What Counts? Developing a Communal Classroom Criteria of Proof

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The Sticky Gum Problem[1]

Ms. Hernandez came across a gumball machine one day when she was out with her twins. Of course, the twins each wanted a gumball. What's more, they insisted on being given gumballs of the same color. The gumballs were a penny each, and there would be no way to tell which color would come out next. Ms. Hernandez decides that she will keep putting in pennies until she gets two gumballs that are the same color. She can see that there are only red and white gumballs in the machine.

1) Why is three cents the most she will have to spend to satisfy her twins?
2) The next day, Ms. Hernandez passes a gumball machine with red, white, and blue gumballs. How could Ms. Hernandez satisfy her twins with their need for the same color this time? That is, what is the most Ms. Hernandez might have to spend that day?
3) Here comes Mr. Hodges with his triplets past the gumball machine in question 2. Of course, all three of his children want to have the same color gumball. What is the most he might have to spend?
4) Generalize this problem as much as you can. Vary the number of colors. What about different size families? Prove your generalization to show that it always works for any number of children and any number of gumball colors.

[1] Fendel, D., Resek, D., Alper, L., & Fraser, S. (1996). *Interactive mathematics program year 1 - unit 2: The game of pig* (p.99). Emeryville, CA: Key Curriculum Press.

Student A

Ms. Hernandez came across a gumball machine one day when she was out with her twins. Of course, the twins each wanted a gumball. What's more, they insisted on being given gumballs of the same color. The gumballs were a penny each, and there would be no way to tell which color would come out next. Ms. Hernandez decides that she will keep putting in pennies until she gets two gumballs that are the same color. She can see that there are only red and white gumballs in the machine. (*The following questions are to help guide you towards answering question 1. You should complete all of the bulleted questions, but you only need to email complete responses for Questions 1-4 below. Based on your solution to Question 1 below, I may ask you to turn in your bulleted responses in class.)*

- Why is three cents the most Ms. Hernandez will have to spend to satisfy her twins?
- The next day, Ms. Hernandez passes a gumball machine with red, white, and blue gumballs. How could Ms. Hernandez satisfy her twins with their need for the same color this time? That is, what is the most Ms. Hernandez might have to spend that day?
- Here comes Mr. Hodges with his triplets past the same gumball machine with red, white, and blue gumballs. Of course, all three of his children want to have the same color gumball. What is the most he might have to spend?

Question 1. Generalize this problem as much as you can. Vary the number of colors. What about different size families? Prove a generalization that always works for any number of children and any number of gumball colors. I encourage you to think about using alternative representations within your proof in order to create your most convincing/compelling argument.

# of colors = n , # of children = X # of times to get x amount of similar colors = T

$$n(x-1)+1 = T(n,x)$$

- Mrs. Hernandez w/ R, W gumballs: $n=2$, $x=2$

  $T = 2(2-1)+1 = 3$

- Mrs. Hernandez w/ R, W, B gumballs: $n=3$, $x=2$

  $T = 3(2-1)+1 = 4$

- Mr. Hodges w/ R, W, B gumballs: $n=3$; $x=3$

  $T = 3(3-1)+1 = 7$

The expression $T(n,x) = n(x-1)+1$ is valid for the values of $(n=2, x=2)$ and $(n+1=3, x=2)$ as well as $(n+1=3, x+1=3)$, so we can assume the expression is valid for all positive integers of n and x.

Student B

# of Children	# of Colors	Amount Spent (¢)
2	2	3
2	3	4
2	4	5
2	5	6
3	3	7
3	4	9
3	5	11
3	6	13
4	4	13
4	5	16
4	6	19

From the table above we can see a pattern going on. When we have 2 children the amount spent goes up by one penny. When we have 3 children and increase the amount of colors the amount of pennies spent goes up by two. Example: When we have 4 children and 6 colors the first 6 pennies will give me one of each different color. The next 6 pennies will give me one of each different color again. This process is the same for the third time 6 more pennies will be spent. Until the fourth time we know we will get any color matching 4 color gumballs.

Amount Spent=A
Children=K
Colors =k
A=(K-1)(C))+1
When we have 2 children we see that the amount spent is simply C+1. But then when we have 3 children the amount spent is by two pennies. This is where the K-1 comes in place.

Student C

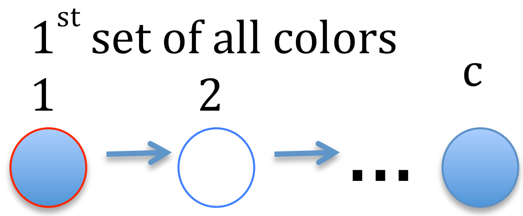
Children = c
Gumballs = g
Pennies = p
P = g (c -1) +1

In order to make sure you get enough gumballs for every child to have one of the same color you must buy one of all the colors for every child but one. Then no matter what color you get next every child will have one of the same color. Therefore multiply the number of gumball colors by one less than the number of children, after that add one.

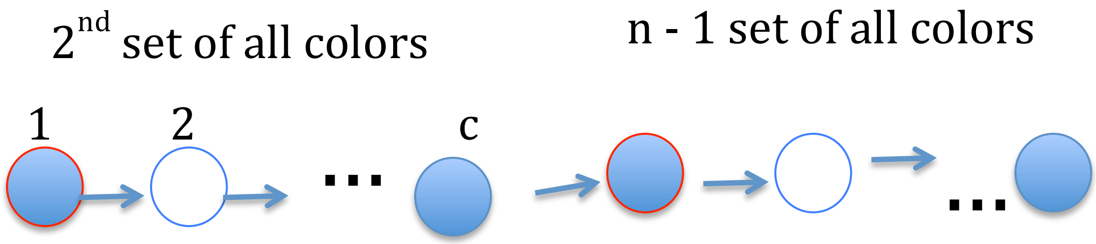
2c and 2g = 3p
2c and 3g = 4p
2c and 4g = 5p
3c and 2g = 5p
3c and 3g = 7p
7c and 2g = 13p

Student D

If we always consider the longest possible chain of events that gets us to our desired end and we count each draw as ¢, we can find the maximum number of cents needed to get n balls of the same color.



Interchange as long as exactly one of each color



Variations on color combinations

Once you have n – 1 of each color you have drawn c(n – 1) times on your next draw no matter what color you draw you will have n colors of that color. Satisfying the requirement that each child gets a gumball of the same color. At this point you have spent c(n – 1) + 1 cents.
We know this is the maximum because completing a set of some number of gumballs of the same color any earlier means you will have n gumballs of one color sooner than in the model above; meaning you have spent fewer cents than in the model above. This rule works for n ≥ 1

Student E

Please complete this assignment by answering the three questions below with a full description. All responses must be submitted digitally. You may scan and email your response.

Sticky Gum Problem

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- Why is three cents the most Ms. Hernandez will have to spend to satisfy her twins?
- The next day, Ms. Hernandez passes a gumball machine with red, white, and blue gumballs. How could Ms. Hernandez satisfy her twins with their need for the same color this time? That is, what is the most Ms. Hernandez might have to spend that day?
- Here comes Mr. Hodges with his triplets past the same gumball machine with red, white, and blue gumballs. Of course, all three of his children want to have the same color gumball. What is the most he might have to spend?

Question 1. Generalize this problem as much as you can. Vary the number of colors. What about different size families? Prove a generalization that always works for any number of children and any number of gumball colors. I encourage you to think about using alternative representations within your proof in order to create your most convincing/compelling argument. R-red, W-white, B-blue, Y-yellow, G-green these are the color of gumballs

k RWB/RWB/RWB/RWB/RWB

3 gum balls, 2 kids, spend $0.04

3 gum balls, 3kids, spend $0.07

3 gum balls, 4 kids, spend $0.10

RWBY/RWBY/BYWR/YBRW/RWYB

4 gum balls, 2 kids, spend $0.05

4 gum balls, 3 kids, spend $0.09

4 gum balls, 4 kids, spend $0.13

4 gum balls, 5 kids, spend $0.17

RWBYG/RGBYW/YBGRW/WBYRG/GBWRY

5 gum balls, 2 kids, Spend $0.06

5 gum balls, 3 kids, Spend $0.11

5 gum balls, 4 kids, Spend $0.16

5 gum balls, 5 kids, Spend $0.21

For every child you add, the amount of money you spend increases by the number of colored gum balls present.

Argument Analysis Sheet

Arguments	**Yes or no**	**Rationale**
A		
B		
C		
D		
E		

List Characteristics of Proof