The 21st Century Wireless Classroom Network for AP Calculus

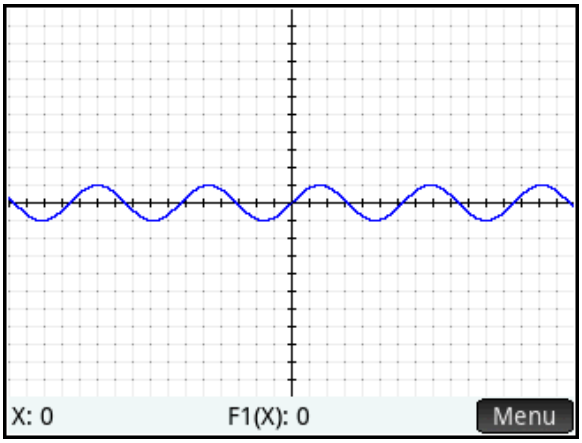
In this exploratory hands-on workshop, we will be solving Calculus problems with the HP Prime Graphing Calculator and the HP Wireless Classroom Network. HP Prime is an app-based graphing calculator with a touch-gesture user interface.

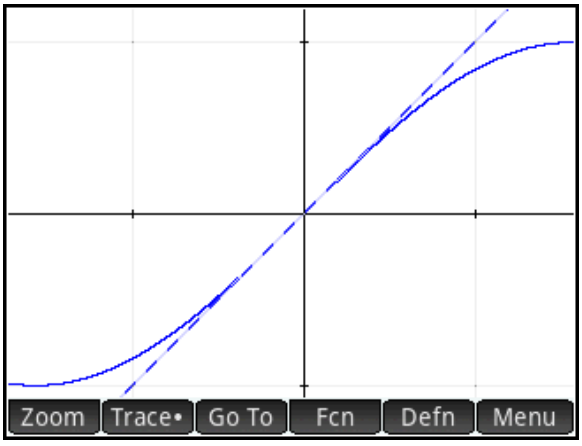
Problem 1: The slope of y=sin(x) at x=0

Part A: Getting Started

In this introductory problem, we pinch to zoom in on the graph of y=sin(x) at x=0 to establish local linearity with a slope of one.

1. Press Apps Info to open the App Library and select the Function app. The app opens in Symbolic view.
2. Enter our function in F1(X).
3. Press Plot Setup to open Plot view where the graph is displayed. You can drag with your finger to scroll the viewing window.
4. The tracer is at the origin. Press + Ans ; to zoom in on the tracer and − Base ; to zoom out on the tracer.
5. You can also pinch to zoom. A diagonal pinch zooms square. You can pinch horizontally to zoom on just the x-axis or pinch vertically to zoom on just the y-axis.
6. Pinch to zoom in on (0, 0).
7. To draw the tangent at X=0, press 0 Notes " " Enter ≈ to place the tracer at X=0, tap Fcn and select `Tangent`. This is a toggle, so you remove the tangent the same way.

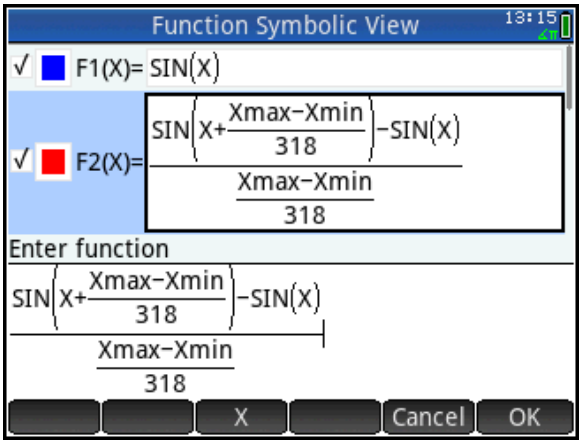




The grid lines, tick marks, and tangent all indicate the slope of sin(x) at x=0 is 1. These are new and authentic technological experiences that are beneficial in developing student conceptual understanding.

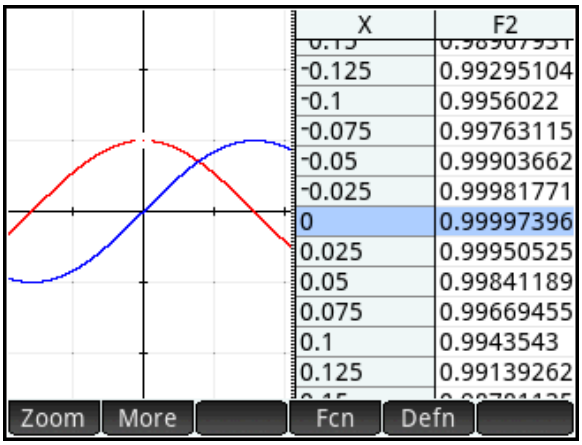
Part B: The corresponding numeric view

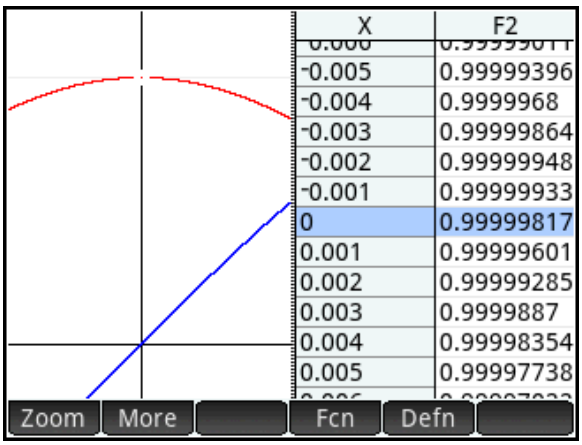
1. Press Shift Plot Setup (Plot Setup). Press Shift Esc Clear to reset all plot view parameters to their defaults.
2. Press Symb Setup to return to Symbolic view
3. In F2(X) enter the difference quotient for F1(X), as shown in the figure to the right, and press Enter.



Our definition for h is $h = \frac{X\ \min - X\ \max}{318}$, which is the width of 1 pixel regardless of any zoom you might make. To retrieve the variables Xmin and Xmax, press Vars and tap App; then tap `Function`, `Plot`, and select the variable `Xmax`.

4. Press View Copy and select `Split Screen: Plot Table`.
5. In the table, tap on X=0 to select that row. Now press ▼ to move the tracer to F2(X).
6. Press + Ans a few times to zoom in on the tracer, or use your finger to pinch the Plot view. You will see the value of F2(0) approach 1 rather quickly.
7. You can also pinch vertically in the table to zoom in or out on the selected x-value (in this case x=0).



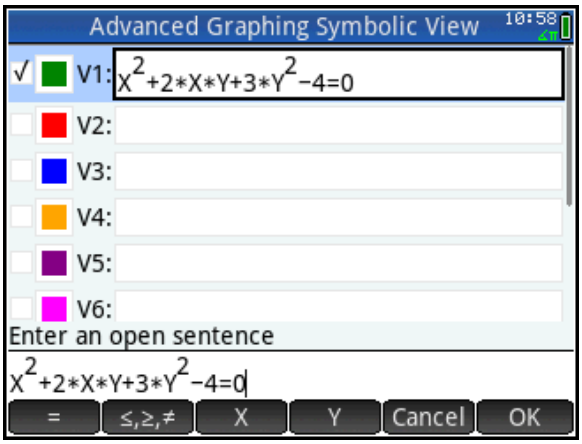


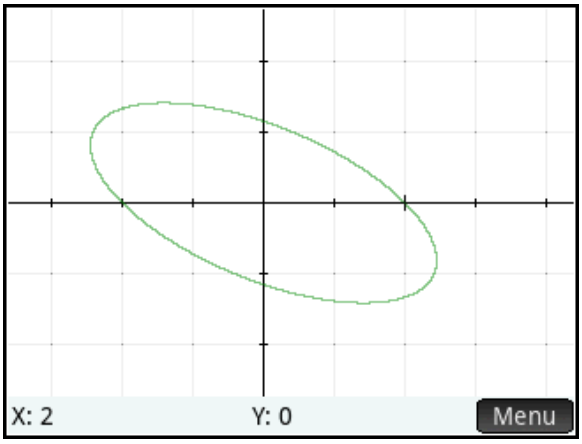
These are technological experiences for the 21st Century!

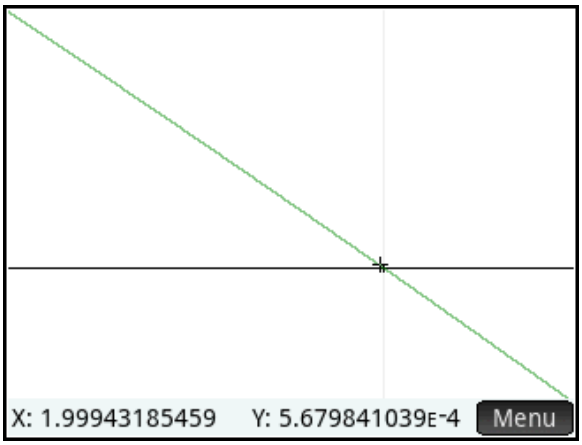
Problem 2

The point whose coordinates are (2,0) is a point on the ellipse $x^2 + 2 \cdot x \cdot y + 3y^2 - 4 = 0$. What is the equation of the line tangent to the ellipse through that point?

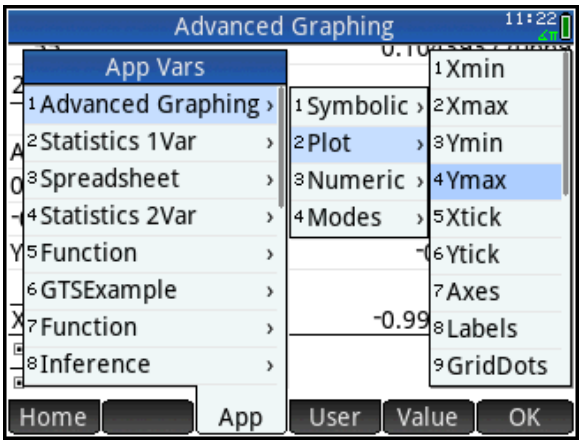
1. Press Apps to open the App Library and select the `Advanced Graphing` app.
2. The app opens in Symbolic view, where you can enter up to 10 equations or inequalities. In V1, enter the equation for our ellipse. Tap the menu keys at the bottom to enter =, X, and Y. Tap OK or press Enter when you are done.
3. Press Plot Setup to see the graph of the ellipse.
4. Move the tracer to the point (2,0) and press + to zoom in on this point. Drag the display with your finger to scroll the window. Continue to zoom and scroll until the plot fills the window as in the figure to the right.
5. Zoom in on the point (2,0) using the open pinch gesture. Place two fingers on the display at the same time and move them apart to zoom in. A vertical or horizontal pinch will zoom in one dimension only, while any diagonal pinch will zoom in both dimensions. Zoom in to establish local linearity and then zoom vertically or horizontally to get the line to pass through two opposite corners of the display.



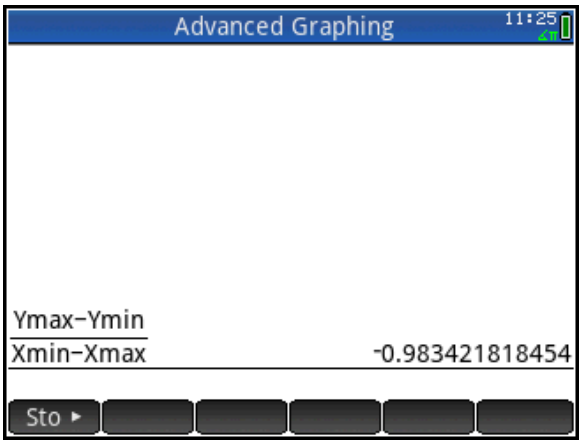


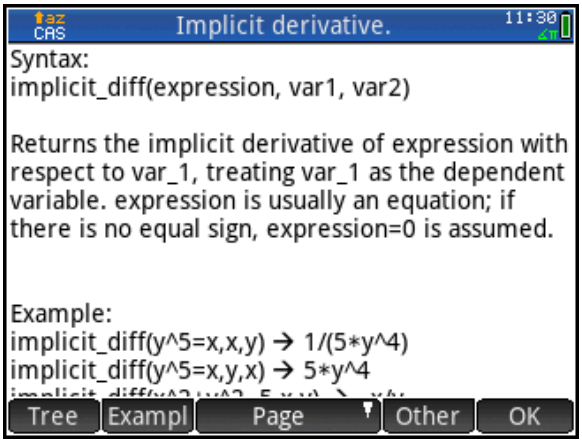


6. We will now use the window dimensions to estimate the slope of the tangent line. Press Settings to open the Home view. Press Units and choose the fraction template. Press Vars, tap App, tap `Advanced Graphing`, tap `Plot`, and select `Ymax`. Continue in this fashion to build the expression $\frac{Y\max - Y\min}{X\min - X\max}$. Press Enter ≈ to evaluate. Our estimate for the slope is close to -1.



7. We will now use implicit differentiation to find the slope exactly. Press CAS Settings to open the CAS view. Press Mem to open the Toolbox Menus, tap Catlg, then press TAN ATAN (IM) to jump to commands that start with those two letters. Scroll down to `implicit_diff`. Press Help User to view the help page for this command. Notice the menu keys:
   - Tree: opens the entire help tree
   - Exampl: opens a menu of examples to paste into the CAS
   - Page: page by page navigation
   - Other: views related commands
   - OK: closes the help page



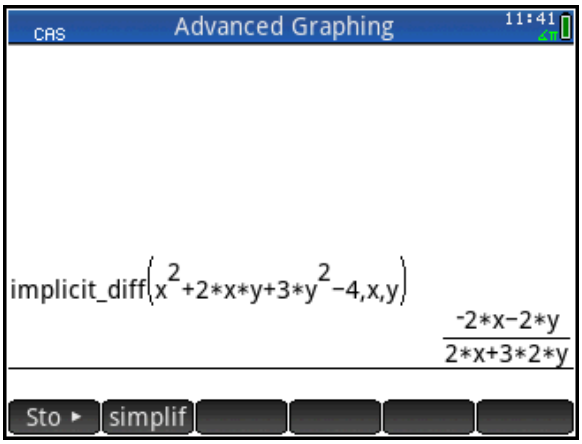


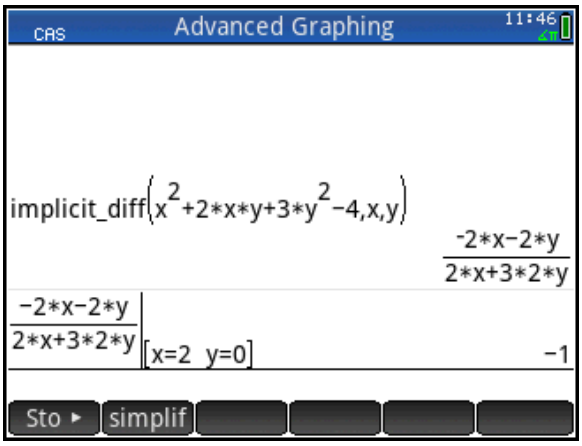
We can see that the command takes an expression, followed by the variables for differentiation.

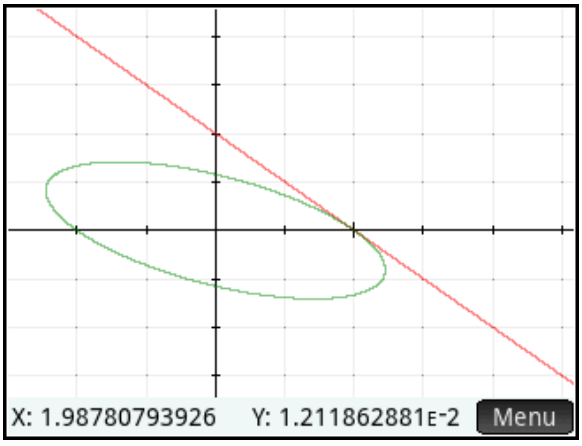
8. Tap OK to close the help page. Tap OK to paste the command into the CAS.

The CAS uses lower-case variable names, so use *x* and *y* here.

9. Enter the expression for our ellipse, followed by both x and y, and press Enter to see the result.
10. Press Units and select the third template in the first row (called the where() command. Tap on the first square, then tap on our last result and tap Copy to copy it into the square. Tap on the second square and enter x=2, y=0. Press Enter to see the result: the slope is -1.
11. Press Symb Setup to return to Symbolic view and enter the equation of the tangent line. The line whose slope is -1 and contains (2,0) is y-0= -1(x-2) or y= -x+2. Enter this equation in V2 and press Plot Setup to see the graph. You can enter the equation in either point-slope form or slope-intercept form. Press Base to zoom back out so you can see the whole ellipse.





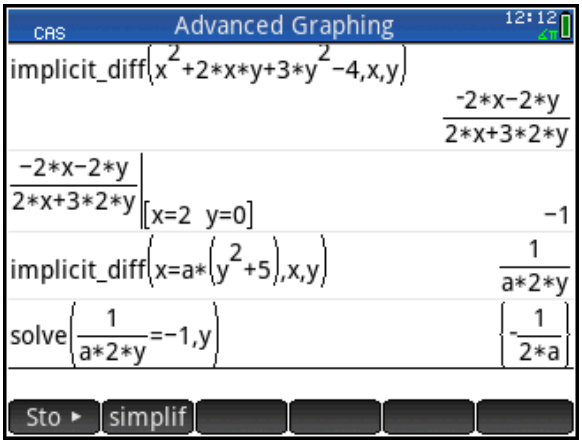


HP Prime delivers technological experiences that promote hands-on learning, as well as tools that open new areas for mathematical exploration, as you have seen in this example.

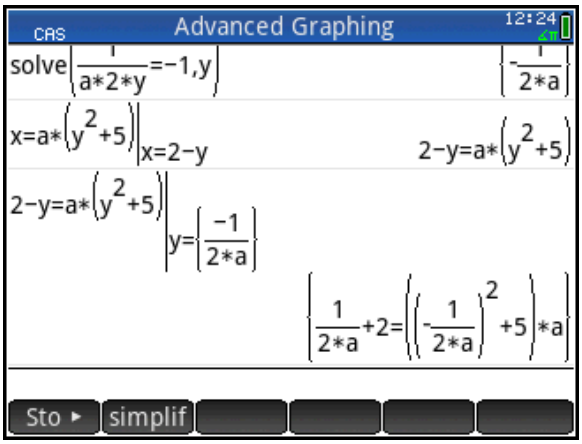
Problem 3

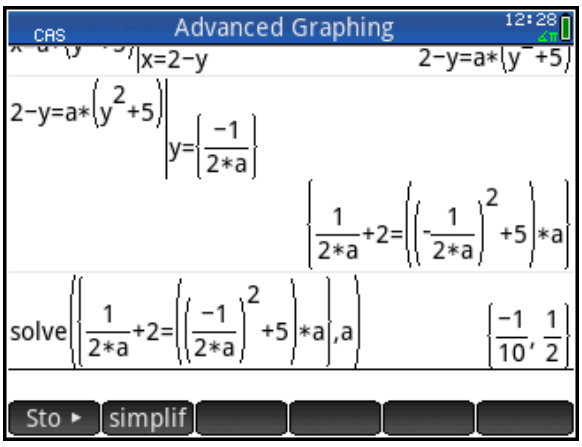
Find any values of *a* for which the parabola $x = a \cdot \left(y^2 + 5\right)$ is also tangent to our line y= -x+2.

1. Press CAS Settings to return to the CAS. Differentiate our equation implicitly with respect to both x and y, as shown in the figure to the right.
2. Press Mem, tap CAS, tap `Solve`, and select `solve`. Now copy the expression (just double-tap it) and set it equal to -1 as shown. Press Eval, followed by *y* to solve for *y* and press Enter to see the result.

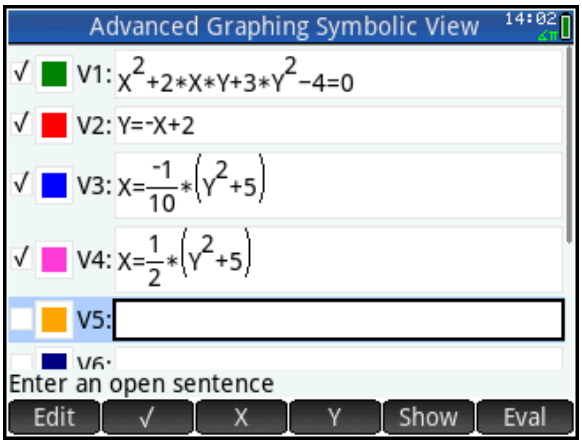


3. We know that the point of tangency satisfies both equations (the parabola and the line). Return to the where() command and enter the parabola equation as shown in the figure to the right. In the second square enter the substitution x= 2-y to get an equation in *a* and *y*.
4. Repeat with the result and our substitution for *y* in terms of *a*.
5. Now solve for *a*. The result shows that there are two values of *a*, -1/10 and 1/2, for which the graph of $x = a \cdot \left(y^2 + 5\right)$ is tangent to the line y= -x+2.

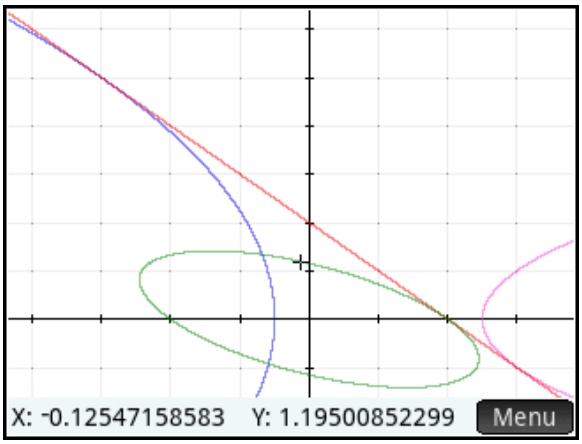




6. Return to Symbolic view and enter these equations in V3 and V4.



7. Press Plot Setup to see the graphs.



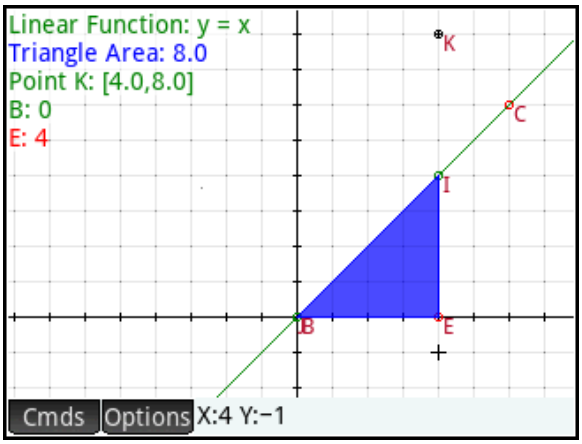
HP Prime's winning combination of CAS and the Advanced Graphing app extend the visualization of solutions to problems involving non-functions. And the touch interface makes it easy to copy previous results, get a nice viewing window, and just generally makes exploration and problem-solving easier.

Come visit us at Booth #1209 to learn more!

Problem 4

The instructor will use the HP Prime Wireless Network to send you an app called `AreaFunction1`. This app, based on the Geometry app, was designed to introduce students to the notion of area under a graph and an area function. We will explore this app together.

Press Apps and select `AreaFunction1`. The app opens in Plot view. Line BC is shown, along with a point E on the x-axis. The area under line BC, from B to E, is shown as a blue triangle. The equation of line BC is shown at the top left, along with the area of the triangle. Point K has the same x-coordinate as point E and its y-coordinate is the area of the triangle. Thus, point K is a point on the area function for line BC.



Controlling the app:

- Objects under the cursor turn red to show they are below the cursor.
- Tap and drag a point to move it quickly
- Tap a point and press Enter to select it. It will turn light blue to show it is selected. Now you can drag it OR use the cursor keys to move it one pixel at a time. Press Esc to deselect a selected point.

Exploration 1:

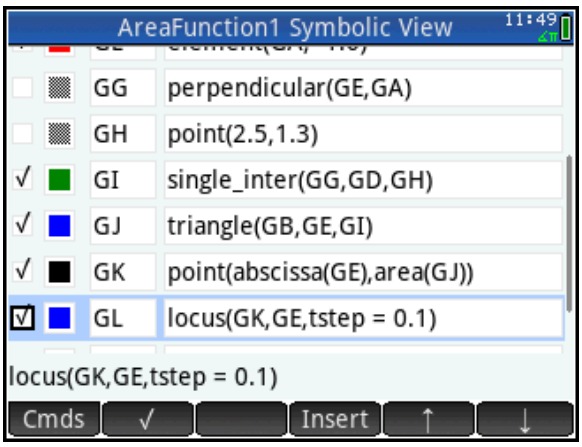
Tap point E and press Enter to select it. Now use ◀ and ▶ to move the point along the x-axis. Students should see that the triangle is always an isosceles right triangle, so its area is always of the form x^2/2. So the area function must be y=x^2/2. Focus on the path of point K as point E moves. Does its path appear parabolic? Students must see that the y-coordinate of point K is the same as the triangle area; that is, that the y-coordinate of point K can always give you the area of the triangle for a given x-value.

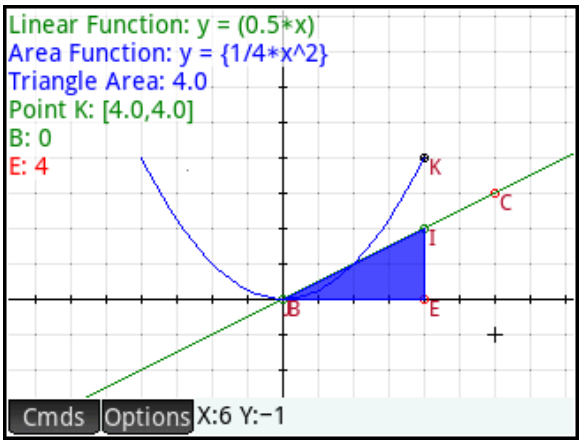
Things you can do:

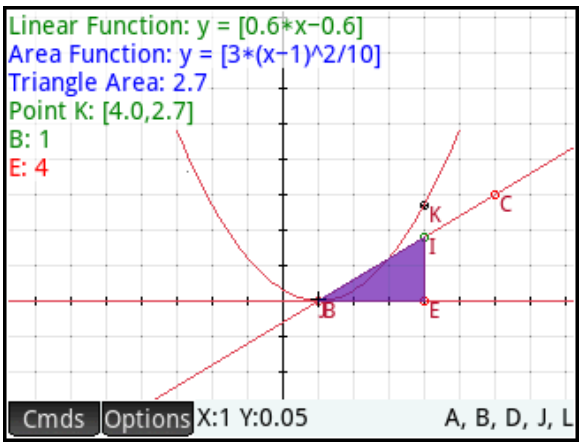
- Tap point K (it will turn red); then tap Options and select `Trace`. Now when you move point B, the trace of point K is shown. This option is a toggle; repeat to turn off tracing. Turning off tracing does not clear any existing trace. To clear the trace, tap point K, tap Options and select `Clear Trace`.
- Tap point K; tap Options and select `Animate`. Point E will now move from x= -3 to x=4 in steps of 0.1.

Exploration 2

1. Press Symb to open Symbolic view. Here, each geometric object in Plot view has its definition.
2. Notice GL, the locus of point K, does not have a checkmark beside it. Check GL so that it is visible in Plot view.
3. Similarly, press Num to open Numeric view and check `Area Function` so that it is visible in Plot view.
4. Press Plot to open Plot view.







Things you can do:

- Tap point C and press Enter to select it. Press ▼ to move it down one pixel at a time. Note the relationship between the coefficients of the linear and area functions.
- Move point C so that it is below the x-axis. Discuss signed area at this point. Take point C back above the x-axis and move point E to the left of the y-axis. Discuss why the triangle area here is still positive.
- Press Esc to deselect point C
- Tap point B and press Enter to select it. Now press ▶ to move it right one pixel at a time. Note the relationship between the linear and area functions.
- Press Esc to deselect point B

The AreaFunction app can be used to give students a basic conceptual understanding of area under a graph and accumulation functions (integrals). With the HP Prime Wireless Network, it is easy to send these custom apps to students to enhance their ability to explore essential Calculus concepts. These custom apps are based upon the built-in HP Prime apps and are basically HP Prime interactive documents!