Problems Worthy of your Effort and your Students' Engagement

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The NCTM *Reasoning and Sense Making* Initiative

Reasoning—the process of drawing conclusions based on evidence or previous knowledge and/or prior assumptions.

Sense Making—developing an understanding of mathematics concepts in a situation or context.

-- from *Focus on Reasoning and Sense Making* (NCTM, 2009)

A Problem is 'Worthy of Effort' if:

- It has *multiple entry points*
- It has mathematical punch and power -- involves mathematical *connections*, *structure, big ideas, problem solving heuristics*, requires students to *justify* their thinking
- Opens the door for *conjectures, extensions, generalizations*
- Taps multiple Effective Teaching Practices from NCTM's *Principles to Actions*

Mathematics Teaching Practices Prominent in Tasks Worthy of Effort

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

Today's Habits of Interaction

- Private Think Time
- Elbow pair share—Notices, Wonders, Ways to begin…
- Go Round once at Table
- Table Broadcasts— "We noticed, we wondered, we conjectured, we tried…"

PTP 1. The Grouchy Customer

Where Would You Sit in Your Neighborhood Café?

* In a neighborhood café there are 10 seats in a row at the counter. Each morning, customers enter the café for their morning coffee. They don't really want to have a conversation, so they prefer not to sit next to one another at the counter.

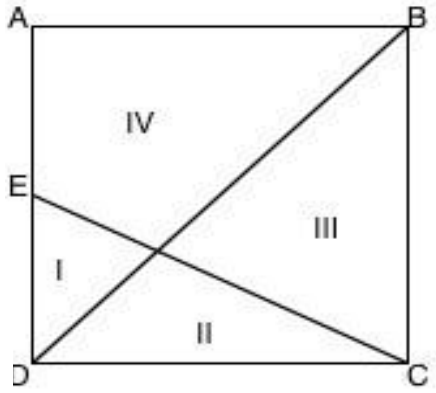
* Two people enter the café when it opens. How many different ways can these two customers sit at the counter so that they are not next to each other?

Extensions of Grouchy Customer PTP

- What if there were *three* grouchy customers in the café first thing in the morning. How many ways could they seat themselves so none of them had to sit next to another?
- What if the café had 12 seats instead of 10? Or had 15 seats? Or *N* seats? *N* seats and *P* customers*?*
- What if the café had a circular counter?
- What if we are concerned about the order in which the customers are sitting?

PTP2: **Comparing Regions in a Square**

- In the figure below, quadrilateral *ABCD* is a square, and *E* is the midpoint of the side *AD*.
- How do the areas of regions I, II, III, and IV compare?



Extensions of Regions in a Square

- What if E were at the 1/3 point?
- The 1/4th point? The 1/nth point?
- What if the the figure were a rectangle instead of a square?

Love this problem, especially because algebraic approaches, though possible, are *really* messy!

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PTP 3: **Looking Squarely at the Difference!**

- Here's a "kick back and enjoy" Problem to Ponder.

Which whole numbers can be expressed as the difference of two perfect squares?

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PTP 4: The Jacobean Locks Problem

- In medieval times, the inhabitants of a remote village decided to put a number of locks on a giant chest to protect the village valuables from marauding thieves.
- For additional security, the villagers created a enough locks and keys so that no two people from the village had enough keys to open the chest, but amongst them any group of three people always had enough keys to open all the locks on the chest.

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The problem:

How many locks, and how many keys, are needed to insure that no two people can open the chest, but any three people can?

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Links to Todays P2P's Below

Grouchy Customers https://www.nctm.org/P2P-cafe.aspx

Regions in a Square https://www.nctm.org/P2P-Regions.aspx

Differences of Perfect Squares https://www.nctm.org/P2P-SquareDifferences.aspx

Jacobean Locks https://www.nctm.org/P2P-Lock.aspx

Dear Participants,

These might not work well yet—write me if you can't get them and I'll send them to you in a form that should provide hot links. NCTM website is undergoing major renovation.

Mike

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