All Students Should "Talk to Learn" Proportionality and Academic Language

Session 129

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Session description: Students, including English language learners, benefit from lessons that are designed to develop concepts, vocabulary, and problem solving through structured student talk, vocabulary development, …with a focus on the concepts compare, proportional, and times as many.

While working on Activities, please pay attention to student opportunities to address our conference theme -
Common Core Mathematics: Building Competent and Confident Problem Solvers.

Provide All Students Opportunities to Learn

While working on activities, pay attention to student opportunities to:

1) Informally explore a mathematical relationship
2) Expand enthusiasm and confidence
3) Expand understanding of a major mathematical concept
4) Become familiar with new vocabulary
5) Work through confusing or frustrating moments
6) Inadvertently, remain only minimally engaged with mathematical ideas

Session Warm-up

Brandon has 96 special beads. He likes to make bracelets that each have 16 special beads and 10 metal rings. If he wants to use all of his 96 special beads, how many metal rings does he need?
How many bracelets can he make?

What would you suggest that a teacher do to make this problem accessible to all 6th or 7th grade students?

Session Goals

Consider together the benefits and challenges of using an instructional approach that emphasizes student talk and collaboration while:

- developing essential, new mathematical concepts and vocabulary, particularly in linguistically diverse classes

- discussing the impact of focusing on challenging content and problems

- considering sample representations

- expanding students' abilities to solve extended tasks

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5. Remain only minimally engaged with mathematical ideas
6. Work through confusing or frustrating moments

Sample Lesson Discussion

Exploring Models with Partners to Learn about Proportional Relationships

Do you think that this toy car is proportional to a real car?
Are Hot Wheels Cars "like real"? How would you find out?

Talk to your partner(s). **Work with your primary language partners.**
I notice that…
We could …
It would help if we…

Are Hot Wheels Cars "Like Real"?

What would it mean to say that a toy car is "like real"?

How could students investigate this question?

Discuss in small groups.

Is this a reasonable assignment for a class that includes students at all levels of English speaking and listening ability?

Talk with your neighbors about how a class could use mathematics to decide if they are accurate models.

Structures for Supporting Concept Development and Student Talk

Susan Mercer

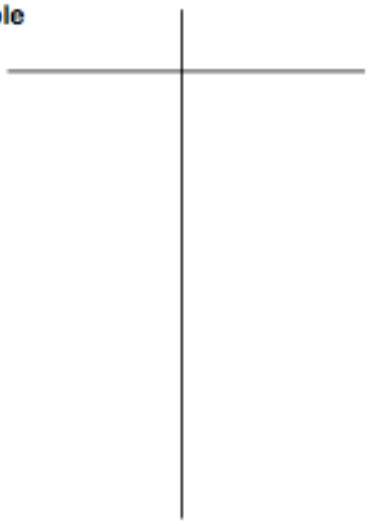
Word Problem: Melody gets paid $4 per hour for baby-sitting. If she saves all her money, how many hours does she have to work in order to buy a $30 CD player?

Rate or Ratio

Picture

Clearly label the answer to the problem.

Table



Clearly label the answer to the problem.

Proportion

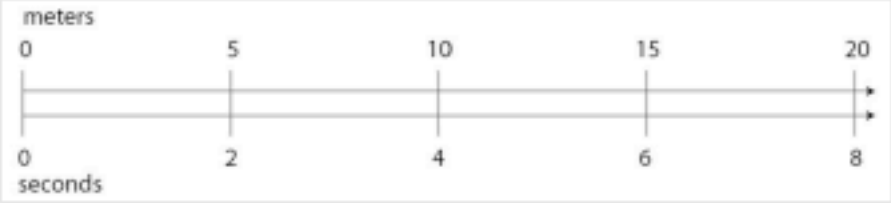
Clearly label the answer to the problem.

Study and discuss what is being represented in these tape diagrams.

Representing ratios with tape diagrams

This diagram can be interpreted as representing any mixture of apple juice and grape juice with a ratio of 3 to 2. The total amount of juice is represented as partitioned into 5 parts of equal size, represented by 5 rectangles. For example, if the diagram represents 5 cups of juice mixture, then each of these rectangles represents 1 cup. If the total amount of juice mixture is 1 gallon, then each part represents $\frac{1}{5}$ gallon and there are $\frac{3}{5}$ gallon of apple juice and $\frac{2}{5}$ gallon of grape juice.

Representing ratios with double number line diagrams



On double number line diagrams, if A and B are in the same ratio, then A and B are located at the same distance from 0 on their respective lines. Multiplying A and B by a positive number p results in a pair of numbers whose distance from 0 is p times as far. So, for example, 3 times the pair 2 and 5 results in the pair 6 and 15 which is located at 3 times the distance from 0.

(NCTM book 2)

Conversational and Academic language

When developing students' mathematical concepts, what are the roles of each type of language?

How can academic vocabulary be developed for all students, including second language learners?

Proportionality

"When two quantities are related proportionally, the ratio of one quantity to the other is invariant, or the numerical values of both quantities change by the same factor."

Developing Essential Understanding of Ratios, Proportions and Proportional Reasoning for Teaching Mathematics: Grades 6-8 NCTM. Joanne Lobato, Amy Ellis, Rose Mary Zbiek 2010)

How can we informally introduce middle school students, including English learners, to this important concept?

Possible comments:

Point out mathematical similarities across problems

Support through use of primary language and cognates

Gestures, models

Visual and tactile scaffolding

Linguistic scaffolding, such as sentence stems and sentence frames

Rephrasing

Middle School Math Task 24: Similar Figures *

Source: English Learners in the Mathematics Classroom (Corwin Press, Debra Coggins, 2014)

POD Task 24 Sheet-Part 1

With your partner, *draw and label 2 similar figures* (A and B) that are different from the similar figures shown in today's lesson. Your teacher will assign you length and width dimensions such as (2.5, 3.0), (3.4 cm, 0.8 cm), (4, 5) and a scale factor such as 5 or 3 times as long, or ½[1] as long.

Record your length and width in the chart.

Quadrilateral Name	Length	Width	L:W Ratios
A	ℓ_A=	w_A[2]=	
B	ℓ_B =	w_B[3]=	

*Background: This is a project/lesson that is a component of a middle school unit on proportional relationships. It serves as part of a discussion about attending to comprehensible input.

POD Task 24 Sheet-Part 2

1. Work and discuss with a partner. (My partner's name = _____)

2. Use the sentence frames:

a. "I know Mr. Cruz's shapes are similar because . . ."

b. "I think you have similar figures because I see that _____."

"Please explain why _____."

c. "The length of this line is ___ times as long as my original line."

3. Side measurements of Rectangle A: ____ , ____

Scale factor s = _____

4. Help all group members have correct examples. Trade drawings with others. Repeat Steps 1 to 5 to check work.

5. Choose one group member's Figure B and use a scale factor to turn it back into Figure A. Work together to *discover the scale factor* to use to change it to be exactly the same as Figure A. Show and describe your method.

Students: Write on your Task Sheet - My group's side measurements of Rectangle B = ____ , ____ Our scale factor = _____

POD Task 24 Sheet

With your partner, *draw and label 2 similar figures* (A and B) that are different from the similar figures shown in today's lesson. Your teacher will assign you length and width dimensions such as (2.5, 3.0), (3.4 cm, 0.8 cm), (4, 5) and a scale factor such as 5 or 3 times as long, or $\frac{1}{2}$ as long.

Record your length and width in the chart.

Quadrilateral Name	*Length*	*Width*	*L:W Ratios*
A	l_A= l_A=	W_A=	
B	l_B=	W_B=	

1. Work and discuss with a partner. (My partner's name = _____)
2. Use the sentence frames:
   a. "I know Mr. Cruz's shapes are similar because . . ."
   b. "I think you have similar figures because I see that _____."
      "Please explain why _____."
   c. "The length of this line is ___ times as long as my original line."
3. Side measurements of Rectangle A: ____ , ____ Scale factor **S** = _____
4. Help all group members have correct examples. Trade drawings with others. Repeat Steps 1 to 5 to check work.
5. Choose one group member's Figure B and use a scale factor to turn it back into Figure A. Work together to discover the scale factor to use to change it to be exactly the same as Figure A. Show and describe your method.
   a. Students: Write on Task Sheet

Side measurements of Rectangle B: ____ , ____ Scale factor = _____

Use these Talk Structures:

- I notice that…
- I wonder if…
- Do you understand…
- Let's all …

Work in groups of 2 or 3.

Plan to discuss:. ..scale up

- …scale down
  - What possible challenges (difficulties) do you anticipate if using this task in a classroom of diverse learners?
  - When would you envision students successfully engaging with this task?
  - What prior experiences would be helpful?
  - *What problems like this one have you studied in the past?

“Talk to Learn”

Math Pathways and Pitfalls Lesson: How Many for 1?

[Insert Lesson]

Read the Two Student Responses. Introduce yourself to a neighbor and list ways in which this lesson can support diverse students as they expand their proportional thinking abilities.

(Math Pathways and Pitfalls Grades 6-8 Unit 7 Lesson 5. WestEd)

Middle School Math Task 24: Similar Figures

Source: English Learners in the Mathematics Classroom (Corwin Press, Debra Coggins**)**

(Background: This is a project/lesson that is a component of a middle school unit on proportional relationships. It serves as part of a discussion about attending to comprehensible input.)

What possible challenges do you anticipate if using this task in a classroom of diverse learners?

- When would you envision students successfully engaging with this task?
- What prior experiences would be helpful?
- What relationships did you find?

Use these Talk Structures:

- I notice that…
- I wonder if…
- Do you understand…
-

Work in groups of 2 or 3.

Word Problem: Melody gets paid $4 per hour for baby-sitting. If she saves all her money, how many hours does she have to work in order to buy a $30 CD player?

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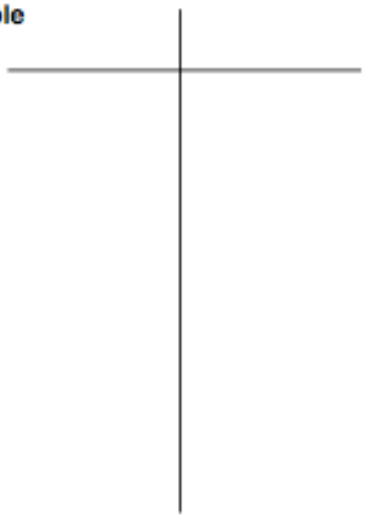

Clearly label the answer to the problem.

Proportion

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Stretchy Critters

Identify and describe mathematical relationships using Stretchy Critters and paper Grid Strips

Make a table of equal lengths using
Single Critters and Stretched Critters
What relationships can we find?
$5\ Frog_{0} = 1 Lizard_{Stretched}$

Can you find examples of Multiplicative Relationships and also of Additive Relationships?

Option: Use grid strips of different scales and find relationships between strips of equal lengths

Create a Possible Problem

_T_____ 8 _____ for _e_____ 20 _____.

How many _____for 45 _____?

What context might relate to this problem?

If a teacher leads a class to create such a problem, the underlying understanding to help students develop is…

A 5th or 6th grader might mathematically solve this problem by …

When or why is it inefficient to cross-multiply and divide for this situation?

Standards for Mathematical Practice

Mathematical practices
CCSS.Math.Practice.MP7 Look for and make use of structure.

Standard for Mathematical Practice # 6
Attend to precision

Closing

- Reflect on how all students can be taught in a manner that addresses this conference's theme: "Common Core Mathematics: Building Competent and Confident Problem Solvers."

- Jot down, or discuss quietly with a neighbor, reasons why more complex lessons are beneficial to students, including students with limited English language skills.

Resources:

Developing Essential Understanding of Ratios, Proportions and Rates. NCTM Yearbook G6-8

Developing Essential Understanding of Ratios, Proportions and Proportional Reasoning for Teaching Mathematics: Grades 6-8 NCTM(2)

Joanne Lobato, Amy Ellis, Rose Mary Zbiek 2010

English Learners in the Mathematics Classroom. Debra Coggins, Corwin Press, 2014

Jump-Starting Effective and Equitable Mathematics Instruction": The Design and Efficacy of Math Pathways & Pitfalls AERA 2009

Math Pathways & Pitfalls: Algebra Readiness.

Carne Barnett-Clarke, Alma B. Ramirez, with Debra Coggins WestEd, 2010

A Mathematics SourceBook , Debra Coggins, Maria Carroll, Grace Coats and Drew Kravin, Arena Press 2000

Ratios and Rates (student booklet). Susan Mercer www.supermathunits.com/units