



# Developing and Assessing Addition Fact Fluency

Gina Kling and Jennifer Bay-Williams  
NCTM Annual Meeting  
April 17, 2015



# Our Session

- Overview of basic fact *fluency*
- Games to support basic fact *fluency*
- **Assessment overview**
- Assessment strategies for basic fact *fluency*
- Results of a strategy-based approach to learning basic facts



# What do we mean by fluency with basic facts?



# CCSS-M Descriptions



## Grade 1 (1.0A.C.6):

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

## Grade 2 (2.0A.B.2):

Fluently add and subtract within 20 using mental strategies (reference to **1.0A.C.6**). By end of Grade 2, know from memory all sums of two one-digit numbers.



# CCSS-M Descriptions



## Grade 1 (1.0A.C.6):

Add and subtract within 20, *demonstrating fluency* for addition and subtraction within 10. *Use strategies* such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

## Grade 2 (2.0A.B.2):

*Fluently* add and subtract within 20 *using mental strategies* (reference to **1.0A.C.6**). By end of Grade 2, *know from memory* all sums of two one-digit numbers.



This would suggest that *fluency is different* from automatic retrieval. Research heavily supports this...



**So, what does fluency *really* mean?**



# The Common Core State Standards for Mathematics (CCSS-M)

describes procedural fluency as

“skill in carrying out procedures

**flexibly**, **accurately**, **efficiently** and  
**appropriately**”

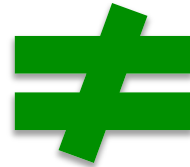
(CCSSO, 2010, p. 6).



# Procedural Fluency



Knowing  
from  
Memory



Memorization





# Developing Addition Fact Fluency

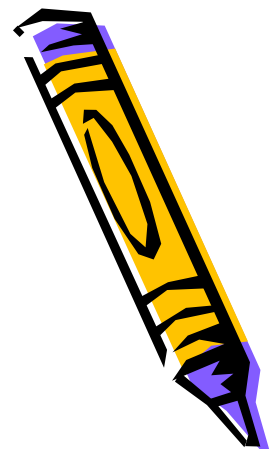


# Mastering Basic Facts

**Phase 1: Counting**  
(counts with objects or mentally)

**Phase 2: Deriving**  
(uses reasoning strategies based on known facts)

**Phase 3: Mastery**  
(efficient production of answers)



# Mastering Basic Facts

Phase 1 Example: Solving  $5 + 7$  by counting on from 5.

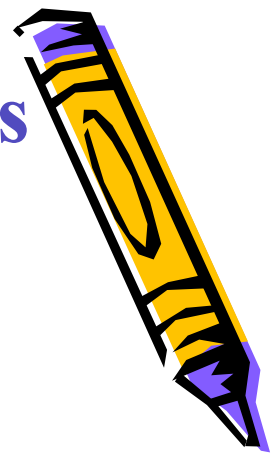
Phase 2 Example: Solving  $5 + 7$  by starting from  $5 + 5 = 10$  and then adding 2 more to get 12.

Phase 3 Example: Answering  $5 + 7 = 12$  with little/no hesitation.



# Reasoning Strategies for Addition Facts

## K-2 Trajectory



### Foundational Fact Strategies

- Sums within 5 **K**
- $\pm 1$  or 2 **K**
- Doubles **K 1**  
(2 + 2, 6 + 6, etc.)
- Combinations of Ten  
(3 + 7, 8 + 2) **K 1**

### Derived Fact Strategies

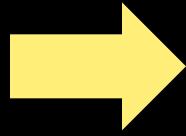
- Near Doubles **1 2**  
(6 + 7, 8 + 7)
- Making Ten **1 2**  
(8 + 3, 9 + 5)



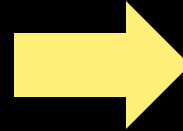
# Basic Fact Fluency: A 3-yr. Progression



**K**



**1**



**2**

Sums within 5

+/- 1

+/- 2

Doubles

Combo's of 10

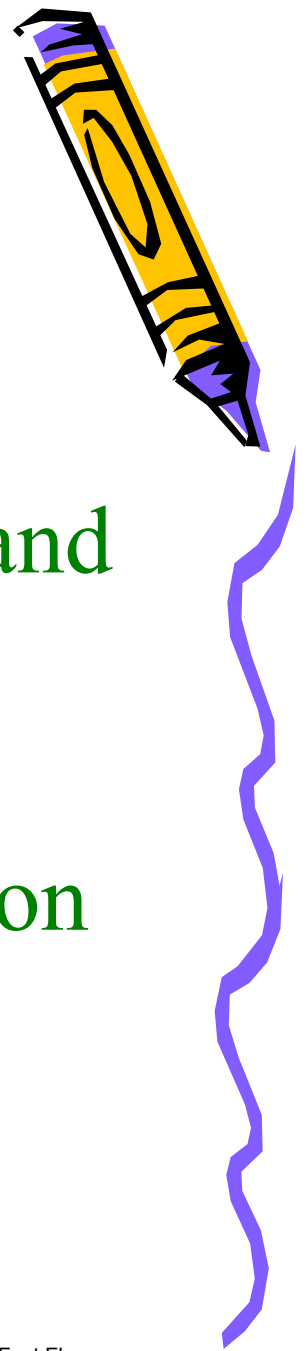
Sums within 10

Near Doubles

Making 10



# Meaningful Activities for Learning Basic Facts



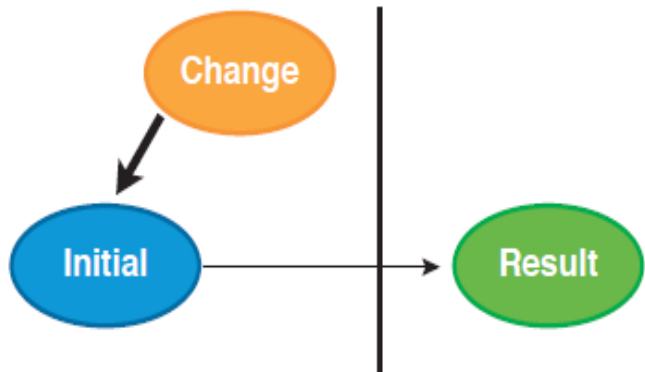
- Solve Number Stories
- Use Quick Looks with dot patterns and ten frames
- Discuss/write about strategy use
- Play basic facts games with a focus on reasoning strategies



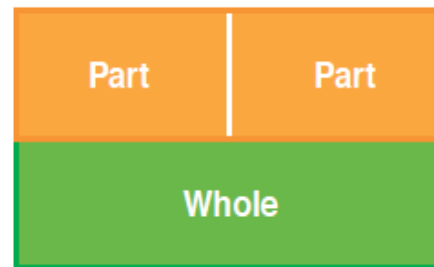
# Story Telling



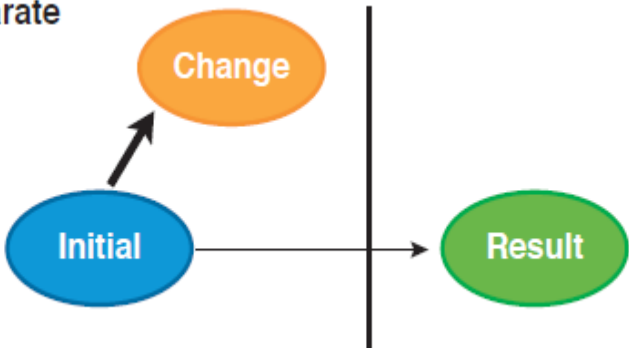
(a) Join



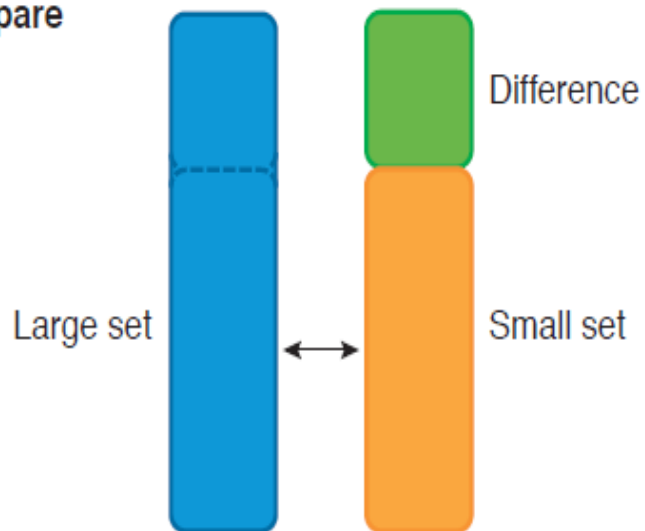
(c) Part-part-whole



(b) Separate



(d) Compare



# Phase 1 Phase 2

## Quick Images

### PreK:

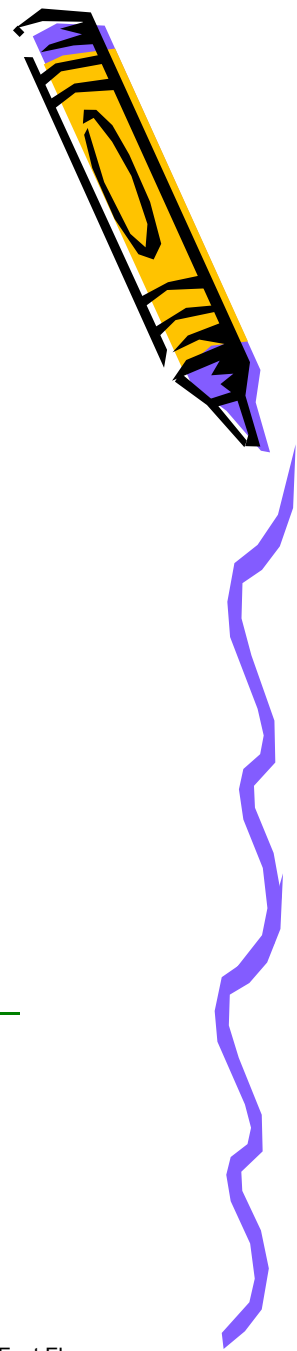
- ✓ number recognition
- ✓ representation

### Kindergarten:

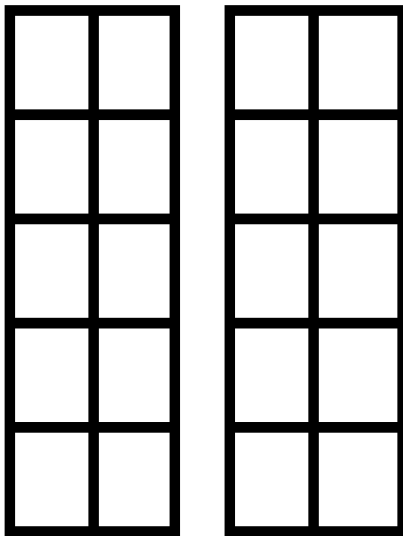
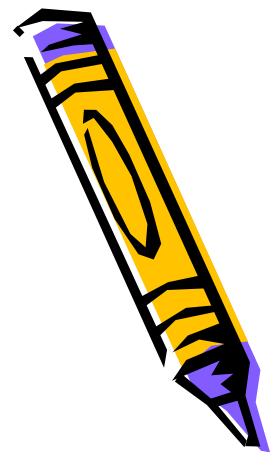
- ✓ decomposing and recomposing numbers
- ✓ subitizing

### Grade K- 2:

- ✓ recognizing and developing strategies for basic  $+/-$  combinations



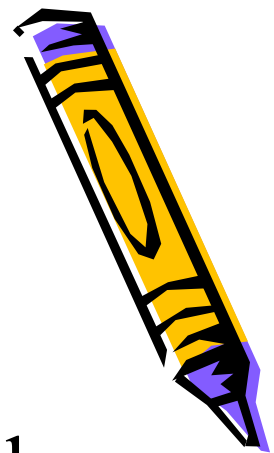
# Developing Fact Strategies through Quick Looks



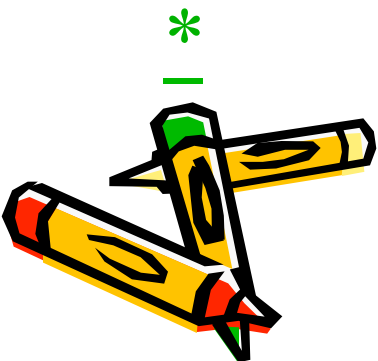
What arrangements of dots on double ten frames could encourage strategies such as near doubles and making ten?



# Developing Fact Strategies through Quick Looks



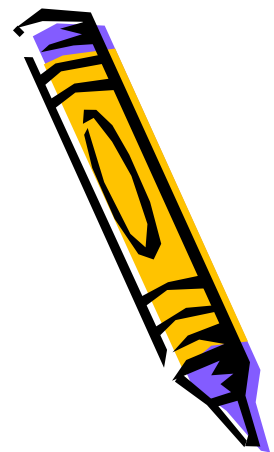
Teachers need to help children transition from developing strategies with Quick Looks to symbolic notation for strategies...



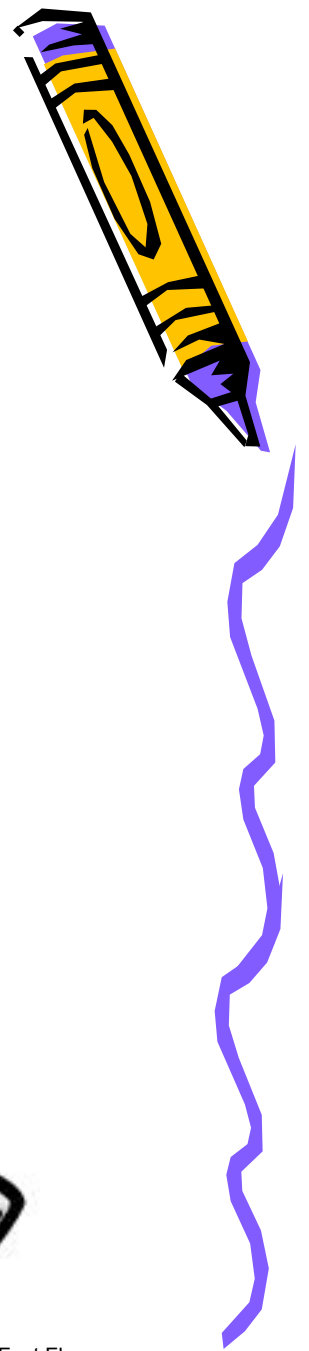
# Phase 2 Phase 3

## Meaningful Practice

“Practice that follows substantial initial experiences that support understanding and emphasize ‘thinking strategies’ has been shown to improve student achievement with single-digit calculations.” (NRC, 2001).



# Games Across Phases



- Close to 20
- Double It
- Name that Number
- Roll and Total
- Tens Go Fish
- Top-It



# Games as Meaningful Practice

As you explore each game, consider the following:

- What is the mathematical content of the each game?
- What makes these games examples of “meaningful practice”?
- What questions do you have about these games?
- Which games might work best with your students?



# Where does each game fit in?

Post it!

Phase 1:  
Counting

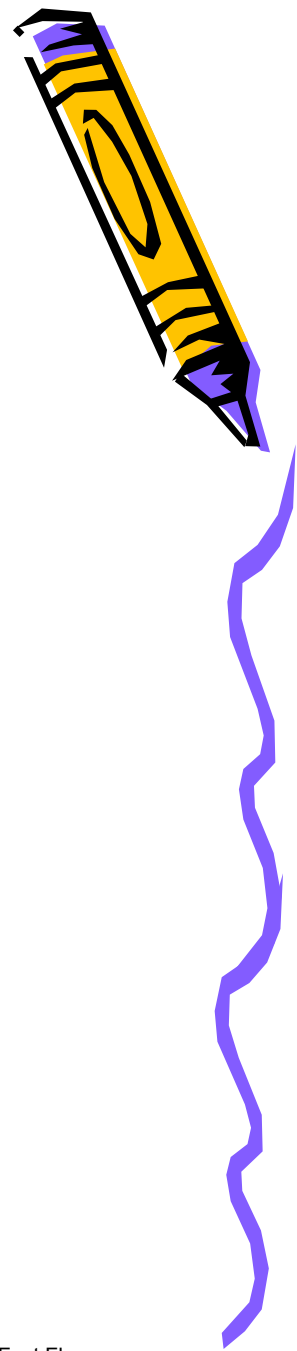


Phase 2: Deriving



Phase 3:  
Mastery

Game



# Meaningful Practice

## Games:

- Are engaging.
- Provide opportunities for strategy discussion and assessment.
- Should be sequenced developmentally (for example, playing combinations of ten games before exploring making ten strategies).
- Can be targeted practice or general practice.
- Lead to differentiation.



November  
2014 article  
discusses  
these  
games



NCTM 2015

ENRICHING  
ADDITION  
AND  
SUBTRACTION  
FACT  
MASTERY  
THROUGH

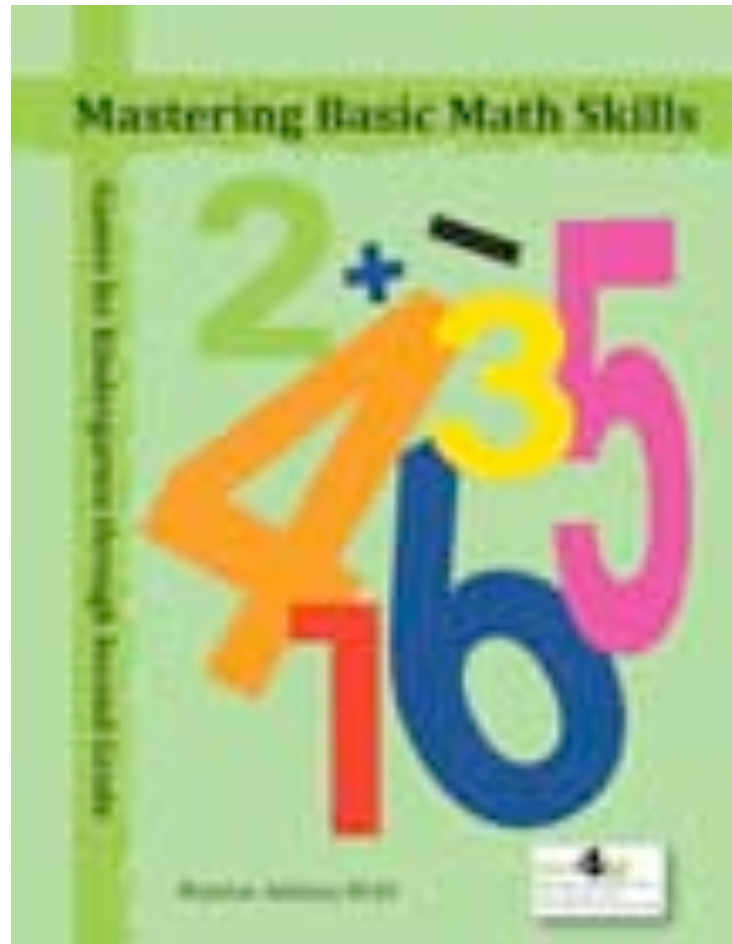
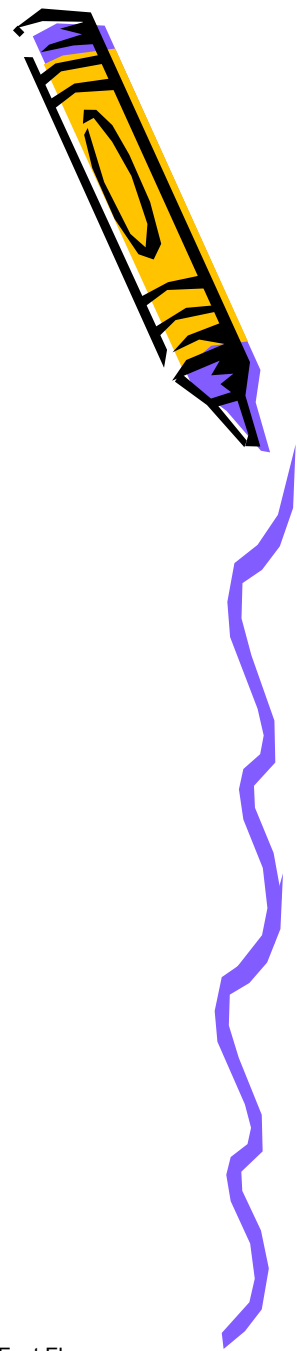
Gam



Developing and Assessing Addition Fact Fluency

Gina Kling and Jennifer Bay-Williams

# NCTM NEW Book with more Games





# Assessing Addition Fact Fluency



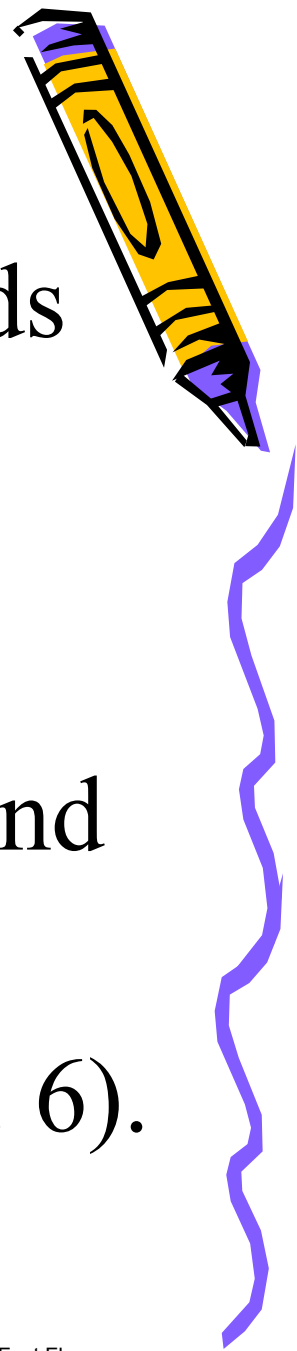
# The Common Core State Standards for Mathematics (CCSS-M)

describes procedural fluency as

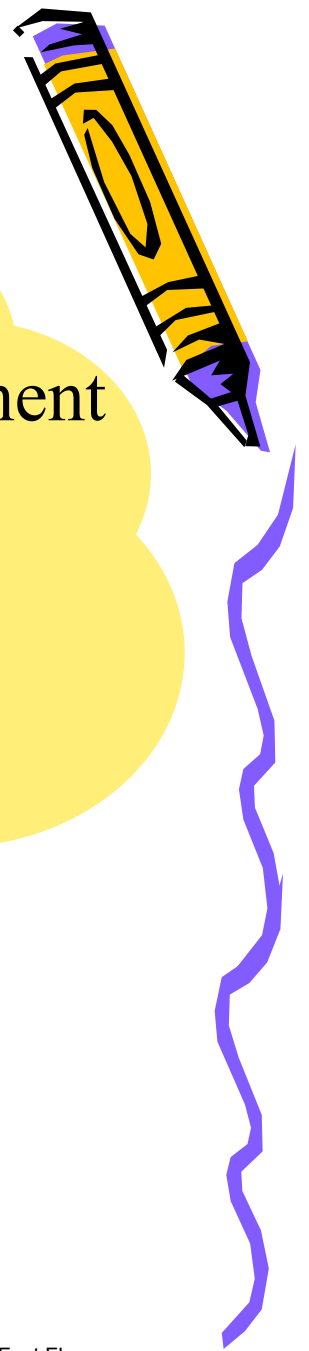
“skill in carrying out procedures

**flexibly**, **accurately**, **efficiently** and  
**appropriately**”

(CCSSO, 2010, p. 6).



# Assessing Basic Fact Fluency



What can we learn from this assessment related to:

- ✓ Flexibility
- ✓ Accuracy
- ✓ Efficiency
- ✓ Appropriate Strategy Use

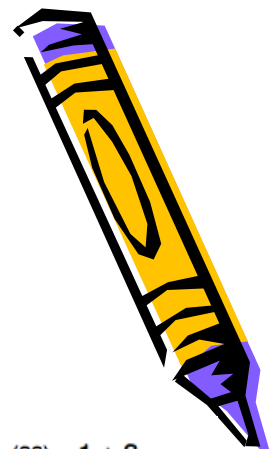


# Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

## Timed Tests

- (1)  $19 + 1 =$
- (2)  $9 + 4 =$
- (3)  $5 + 4 =$
- (4)  $8 + 4 =$
- (5)  $14 + 4 =$
- (6)  $15 + 3 =$
- (7)  $14 + 4 =$
- (8)  $15 + 1 =$
- (9)  $14 + 2 =$
- (10)  $2 + 4 =$
- (11)  $9 + 1 =$
- (12)  $15 + 1 =$
- (13)  $16 + 1 =$
- (14)  $16 + 1 =$
- (15)  $8 + 4 =$
- (16)  $15 + 2 =$
- (17)  $9 + 1 =$
- (18)  $15 + 4 =$
- (19)  $10 + 2 =$
- (20)  $5 + 3 =$
- (21)  $14 + 3 =$
- (22)  $5 + 3 =$
- (23)  $7 + 2 =$
- (24)  $5 + 1 =$
- (25)  $9 + 2 =$
- (26)  $2 + 1 =$
- (27)  $12 + 1 =$
- (28)  $1 + 3 =$
- (29)  $5 + 4 =$
- (30)  $2 + 2 =$
- (31)  $11 + 1 =$
- (32)  $14 + 1 =$
- (33)  $7 + 4 =$
- (34)  $6 + 4 =$
- (35)  $4 + 2 =$
- (36)  $10 + 1 =$





# Timed Testing: Issues



**The issues with timed testing include:**

- 1) Limitations as an assessment tool
- 2) Can impede progress when mastering facts
- 3) Psychological effects





## Timed Testing: Issues



### 1) Limitations with respect to the four components of fluency.

A child finishes a 20-fact timed test in 60 seconds.

- Did the child spend 3 seconds on each fact?

*Or...*

- Did the child spend 1 second on 16 facts and 10 seconds each on 4 of the facts?





## Timed Testing: Issues

### 2) Can impede progress in mastering facts

A study of nearly 300 first graders found that children who were *more frequently* exposed to timed testing demonstrated *lower* progress towards knowing facts from memory than their counterparts.

Henry & Brown, 2008





## Timed Testing: Issues



### 2) Can impede progress in mastering facts

A study of 2<sup>nd</sup> and 4<sup>th</sup> graders showed that children in experimental classrooms with a focus on strategy development vastly outperformed those in the control classrooms, even on traditional timed assessments.

Thornton, 1978





## Timed Testing: Issues



### 3) Can have negative psychological effects

- The stress that children experience with timed testing is not experienced when they complete the same tasks in untimed conditions.
- “Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range.”

Boaler, 2014





## Timed Testing: Issues



### 3) Can have negative psychological effects

Anxiety over timed testing is often not related to achievement. Even high-achieving children share concerns such as “I feel nervous. I know my facts, but this just scares me.”

Boaler, 2012





## Timed Testing: Issues



### 3) Can have negative psychological effects

Children experience math anxiety as early as first grade and this anxiety is not correlated with reading achievement. This suggests that the children's anxiety is specific to mathematics, not general academic work.

Ramirez et al. 2013





## Timed Testing: Issues



### 3) Can have negative psychological effects

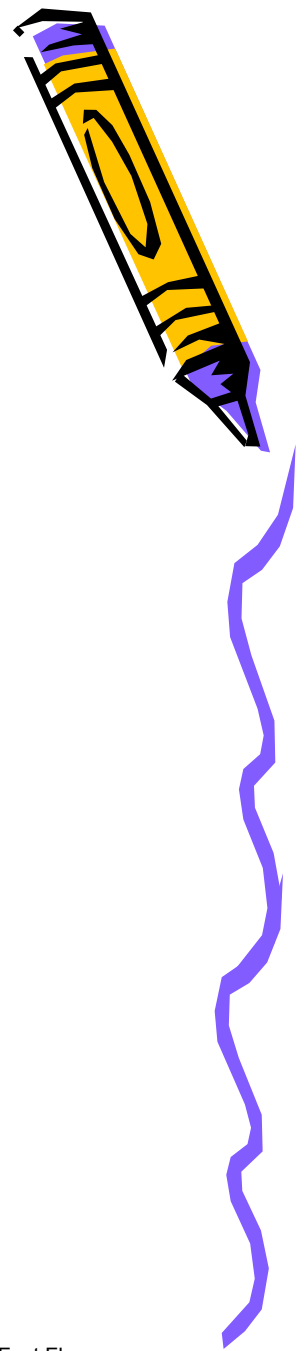
Children who tended to use more sophisticated mathematical strategies experienced the most negative impact on achievement due to math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively impacted by timed testing.



Ramirez et al. 2013



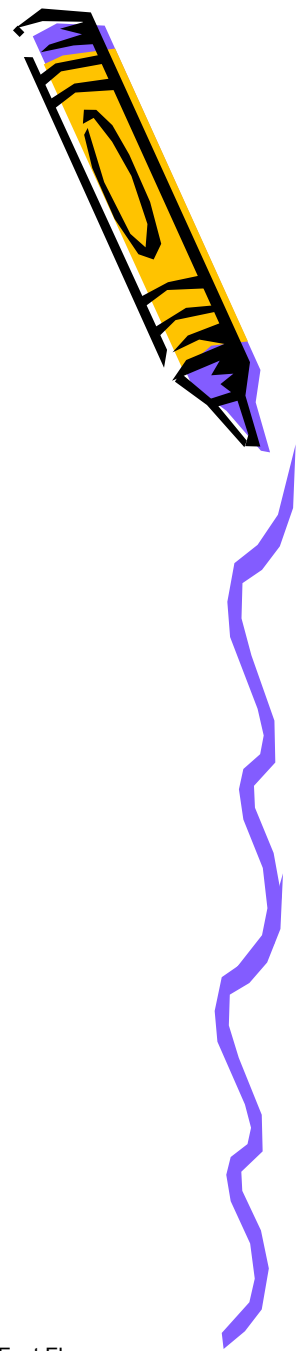
## Timed Tests: Alternatives



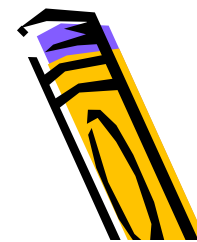
- ✓ Observation
- ✓ Interviews
- ✓ Writing prompts
- ✓ Strategy quizzes
- ✓ Self-assessment



# Salute!



# Observation Checklist



Addition Facts Fluency Chart

Date: \_\_\_\_\_

Game: \_\_\_\_\_

Student	Models and counts all	Counts on	Derived Fact	Recall (double or combo of 10)	Recall	Comments

## Aspects of Fluency

Flexibility

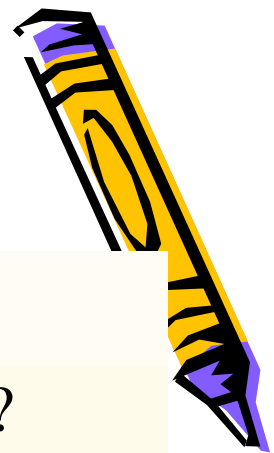
Accuracy

Efficiency

Appropriate Strategy Use



# Interviews



## Flexibility

Solve  $6 + 7$  using one strategy. Now try solving it using a different strategy.

## Accuracy

What is the answer to  $7 + 8$ ?  
How do you know it is correct (how might you check it)?

## Efficiency

For which facts did you **just know**?

For which facts did you **use a strategy**?

## Appropriate Strategy Selection

Emily solved  $6 + 8$  by changing it in her mind to  $4 + 10$ . What did she do? Is this a good strategy? Tell why or why not.





## Timed Tests: Alternatives



**Interviews:** “Riley” at the end of Grade 1

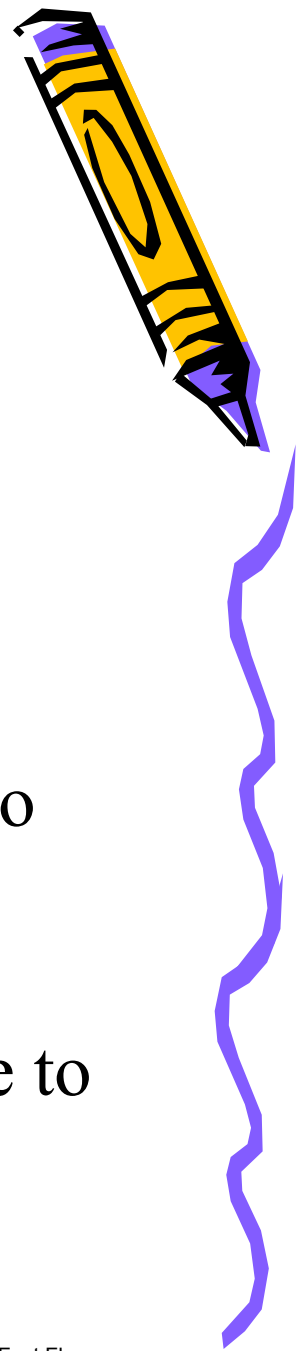
At first, Riley incorrectly answers that  $8 + 7 = 17$ .

- Which components of fluency were you able to assess through the follow-up questions?
- How does this assessment experience compare to traditional facts assessments?





## Timed Tests: Alternatives



Interviews: “Riley” at the end of Grade 2

Riley solves  $5 + 9$  in one second.

- Which components of fluency were you able to assess through the follow-up questions?
- How does this assessment experience compare to traditional facts assessments?

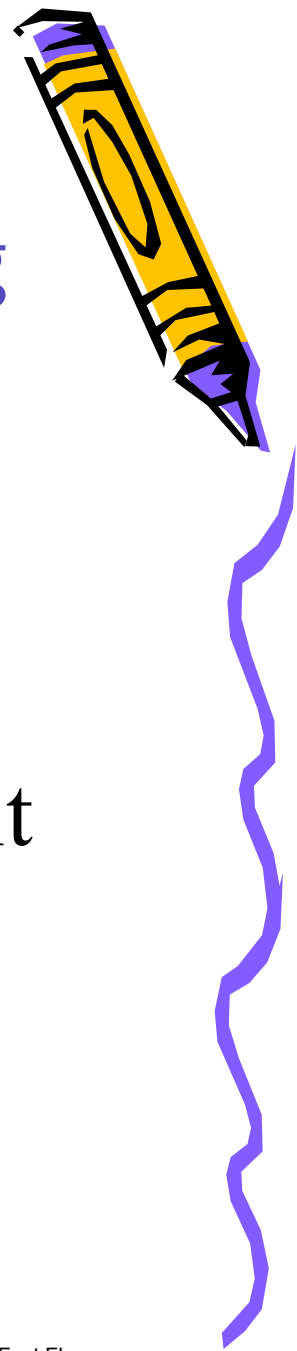


# Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

# Journal Writing

If your friend didn't know the answer to  $4 + 5$ , how would you tell him to figure it out?



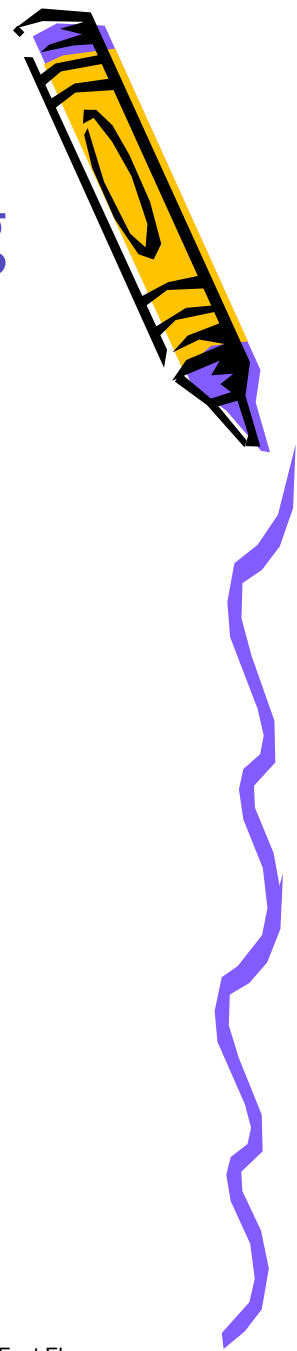
# Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

# Journal Writing

Review the four student responses:

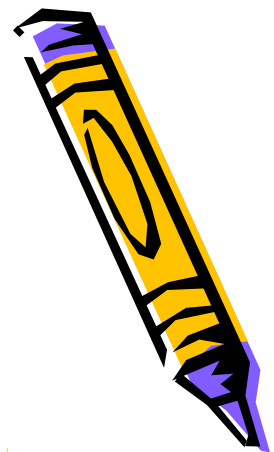
What might you infer about each child's level of fluency?





# Timed Tests: Alternatives

## Journal Writing Samples



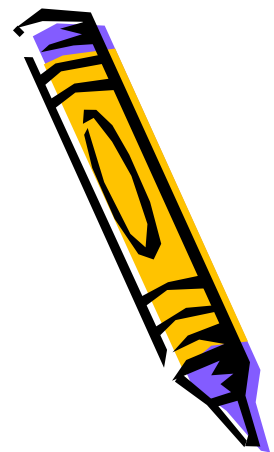
MAY 10, 2012  
I would tell my friend  
to take 5 and  
count 4 in your hand





# Timed Tests: Alternatives

## Journal Writing Samples



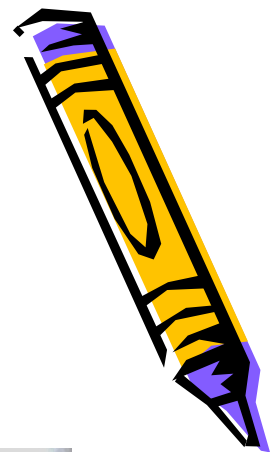
I would tell my friend to start with 5 then add 2 then one more 2 and then you have 9.





# Timed Tests: Alternatives

## Journal Writing Samples



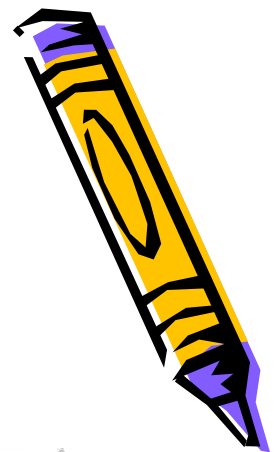
I would tell my friend to pass a double plus 1.  $4+4=8$  so count it up. now you get your answer





# Timed Tests: Alternatives

## Journal Writing Samples

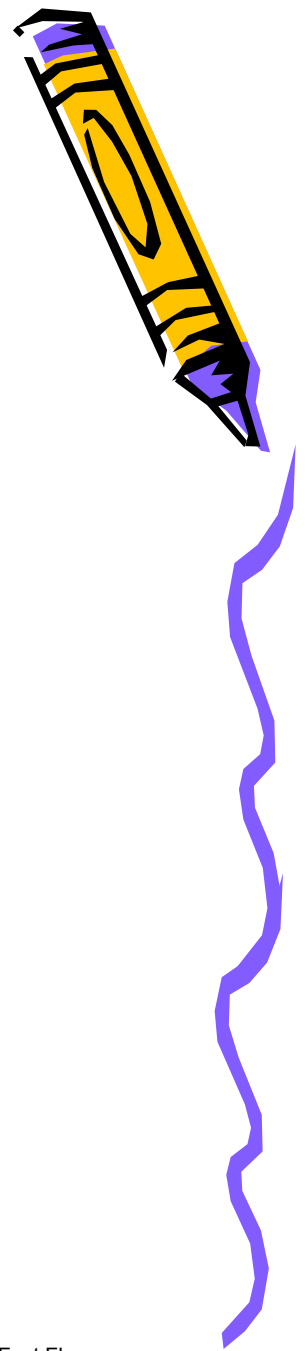


I would tell my friend  
to take away one  
number from ten.  
And that is nine.  
I know that five plus  
five equals ten.





# Tests: Alternatives



## Addition Fact Fluency Quiz

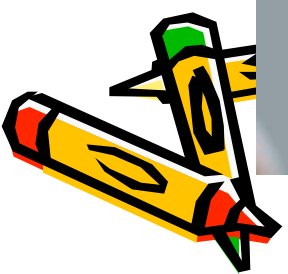

Solve these problems and tell how you solved it.

$4 + 5 =$  \_\_\_\_\_ Check one: \_\_\_\_\_ I used this strategy: \_\_\_\_\_  
 \_\_\_\_\_ I just knew.

$10 + 6 =$  \_\_\_\_\_ Check one: \_\_\_\_\_ I used this strategy: \_\_\_\_\_  
 \_\_\_\_\_ I just knew.

$6 + 2 =$  \_\_\_\_\_ Check one: \_\_\_\_\_ I used this strategy: \_\_\_\_\_  
 \_\_\_\_\_ I just knew.





# Assessing Basic Fact *Fluency*

**Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.**

By Gina Kling and Jennifer M. Bay-Williams



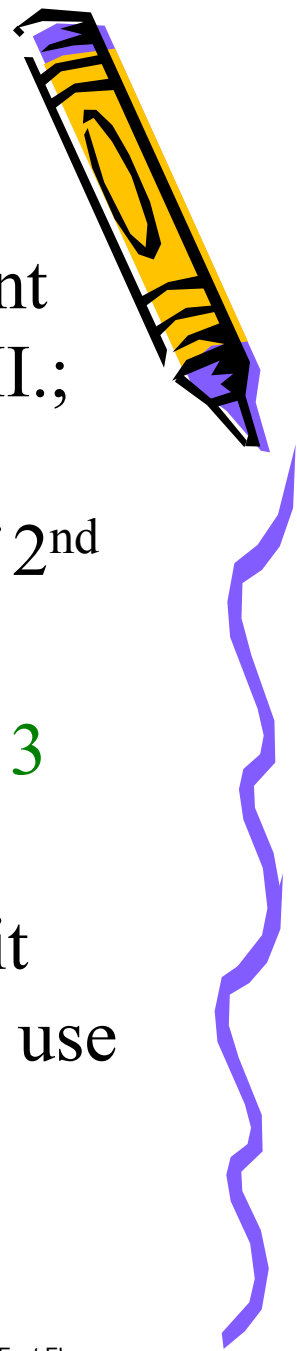
# What Works with Addition Fact Fluency



# Promising Results

Kling (2013) followed 30 children from 2 different schools, 4 different classrooms, in Kalamazoo, MI.; 21 had no exposure to timed testing or drill in the classroom in either 1<sup>st</sup> or 2<sup>nd</sup> grade. By the end of 2<sup>nd</sup> grade those 21 children demonstrated:

- automaticity with addition facts (solved within 3 seconds) 95% of the time.
- Strategy use (e.g., making ten) so quickly that it was impossible to distinguish between strategy use and “knowing from memory.”



## ... and RETENTION



18 of the children were interviewed once more in the first week of 3<sup>rd</sup> grade *prior to any fact strategy review*.

- Children demonstrated automaticity 91% of the time.
- Were accurate and used strategies (not counting) 99.99% of the time.

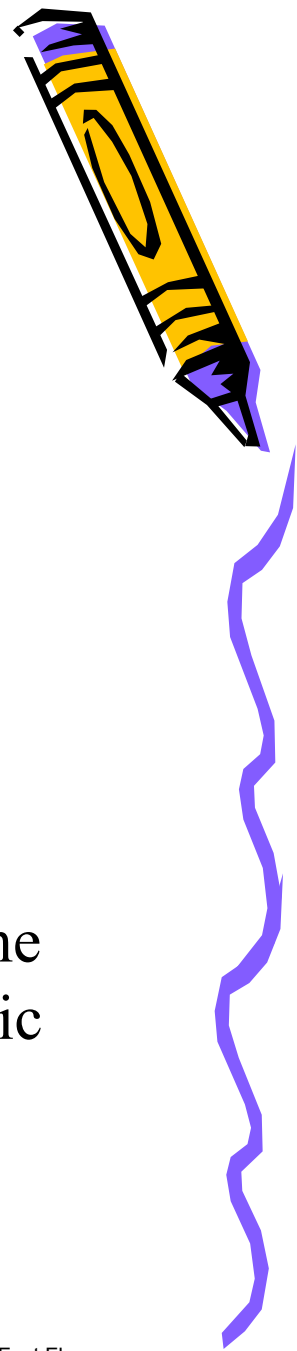


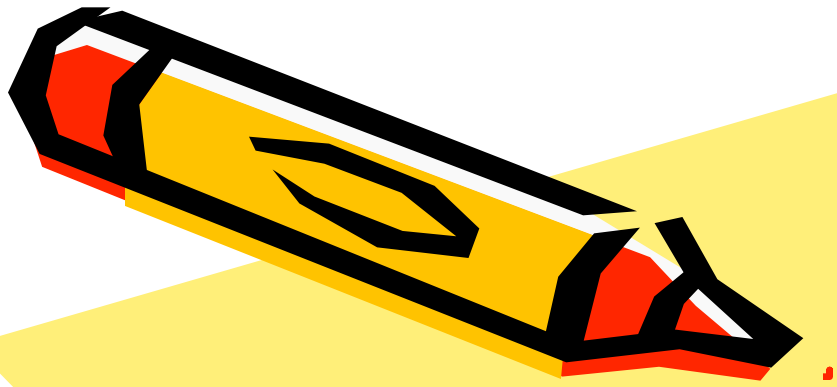
# Conclusions

Basic facts instruction, practice, and assessment must truly encompass all four components of fluency:

- **Flexibility**
- **Accuracy**
- **Efficiency**
- **Appropriate strategy use**

Traditional approaches to teaching and assessing basic facts do not support these goals. But when *fluency* is the focus, children can achieve meaningful mastery of basic facts.





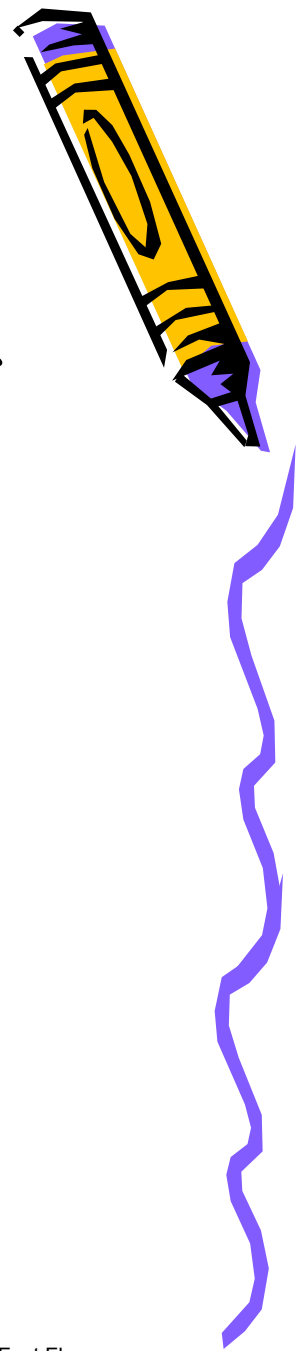
# Assessing Your Fact [teaching] Fluency



# Whip Around

At your table, share your answer to one of these prompts:

- An activity I will use is...
- An assessment strategy I will use is...
- Something surprising I heard is...
- I am going to/not going to...



# Bibliography

Bay-Williams, J. and G. Kling (2014). Enriching Addition and Subtraction Fact Mastery through Games. *Teaching Children Mathematics* 21(4): 238-247.

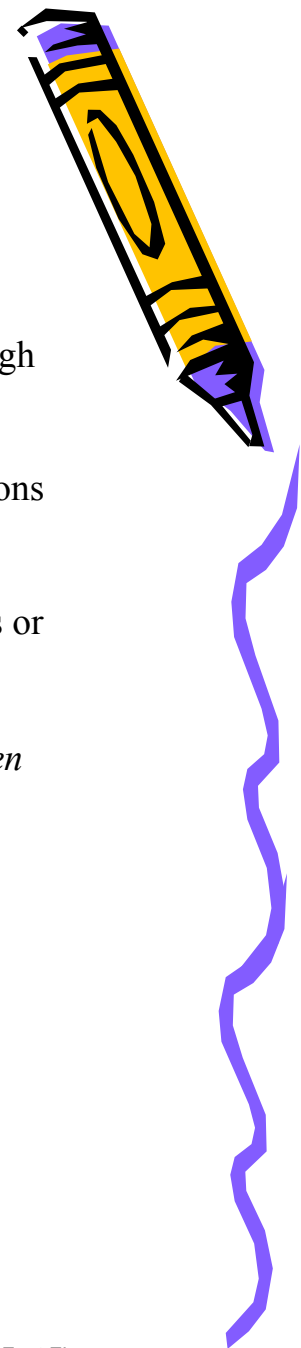
Baroody, A. (2006). Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them. *Teaching Children Mathematics* 13(1): 22-31.

Baroody, A. (1985). Mastery of Basic Number Combinations: Internalization of Relationships or Facts? *Journal for Research in Mathematics Education* 16(2): 83-98.

Boaler, Jo. (2014) Research Suggests That Timed Tests Cause Math Anxiety. *Teaching Children Mathematics* 20(8): 469-474.

Boaler, Jo. 2012. “Timed Tests and the Development of Math Anxiety” *Education Week*, online July 3, 2012.

Carpenter, T. and J. Moser (1984). The Acquisition of Addition and Subtraction Concepts in Grades One through Three. *Journal for Research in Mathematics Education* 15(3): 179-202.



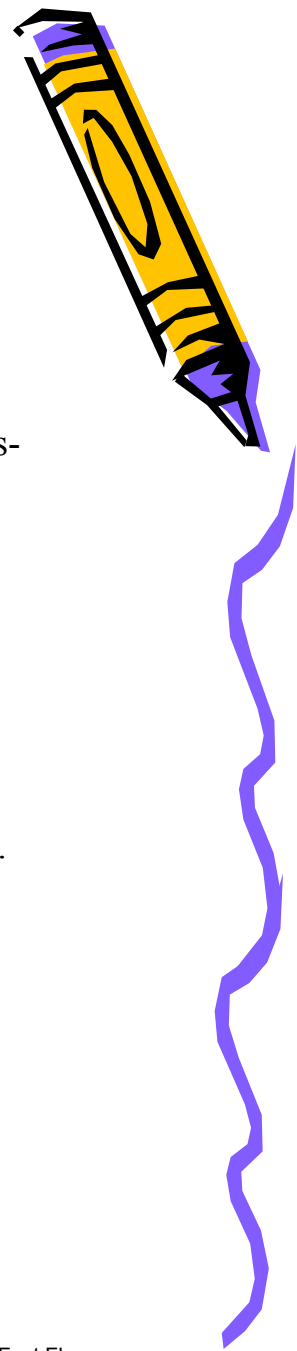
# Bibliography

Grade 1 Mathematics: Operations and Algebraic Thinking: Common Core State Standards Initiative, 2010. <http://www.corestandards.org/the-standards/mathematics/grade-1/operations-and-algebraic-thinking/>

Grade 2 Mathematics: Operations and Algebraic Thinking: Common Core State Standards Initiative, 2010.  
<http://www.corestandards.org/the-standards/mathematics/grade-2/operations-and-algebraic-thinking/>

Fuson, K. and Y. Kwon (1992). Korean Children's Single-Digit Addition and Subtraction: Numbers Structured by Ten. *Journal for Research in Mathematics Education* 23(2): 148-165.

Henry, V. and R. Brown (2008). First-Grade Basic Facts: An Investigation Into Teaching and Learning of an Accelerated, High-Demand Memorization Standards. *Journal for Research in Mathematics Education* 39(2): 153-183.



# Bibliography

Kling, Gina and Jennifer Bay-Williams (2014). Assessing Basic Fact *Fluency*. *Teaching Children Mathematics* 20(8): 488-497.

Kling, Gina (2011). Fluency with Basic Addition. *Teaching Children Mathematics* 18(2): 80-88.

Mokros, J., S. Russell, & K. Economopoulos (1995). Shouldn't Students Memorize the Basic Math Facts? *Beyond Arithmetic*. White Plains, NY: Dale Seymour Publications.

National Council of Teacher of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, Inc.

National Council of Teachers of Mathematics (2003). *A Research Companion to Principles and Standards for School Mathematics*. J. Kilpatrick, W. Martin, D. Schifter (Eds.). Reston, VA: National Council of Teachers of Mathematics, Inc.



# Bibliography

National Research Council (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Ramirez, Gerardo, Elizabeth A. Gunderson, Susan C. Levine, and Sian L. Beilock. 2013. "Math Anxiety, Working Memory, and Math Achievement in Early Elementary School." *Journal of Cognition and Development* 14(2): 187-202.

Steinberg, R. (1985). Instruction on Derived Facts Strategies in Addition and Subtraction. *Journal for Research in Mathematics Education* 16(5): 337-355.

Thornton, C. (1978). Emphasizing Thinking Strategies in Basic Fact Instruction. *Journal for Research in Mathematics Education* 9(3): 214-227.

Wheatley, G. and A. Reynolds (1999). Thinking in Units. *Coming to Know Number: A Mathematics Activity Resource for Elementary Teachers*. Bethany Beach, DE: Mathematical Learning.

