

## **Session 504**

# **Developing a Conceptual Understanding of Multiplication of Fractions**

**Presented by**

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## What does multiplication of fractions mean?

- Finding a portion of a part of a whole
- Finding a part of a portion of a whole
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### **Example:**

Problem:

The fourth-grade class conducted a survey to learn how students get to our school every day. The results of the survey showed that two-thirds of the students get to school by riding in a vehicle. The results also showed that three-fourths of the students who ride in a vehicle ride the bus to school. What fraction of all the students at our school ride the bus to school?

Equation:

$$\frac{3}{4} \times \frac{2}{3} = ?$$

Interpretation:

- Find three-fourths of the two thirds of the whole class.
- Find three-fourths of two-thirds of one whole.

Multiplication of Fractions Algebraic Representation	Numerical Examples	Corresponding Contextual Situation Example	Important Ideas Students Need to Comprehend About Units and Partitioning Different Units
$\frac{1}{b} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} \text{ and } \frac{1}{4} \times \frac{1}{2}$	One half of all the students in our class have on red shirts today. One-fourth of the students with red shirts have buttons on their shirts. What fraction of the whole class is wearing red shirts with buttons today?	<ul style="list-style-type: none"> <li>• There can be more than one unit in a problem.</li> <li>• A quantity other than one whole can serve as a unit.</li> <li>• What it means in general to partition a unit into a fractional amount of one-half or one-fourth.</li> </ul>
$\frac{a}{b} \times \frac{b}{d}$	$\frac{3}{5} \times \frac{5}{8} \text{ and } \frac{2}{3} \times \frac{3}{4}$	Three fourths of the students in our class brought a sack lunch today. Two thirds of the people who brought a sack lunch brought a banana in their lunch. What fraction of all	<ul style="list-style-type: none"> <li>• What it means in general to partition a fractional unit other than one-half into any fractional amount, such as halves, thirds, fourths, fifths, and etc.</li> <li>• View <math>\frac{b}{d}</math> as a unit as that is</li> </ul>

		<p>the students in our class brought a banana for lunch today?</p>	<p>already partitioned into the needed fractional amount, or <math>b</math> pieces.</p>
$\frac{a}{nb} \times \frac{b}{d}$	$\frac{3}{4} \times \frac{2}{5} \text{ and } \frac{3}{4} \times \frac{2}{3}$	<p>Our class planted a garden. We planted peppers in two fifths of the whole garden. Three fourths of the peppers we planted were red peppers. How much of the whole garden was planted with red peppers?</p>	<ul style="list-style-type: none"> <li>• View <math>\frac{b}{d}</math> as a unit that can be repartitioned into a desired fractional amount.</li> <li>• Realize <math>\frac{b}{d}</math> can be repartitioned by thinking <math>b</math> times what other number equals <math>nb</math>.</li> </ul>
$\frac{a}{b} \times \frac{nb}{d}$	$\frac{2}{3} \times \frac{9}{11} \text{ and } \frac{2}{3} \times \frac{9}{10}$	<p>Nine elevenths of all the students in our school voted in the election for the school mascot. Two thirds of the students who voted, voted for an animal as the school mascot. What fraction of all the</p>	<ul style="list-style-type: none"> <li>• View <math>nb</math> as a unit whose pieces can first be grouped to form an equivalent unit of <math>\frac{b}{b}</math>.</li> <li>• Realize the unit composed of grouped pieces is now partitioned into the desired</li> </ul>

		students in our school voted for an animal as the school mascot?	fractional amount.
$\frac{a}{b} \times \frac{c}{d}$ where $b$ and $c$ are relatively prime (share no common factors other than 1)	$\frac{3}{4} \times \frac{7}{8}$ and $\frac{3}{4} \times \frac{5}{6}$	<p>Seven eighths of the students in our class said their favorite snack food is cheese. Three fourths of the students who like to eat cheese for a snack like to eat cheddar cheese for their snack. What fraction of our whole class likes to eat cheddar cheese for a snack?</p>	<ul style="list-style-type: none"> <li>• View <math>\frac{c}{d}</math> as a unit that can be decomposed into <math>c</math> units of <math>\frac{1}{d}</math>.</li> <li>• Realize that each unit of <math>\frac{1}{d}</math> can be partitioned into the desired fractional amount.</li> <li>• Realize that the pieces resulting from partitioning each unit of <math>\frac{1}{d}</math> can be grouped to form a unit of the needed fractional amount.</li> </ul>

Students' Views of Special Cases of Multiplication of Fraction Problems

## Sample Multiplication of Fractions Problems

Please note, I write all fractions in words in contextual problems. Doing so helps students think deeply about what is happening to the quantities involved in the problems.

Problems Corresponding to:  $\frac{1}{2} \times \frac{1}{2}$ ,  $\frac{1}{4} \times \frac{1}{2}$ ,  $\frac{1}{a} \times \frac{1}{b}$

1. You walk one-fourth of a mile from home to school every day. Today, you walked one-half of the total distance from home to school then you stopped to eat a snack. How much of one whole mile did you walk before stopping to eat a snack?
2. One-half of our pizza has only veggies on it. One-fourth of the portion with veggies has only mushrooms on it. How much of our whole pizza has only mushrooms on it?
3. Today, one-half of the students in the class are wearing shirts with buttons. One-third of the students who are wearing a shirt with buttons on it are wearing a blue shirt. What fraction of the whole class is wearing a blue shirt with buttons on it today?
4. The zoo has many different animals. One-third of the animals at the zoo are bears. One-fourth of the bears are polar bears. What fraction of all the animals at the zoo are polar bears?
5. Three-fourths of the earth is water and one-fourth is land. One-half of the land is not habitable. Food can be grown on only one-fourth of the land where people can live. What fraction of the earth is land where food can be grown?

Problems Corresponding to:  $\frac{a}{b} \times \frac{b}{d}$

6. Four-fifths of the books in the library are chapter books. One-fourth of the chapter books are non-fiction. What fraction of all the books in the library are non-fiction chapter books?

7. Three-fourths of the students in the class ate a piece of fruit for lunch today. One-third of the students who ate a piece of fruit ate an apple. What fraction of the whole class ate an apple for lunch today?
8. Three-fourths of the students in the class own a pet. Two thirds of the students who own a pet own a cat. What fraction of all the students in the class own a cat?
9. Five-eighths of the plants in the garden sprouted two days ago. Last night, a rabbit ate four-fifths of the plants that had sprouted. What fraction of all the plants in the garden did the rabbit eat last night?

Problems Corresponding to:  $\frac{a}{nb} \times \frac{b}{d}$

10. Two-thirds of the vehicles in the parking lot were cars. Five-sixths of the cars in the parking lot were red. What fraction of all the vehicles in the parking lot were red cars?
11. Four-tenths of the students in the class play an instrument in the school band. Three-eighths of these students play a brass instrument in the school band. What fraction of the whole class plays a brass instrument in the school band?
12. Five-twelfths of the students in the class have a sibling who is a boy. Seven-tenths of the students who have a brother have an older brother. What fraction of all the students in the class have an older brother?
13. You use a budget to manage your money. Last month, you designated three-eighths of your monthly income for school-related purposes. During the month, you spent five-sixths of the money you designated for school-related purposes on tuition. How much of your monthly income did you spend on tuition last month?

Problems Corresponding to:  $\frac{a}{b} \times \frac{nb}{d}$

14. You had four-fifths of one pound of bird seed. You used one-half of the bird seed that you had to fill the bird feeder. How much of one pound of bird seed did you use to fill the bird feeder?
15. Nine-tenths of the students in the class were born in Michigan. Two-thirds of the students who were born in Michigan were born in Grand Rapids. What fraction of the whole class was born in Grand Rapids, Michigan?
16. You had eight-tenths of a pound of cheese. You used three-fourths of the cheese that you had to make a submarine sandwich. How much of one whole pound of cheese did you use to make the submarine sandwich?
17. Ten-twelfths of the students in the class entered a project in the school science fair. Three-fifths of these students won an award for their science project. What fraction of the whole class won an award for their science project in the school science fair?

Problems Corresponding to:  $\frac{a}{b} \times \frac{c}{d}$

18. You had three-fourths of one yard of polka dot fabric. You used one-half of the polka dot fabric to make a quilt. How much of one yard of polka dot fabric did you use when making the quilt?
19. Five-sixths of the items students in our class collected for the community food drive were canned goods. One-fourth of the canned goods were cans of soup. What fraction of all the items our class collected for the community food drive were cans of soup?
20. You teach at Learning Lane Elementary School. Seven-eighths of the students in your class are bi-lingual. Three-fourths of the bi-lingual students in your class speak Spanish

as their first language. What fraction of all the students in your class speak Spanish as their first language?

21. There are lots of animals available for adoption at the Humane Society. Five-eighths of the animals available for adoption are dogs. Two-thirds of the available adoptable dogs have spots. What fraction of all the animals available for adoption at the Humane Society are dogs with spots?
22. You collected one and one-half bags of cans for recycling. Each full bag weighed three and three-fourths pounds. What is the total weight of the cans you collected for recycling?

### References

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