

Mathematical Habits of Mind— *Practices, Processes, and More*

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DISCUSSION

- Write 3-5 words or phrases that describe the kind of mathematical thinking you want students to be able to do.

In this session...

- Thinking like a mathematician
- Nurturing talent
- Mathematical Habits of Instruction . . .

Being a 'Doer' of Mathematics

“The only way to know mathematics
is to *do* mathematics.”

Paul Halmos, mathematician

“It is pretty hard to *understand* mathematics
without *doing* some mathematics.”

Jordan Ellenberg, mathematician and writer

Thinking Like a Mathematician

- Doing real mathematics--figuring out hard problems
- Exploration that's sometimes messy;
trying things that sometimes work, sometimes don't
- Reasoning, explaining and arguing
- Zooming in and zooming out
- Reflecting, considering, analyzing

What math do all students need?

- **The Big Three:**
 - **Understanding mathematics** (making sense of it)
 - **Doing mathematics** (skills, facts, procedures)
 - **Using mathematics** (applying math, modeling with mathematics, solving a range of problems)
- **Mathematical Thinking: Habits of Mind**
 - Thinking, reasoning, expecting math to make sense
- **The New Basics:** deep transferable skills for *versatilizing*:
 - Problem solving, reasoning, research, communication, creativity

Premise:

What all students need for their future is
as much about how they *think* as
about what they *know* . . .
and helping every student succeed is
as much about *how* we teach as
about *what* we teach.



Math Reasoning Inventory™

mathreasoninginventory.com

Marilyn Burns, PI

Funded by Gates Foundation

DISCUSSION

- How did the teacher find out what Marisa was thinking?
- Had Marisa likely had experience developing mathematical habits of mind?

The difference between...

- Learning clues, keys, and tricks vs. constructively struggling with good problems
- Learning how to do mathematical procedures vs. learning mathematical habits of mind

Mathematical Habits of Mind

- **Performing thought experiments**
- **Finding, articulating, and explaining patterns**
- Generalizing from examples;
articulating generality in precise language
- **Creating and using representations**
- **Expecting mathematics to make sense**

Al Cuoco, E. Paul Goldenberg, June Mark.
“Organizing a Curriculum around Mathematical Habits of Mind.”
Mathematics Teacher May 2010

From a presentation by Cathy Seeley, 2015

NCTM Process Standards

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connections
5. Representations

Principles and Standards for School Mathematics, NCTM 2000
(expanded from *Curriculum and Evaluation Standards for School Mathematics, NCTM, 1989*)

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 3. Construct viable arguments; critique others' reasoning.**
- 4. Model with mathematics.**
- 5. Use appropriate tools strategically.**
- 6. Attend to precision.**
- 7. Look for and make use of structure.**
- 8. Look for and make use of regularity in reasoning.**

Texas Process Standards (TEKS)

- (A) **apply mathematics** to problems arising in everyday life, society, and the workplace
- (B) use a **problem-solving model** that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution
- (C) select **tools**, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems
- (D) **communicate** mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate
- (E) create and use **representations** to organize, record, and communicate mathematical ideas
- (F) analyze mathematical relationships to **connect and communicate mathematical ideas**
- (G) **display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication**

Problem Solving, Applications, Mathematical Modeling

- **Persevere in solving problems**
- Explain to selves; analyze givens, constraints; consider analogous problems; represent the situation in different ways; search for regularity / trends; plan solution path
- **Solve problems that apply the mathematics being learned**
- **Solve problems that may involve mathematics not yet learned**
- Use mathematical modeling to solve problems that may not be well defined; Interpret, reflect, fine-tune the model
- When solving problems, keep an eye on the big picture while also attending to the details (zooming in and out); shift perspective
- **Evaluate the reasonableness of results.**

From a presentation by Cathy Seeley, 2015

Reasoning, Explaining, Justifying

- **Includes quantitative reasoning;** make sense of quantities and relationships in problems
- Make conjectures and explore the truth of conjectures.
- Analyze situations; recognize and use counterexamples.
- **Justify conclusions and communicate/respond to arguments.**
- **Decide whether arguments of others make sense; ask useful questions to clarify or improve the arguments.**

Representing, Connecting, Communicating Precisely

- Look for commonalities / relationships in similar problems or in mathematical ideas
- Coherently represent a problem; decontextualize and contextualize
- **Communicate precisely; make clear, effective arguments; use clear definitions**
- **Put precision in perspective (finding balance . . .)**
- **Does this make sense?**

Tools

- Pencil / paper, manipulatives / concrete models, compass, protractor, calculator, spreadsheet, CAS, software, . . .
- Find and use external information
(Internet, research, etc.)
- **Mental math**
- **Make decisions and understand limitations**

Mathematical Habits of Mind-- Common Themes

- Solving problems
- Thinking, Reasoning, and Reflecting
- Discussing and Communicating
- Justifying and Explaining
- Generalizing from Patterns
- Connecting
- Making sense
- Patience, persistence

DISCUSSION:

Potential

- How smart do you think Marisa is?
- Why don't some students reach their mathematical potential?

Factors to consider

- Student factors: Motivation, intelligence, beliefs
- Teacher factors: Knowledge, expectations, beliefs
- Instructional factors
 - Nature of the task
 - Opportunities to struggle, think, figure things out, discuss
 - A classroom environment of trust, collaboration, respect, and (eventual) success, where perseverance and constructive feedback of each other are valued

Intelligence

- Fixed vs. growth mindset . . .
(Carol Dweck, *Mindset*, 1999)
- Your mindset influences confidence, perseverance, and your willingness to take risks
- From brain research:
The activities a person engages in can change their intelligence.
- Who determines the activities a student engages in?

Targeting beliefs with action

- Students' beliefs matter.
- Teachers' beliefs and actions matter.
- Modest interventions make a difference.

High Expectations means...

- Challenging our habits and beliefs
- Setting challenging standards for all students
- Doing whatever it takes for students to achieve the standards
- Never thinking in advance that you know where they're headed or what they need
- Making sure they all get to struggle and succeed

Answer-getting vs. learning mathematics

- USA:

How can I teach my kids to get the answer to this problem?

- Japanese:

How can I use this problem to teach the mathematics of this unit?

– Devised methods for slowing down,
postponing answer-getting

Phil Daro, 2012

The difference between Japan and the US

- “You quit teaching too soon and go on to the next thing.”
- “We finish.”
- Finishing happens when students have learned.
- And learning is incomplete if students aren't developing mathematical thinking.

Marisa didn't get to finish...

Upside-down teaching

- From: *“I - We - You”*
- To: *“You - We - I”*

Borreo Ferri & Blum on teaching

- Group work / Partner work / Individual work
- Ideal-typical phases of lessons:
 - Task presentation in plenum (whole class)
 - Co-constructive work: single-group-single
 - Presentation of solutions in plenum
 - Comparison of solutions and reflection

Upside-down teaching

- Start with a rich problem
- Engage students in dealing with the problem, constructively struggling with the problem and the mathematics
- Students discuss, compare, interact, question
- Teacher helps students connect and notice what they've learned

Mathematical Habits of Instruction

- Use a problem-centered, upside-down teaching model
- Use appropriate technology appropriately
- Learn (and help students learn) to zoom out, zoom in, and go back and forth
- Help students learn to notice and use patterns, connections, and properties within and across mathematical topics and problems (mathematical structure)
- Use formative assessment to pay attention to learning

Achievement Gap

From a presentation by Cathy Seeley, 2014

Instructional Gap

From a presentation by Cathy Seeley, 2014

Overlooked Talent

From a presentation by Cathy Seeley, 2014

Untapped Potential

From a presentation by Cathy Seeley, 2014

Two Sides of Untapped Potential

- Give all students the opportunity to struggle, learn challenging mathematics, develop mathematical habits of mind, so they can achieve their highest levels of mathematics
- Identify new stars and let them soar
- Untapped potential within each student, within groups of students, and at the school, district, state and national level--potential we haven't reached . . .
YET.

Untapped Potential

From a presentation by Cathy Seeley, 2014

Unlimited Potential

From a presentation by Cathy Seeley, 2014

Even our *best* students...

...will benefit from a strong,
diverse, engaging, relevant
classroom.

Their future is in our hands



...and ours is in theirs

From a presentation by Cathy Seeley, 2014

A pdf of the slides will be posted: cseeley@utexas.edu

Faster Isn't Smarter--

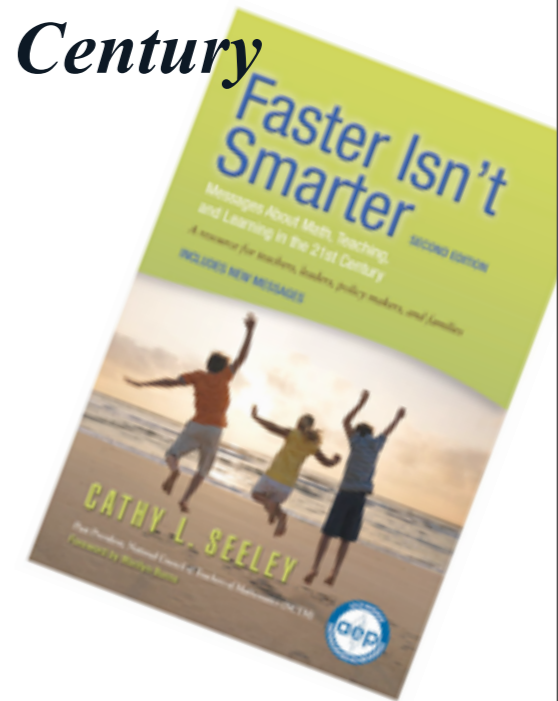
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