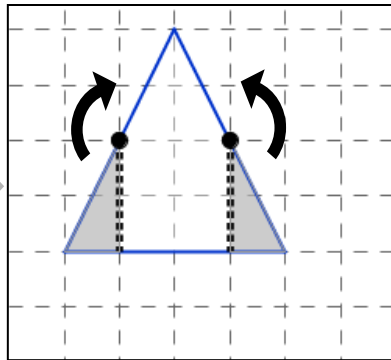
# Transforming a Triangle: Mario



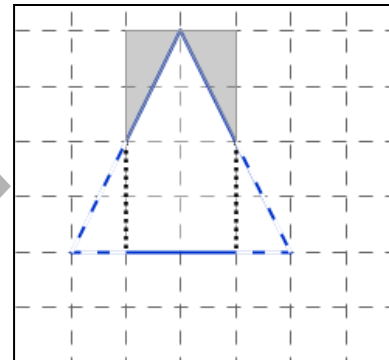
*Given*



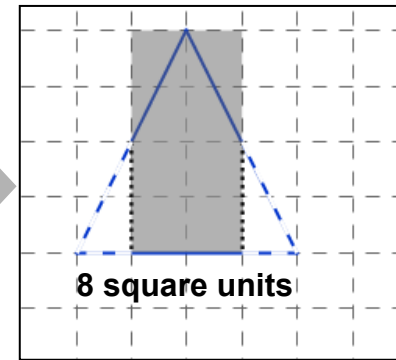
*Step 1*



*Step 2*



*Step 3*



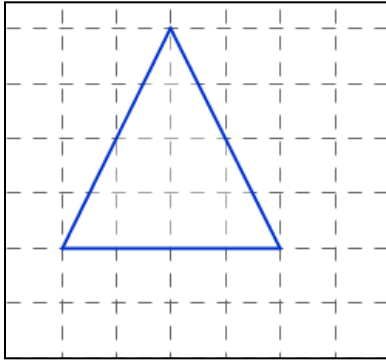
What words will help you describe what Mario did?

# What Changed?

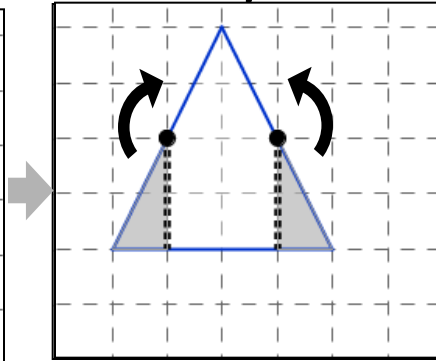
Individual writing and discussion



*Given*



*Step 1*



Questions:

A. What changed:

- From the Given to Step 1?

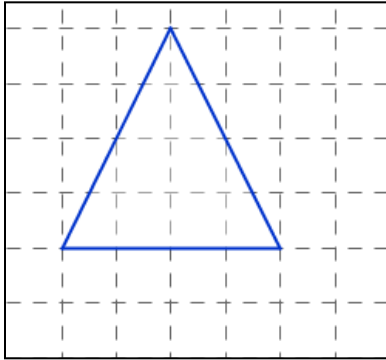


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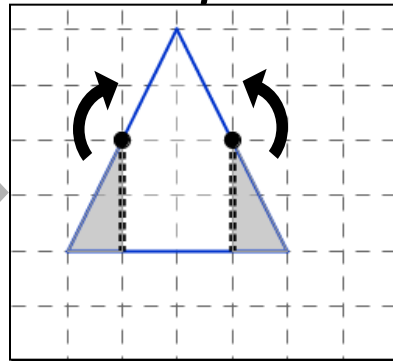
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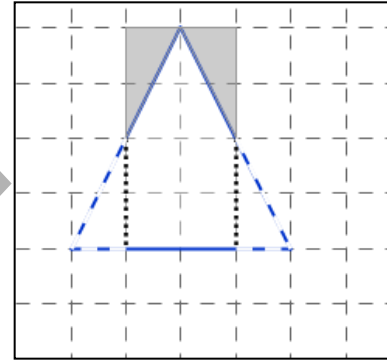
*Given*



*Step 1*



*Step 2*



Questions:

A. What changed:

- From the Given to Step 1?
- From Step 1 to Step 2?

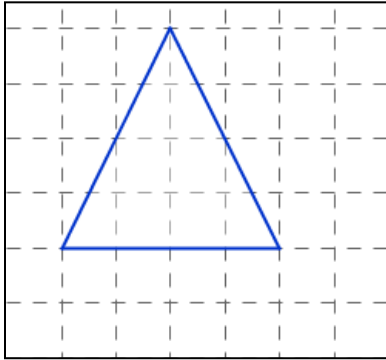


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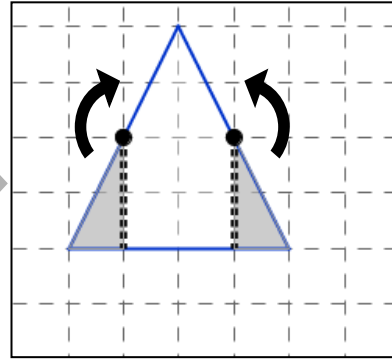
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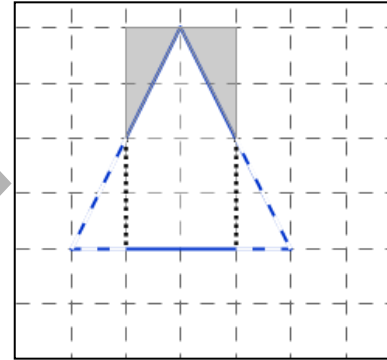
*Given*



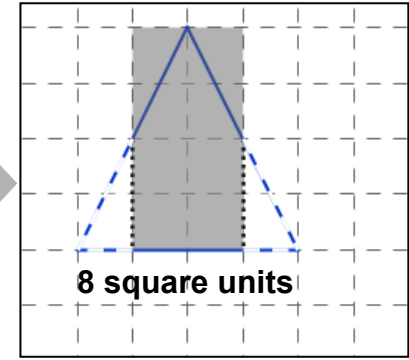
*Step 1*



*Step 2*



*Step 3*



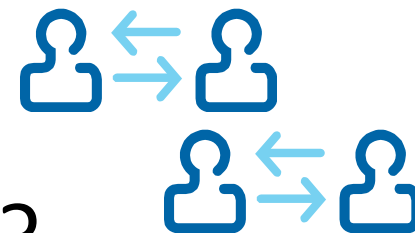
## Questions:

A. What changed:

- From the Given to Step 1?
- From Step 1 to Step 2?
- From Step 2 to Step 3?



# Full Group Share: What was Mario Thinking?



First Mario...

Next he...

Finally he...

Mario discovered...

# Standards for Mathematical Practice

- Mathematical Practice #1: *Make sense of problems and persevere in solving them*
- Mathematical Practice #7: *Look for and make use of structure*



A	<p>What changed from the Given to Step 1?</p> <p>They shaded in the bottom side to make to little triangles</p> <hr/> <p>They cut the edges on the bottom.</p>	<p>What changed from Step 1 to Step 2?</p> <p>What changed was that the rotated the shaded part to the top of the triangle.</p> <hr/> <p>They rotated the two bottom parts to the top</p>	<p>What changed from Step 2 to Step 3?</p> <p>The left everything the same but they shade in <del>the</del> the entire rectangle</p> <hr/> <p>It went to a triangle to a rectangle.</p>
B	<p>I wonder... Why did he cut different shapes</p>		

C Word Bank

Base triangle dotted line rotate rectangle

D	<p>What is given?</p> <p>First mario had a <u>triangle</u></p>	<p>First (step 1), Mario...</p> <p>Mario did to <u>dotted lines</u> to make two small <u>triangles</u>.</p>	<p>Next (step 2), Mario...</p> <p>Mario took the two little triangles and <u>rotated</u> them.</p>	<p>Finally (step 3), Mario...</p> <p>Mario went from a <u>triangle</u> to a rectangle with equal the same area.</p>
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E What did Mario discover?

mario discovers that rectangle, or triangle it has the same area.

# Takeaways

## Tasks that help ELs' reasoning and problem solving:

- Keep cognitive load low without lowering the cognitive demand
- Encourage student use of mathematical reasoning & problem solving
- Emphasize diagramming as a mathematical *thinking* strategy
- Integrate support for language access and production
- Encourage students to leave records of their mathematical thinking and communication
- Incorporate *analysis* of diagrams:
  - Sample diagrams could come from fictional students like Estella, other students in the class, or the teacher.
  - Diagrams for analysis could be incomplete or incorrect.

Thank You