Helping Children Master Multiplication Facts in a Meaningful Way

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Session Overview

- What is fluency?
- Developing multiplication fact fluency
- Moving to Phase 2: Strategies for building fact fluency
- Moving to Phase 3: Meaningful practice
- Assessing multiplication fact fluency

How is *fluency* defined in mathematics?

Read the Fluency article by former NCTM President Linda Gojak

As you read, highlight ideas that:

- are particularly significant to understanding fluency
- provide you with a new way of thinking about fluency

CCSS-M Descriptions

From the Common Core State Standards, Grade 3 (3.OA.7):

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

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This would suggest that ***fluency is different*** from automatic retrieval. Research heavily supports this…

So, what does fluency *really* mean?

The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as "skill in carrying out procedures **flexibly**, **accurately**, **efficiently** and **appropriately**"

(CCSSO, 2010, p. 6).

Procedural Fluency

Knowing from Memory

$\neq$

Memorization

Developing Basic Fact Fluency

Mastering Basic Facts

Phase 1: Counting

(counts with objects or mentally)

Phase 2: Deriving

(uses reasoning strategies based on known facts)

Phase 3: Mastery

(efficient production of answers)

Adapted from Baroody, 2006

Examples

Phase 1: Counting

Solving 6 x 4 by drawing 6 groups of 4 dots and skip counting the dots

Phase 2: Deriving

Solving 6 x 4 by thinking 5 x 4 = 20 and adding one more group of 4

Phase 3: Mastery

Answering that 6 x 4 = 24 within seconds

Mastering Basic Facts

Some programs attempt to push children from Phase 1 directly to Phase 3 through drill and rote memorization.

- What aspects of fluency are/are not developed when this happens?
- What are we communicating to children about what it *means to do mathematics* when we do this?

Mastering Basic Facts

In contrast, to develop true *fluency*, children need adequate time to make sense of multiplication and division and develop strategies in Phases 1 and 2. Through repeated, meaningful practice, children then naturally progress to Phase 3. This transition occurs with different groups of facts at different times throughout the year.

Developing Basic Fact Fluency: Moving to Phase 2

Moving to Phase 2

Children progress to Phases 2 and 3 with different facts at different times.

- Which multiplication facts do children have the most exposure to in earlier grades?
- How do we encourage meaningful memorization of these facts in ways that develop all four aspects of fluency?

Moving to Phase 2

- 2s: Addition doubles or skip counting by 2s
- 10s: Skip counting by 10s or place value experiences
- 5s: Skip counting by 5s or half of the related 10s fact
- 0s and 1s: Equal groups meaning

Foundational Multiplication Facts

- 2s, 10s, 5s (Begin in 2nd grade)
- 1s, 0s
- Squares (3 x 3, 6 x 6)

Once children develop fluency with the above facts, they can then begin to use them as **helper facts** to derive unknown facts. This is an important step for helping children operate within Phase 2.

Games:
Foundational Multiplication Facts

Consider why practice with games such as ***Multiplication Draw*** (*Everyday Mathematics*, 2016) is important before children move onto developing other fact strategies.

Foundational Multiplication Facts

Multiplication Draw

Materials: die labeled with 2, 2, 5, 5, 10, 10; number cards, 1–10

Multiplication Draw Record Sheet

Object of the Game To have the largest sum.

Directions

1. Shuffle the cards and place the deck number-side down.

2. Players take turns. When it is your turn, roll the die and draw 1 card from the deck to get 2 multiplication factors. Record both factors and their product on your Record Sheet.

3. After 5 turns, each player finds the sum of their 5 products.

4. The player with the largest sum wins the round.

Foundational Multiplication Facts

Multiplication Draw Record Sheet

NAME DATE TIME

Partner 1	Round 1	Round 2	Round 3
1st draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
2nd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
3rd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
4th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
5th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
Sum of products:	_____	_____	_____

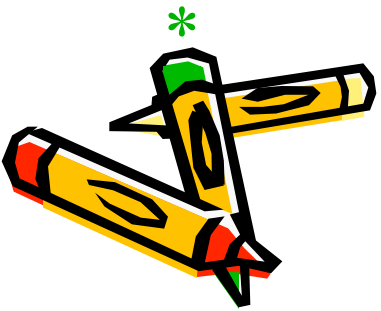
Education. Permission is granted to reproduce for classroom use.

Moving Children to Phase 2: Encouraging Strategy Development

Solve this sequenced number story.

A classroom example:

- What are the instructional goals?
- What prior knowledge do children need to be successful?



Phase 2: Multiplication Fact Strategies

In the video, children used their helper fact of 2 x 8 = 16 and **added a group** to solve the unknown fact 3 x 8 = ?.

What other strategies might children develop?

Phase 2: Multiplication Fact Strategies

- **Adding a Group** (example: 3s from 2s)
- **Subtracting a Group** (example: 9s from 10s)
- **Doubling** (example: double 2 x 7 to solve 4 x 7)
- **Near Squares** (example: solve 8 x 7 from 7 x 7)
- **Break Apart** (example: 7 x 6 = 5 x 6 + 2 x 6)

Properties of Multiplication

Properties of multiplication underlie the facts strategies children develop.

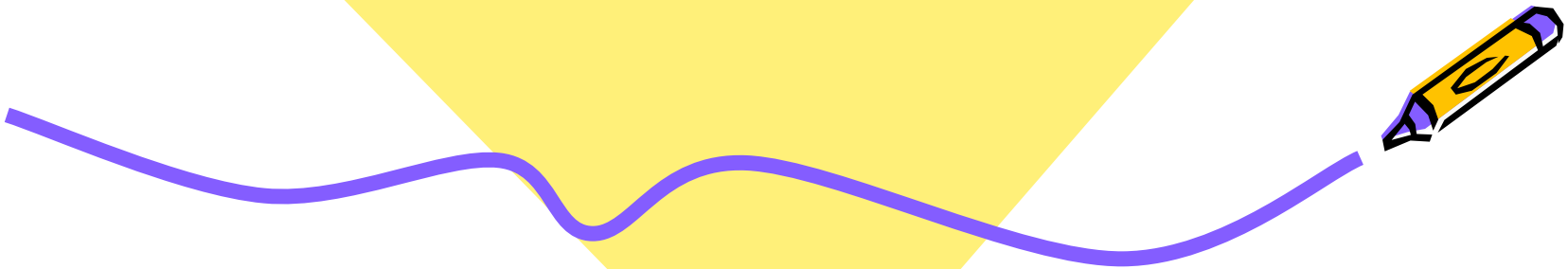
3.OA.5	Apply properties of operations as strategies to multiply and divide. *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)*

Array and area models can help children make sense of and apply these properties through their strategies, as seen in the sample student work for solving 6 x 8.

Phase 2: Multiplication Fact Strategies

- What challenges to you anticipate children having with strategies such as adding and subtracting a group?
- What can teachers do to help children overcome those challenges and make sense of these strategies?

Mastering Basic Facts: Phase 3



Phase 2 ➡ Phase 3

Meaningful Practice

"Practice that follows substantial initial experiences that support understanding and emphasize 'thinking strategies' has been shown to improve student achievement with single-digit calculations." (NRC, 2001).

Meaningful Practice

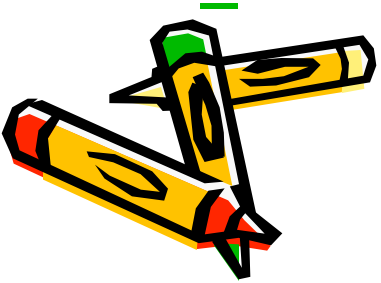
Games:

- Are engaging.
- Provide opportunities for strategy discussion and assessment.
- Should be sequenced developmentally (for example, playing games with 2s, 5s, and 10s and then playing games with all facts after strategy development).
- Can be targeted practice or general practice.
- Lend to differentiation.

Using Games as Meaningful Practice

A classroom example: *Multiplication Top-It*

- What strategies did you observe children using?
- At which phase(s) would you place each child? (children might be at different phases with different facts at any given time).



Assessing Multiplication Fact Fluency

The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as "skill in carrying out procedures **flexibly**, **accurately**, **efficiently** and **appropriately**"

(CCSSO, 2010, p. 6).

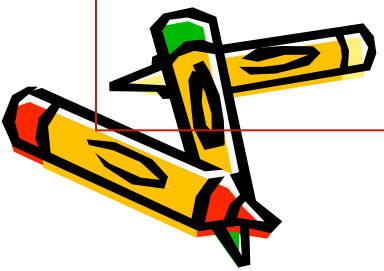
Assessing Basic Fact Fluency

What can we learn from this assessment related to:

- ✓ Flexibility
- ✓ Accuracy
- ✓ Efficiency
- ✓ Appropriate Strategy Use

Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use



Timed Tests

(1) 19 + 1 =	(10) 2 + 4 =	(19) 10 + 2 =	(28) 1 + 3 =
(2) 9 + 4 =	(11) 9 + 1 =	(20) 5 + 3 =	(29) 5 + 4 =
(3) 5 + 4 =	(12) 15 + 1 =	(21) 14 + 3 =	(30) 2 + 2 =
(4) 8 + 4 =	(13) 16 + 1 =	(22) 5 + 3 =	(31) 11 + 1 =
(5) 14 + 4 =	(14) 16 + 1 =	(23) 7 + 2 =	(32) 14 + 1 =
(6) 15 + 3 =	(15) 8 + 4 =	(24) 5 + 1 =	(33) 7 + 4 =
(7) 14 + 4 =	(16) 15 + 2 =	(25) 9 + 2 =	(34) 6 + 4 =
(8) 15 + 1 =	(17) 9 + 1 =	(26) 2 + 1 =	(35) 4 + 2 =
(9) 14 + 2 =	(18) 15 + 4 =	(27) 12 + 1 =	(36) 10 + 1 =

Timed Testing: Issues

The issues with timed testing include:

1) Limitations as an assessment tool
2) Can impede progress when mastering facts
3) Psychological effects

Timed Testing: Issues

1) Limitations with respect to the four components of fluency.

A child finishes a 20-fact timed test in 60 seconds.

- Did the child spend 3 seconds on each fact?

Or…

- Did the child spend 1 second on 16 facts and 10 seconds each on 4 of the facts?

Timed Testing: Issues

2) Can impede progress in mastering facts

A study of nearly 300 first graders found that children who were *more frequently* exposed to timed testing demonstrated *lower* progress towards knowing facts from memory than their counterparts.

Henry & Brown, 2008

Timed Testing: Issues

2) Can impede progress in mastering facts

A study of 2nd and 4th graders showed that children in experimental classrooms with a focus on strategy development vastly outperformed those in the control classrooms, even on traditional timed assessments.

Thornton, 1978

Timed Testing: Issues

3) Can have negative psychological effects

- The stress that children experience with timed testing is not experienced when they complete the same tasks in untimed conditions.
- "Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range."

Boaler, 2014

Timed Testing: Issues

3) Can have negative psychological effects

Anxiety over timed testing is often not related to achievement. Even high-achieving children share concerns such as “I feel nervous. I know my facts, but this just scares me.”

Boaler, 2012

Timed Testing: Issues

3) Can have negative psychological effects

Children experience math anxiety as early as first grade and this anxiety is not correlated with reading achievement. This suggests that the children's anxiety is specific to mathematics, not general academic work.

Ramirez et al. 2013

Timed Testing: Issues

3) Can have negative psychological effects

Children who tended to use more sophisticated mathematical strategies experienced the most negative impact on achievement due to math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively impacted by timed testing.

Ramirez et al. 2013

Timed Tests: Alternatives

- ✓ Observation
- ✓ Strategy quizzes
- ✓ Self-assessment
- ✓ Writing prompts
- ✓ Interviews

Observation Checklist

Strategy Tracking Table: Multiplication Facts

Names	Multiplication Fact Strategies						
	Foundational Facts			Derived Fact Strategies			
	2s and 10s	5s	Squares	Add/Subtract a Group	Doubling	Near Squares	Decompose a Factor

Aspects of Fluency

- ❑ Flexibility
- ❑ Accuracy
- ❑ Efficiency
- ❑ Appropriate Strategy Use

Tests: Alternatives

Fact Strategy Quiz/Self-assessment

Solve these problems and tell how you solved out.

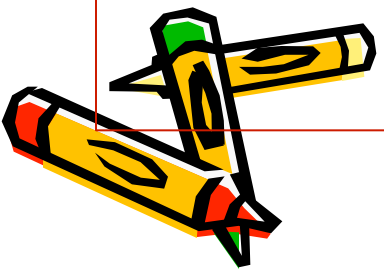
4 x 5 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

10 x 6 = _____ Check one: ____ I used this strategy:_____
____ I just knew.

6 x 2 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

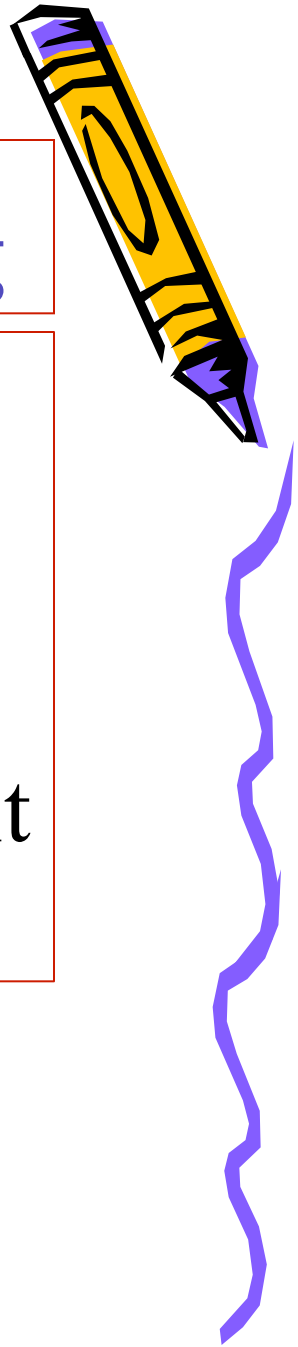
Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use



Journal Writing

If your friend didn't know the answer to 6 x 8, how would you tell him to figure it out?



Journal Writing: Example

6 x 8

I know my 5's well
So if I know 5x8
I just have to add
another group of 8
and I get 6x8=48

or If you know
your 8's really well
or 9's or even
10's you know 8x6=48
or 9x8=72 and
subtract a group
of 8 or 10x8=80
and subtract 2
groups of 8

Interviews

Flexibility	Accuracy
Solve 6 x 7 using one strategy. Now try solving it using a different strategy.	What is the answer to 7 x 8? How do you know it is correct (how might you check it)?
Efficiency	Appropriate Strategy Selection
For which facts did you **just know**? For which facts did you **use a strategy**?	Emily solved 6 x 8 by changing it in her mind to 5 x 8. What did she do? Is this a good strategy? Tell why or why not.

Interview Form

3rd Grade Exit Interview – May 2014 Name:_____

1. 5 x 8

2. 9 x 3

3. 4 x 8

4. 9 x 6

5. 8 x 7

6. 4 x 6

Feedback to Parents

Although it is important for your child to practice **all** multiplication facts this summer to maintain fluency, the end-of-year multiplication facts assessment showed your child,

_____, will benefit from focusing in particular on the following multiplication facts:

Your Turn

At your table, share your answer to one of these prompts:

- An activity I will use is…
- Questions I still have about the strategies are…
- Something surprising I heard is…
- I am going to/not going to…

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