National Council of Teachers of Mathematics
Annual Meeting & Exposition
April 15-18, 2015

Session 618

Supporting the Development of Effective Teaching Practices

Peg Smith
University of Pittsburgh

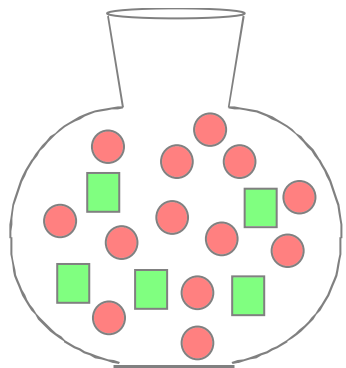
These materials are part of the ***Principles to Actions*** **Professional Learning Toolkit: Teaching and Learning** created by the team that includes: Margaret Smith (Chair) and Victoria Bill (co-chair), Melissa Boston, Fredrick Dillon, Amy Hillen, DeAnn Huinker, Stephen Miller, Lynn Raith, Margaret Smith (chair), and Michael Steele. This project is a partnership between the National Council of Teachers of Mathematics and the Institute for Learning at the University of Pittsburgh.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
www.nctm.org

INSTITUTE for LEARNING
UNIVERSITY OF PITTSBURGH

The Candy Jar Task

The candy jar shown below contains 5 Jolly Ranchers (rectangles) and 13 Jawbreakers (circles). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know.



Jawbreakers and Jolly Ranchers: The Case of Catherine Carlson

Students in Mrs. Carlson's seventh-grade class were solving the following problem: *"A candy jar contains 5 Jolly Ranchers and 13 Jawbreakers. Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know."* Mrs. Carlson told her students that they could solve the problem any way they wanted, but she emphasized that they needed to be able to explain how they got their answer and why it worked.

As students worked in pairs to solve the problem, Mrs. Carlson walked around the room making sure that students were on task and making progress on the problem. She was pleased to see that students were using lots of different approaches to the problem –drawing pictures, grouping manipulatives, making tables, and writing explanations.

She noticed that one pair of students had gotten the wrong answer as shown below.

Jordan and Kate

100 JR is 95 more than the 5 we started with. So we will need 95 more JB than the 13 we started with.

5 JR + 95 JR = 100 JR
13 JB + 95 JB = 108 JB

Mrs. Carlson wasn't too concerned about the incorrect response, however, since she felt that once several correct solution strategies were presented, these students would see what they did wrong and have new strategies for solving similar problems in the future.

When most students were finished, Mrs. Carlson called the class together to discuss the problem. She began the discussion by asking for volunteers to share their solutions and strategies, being careful to avoid calling on the students with an incorrect solution. Over the course of the next 15 minutes, first Owen, then Ellen, Ricardo, Alicia, Jerry, and Kamiko volunteered to present the solutions to the task that they and their partners had created. Their solutions are shown on the back.

During each presentation, Mrs. Carlson made sure to ask each presenter questions that helped the student to clarify and justify the work. She concluded the class by telling students that the problem could be solved in many different ways and now, when they solved a problem like this, they could pick the way they liked best because all the ways gave the same answer.

Owen and Joshua

You have to multiply the five JR by 20 to get 100, so you'd also have to multiply the 13 JB by 20 to get 260. So it has to be 260.

Ellen and Adam

We started drawing candy jars and keeping track of the number of JR we had. Every jar had 5 JR. So when we had 20 jars we knew that we had the 100 JR that would be in the new jar. So we added 20 13's together and got 260. So there would be 260 JB in the new jar.

Ricardo and Melissa

JR	5	10	20	40	80	100
JB	13	26	52	104	208	260

We started by doubling both the number of JR and JB. But then when we got to 80 JR we didn't want to double it anymore because we wanted to end up at 100 JR and doubling 80 would give us too many. So we noticed that if we added 20 JR: 52 JB and 80 JR: 208 JB we would get 100 JR:260 JB.

Alicia and Max

We counted out 100 red cubes because that is how many JR were in the new jar. Then we put them in groups of 5 because each little jar had 5 JR in it. Then we got out green cubes and started to put 13 green cubes with each of the piles of 5 red cubes. But we didn't have enough. But then we saw that there would have to be 13 in each pile and there were 20 piles so we multiplied 13 times 20 and got 260.

Jerry and Nicole

Since the ratio is 5 JR for 13 JB, for each JR you would have 2 JB; that would use up 10 JB. So you have three JB left over. So we had to distribute the three JB to the 5 JR. 3 ÷ 5 = .6 so that would give the ratio of 1 JR to 2.6 JB. So then you just multiply 1 and 2.6 each by 100.

(x100)

1 JR → 100 JR

2.6 JB → 260 JB

(x100)

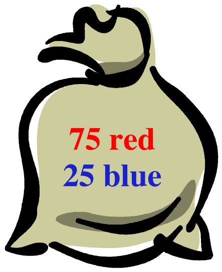
Kamiko and Mike

JR	JB
5	13
10	26
15	39
20	52
25	65
30	78
35	91
40	104
45	117
50	130
55	143
60	156
65	169
70	182
75	195
80	208
85	221
90	234
95	247
100	260

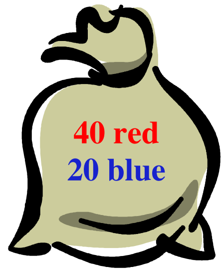
We just kept adding 5 to the JR column and 13 to the JB column. We stopped when we got to 100 JR. So it has to be 260 JB.

Bag of Marbles

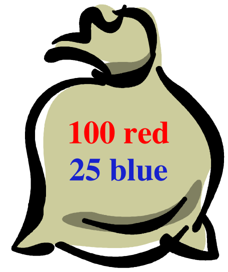
Ms. Rhee's math class was studying statistics. She brought in three bags containing red and blue marbles. The three bags were labeled as shown below:



Bag X
Total = 100 marbles



Bag Y
Total = 60 marbles



Bag Z
Total = 125 marbles

Ms. Rhee shook each bag. She asked the class, "If you close your eyes, reach into a bag, and remove 1 marble, which bag would give you the best change of picking a blue marble?

Which bag would you choose? ____________

Explain why this bag gives you the best choice of picking a blue marble. You may use the diagrams above in your explanation.

The QUASAR assessment team, under the direction of Dr. Suzanne Lane, created this task. It appeared on the QCAI (QUASAR Cognitive Assessment Instrument) at grades 6-7-8.

A

Bag x is 1/3 blue.
Bag y is ½ blue.
Bag z is ¼ blue.

½ is a lot, so it must be Bag y.

B

I found the % of blue marbles in each bag.

X 25/100 = 25%
Y 20/60 = 33 1/3 %
Z 25/125 = 20%

So I chose Y 'cause 33 1/3 % is more than the others.

C

X 75/25 = 3/1 = 3
Y 40/20 = 2/1 = 2
Z 100/25 = 4/1 = 4

I picked bag z because 4 is the biggest which means your chances of getting a blue marble are better.

D

Because bag Y is 1/3 full of blue marbles and bag X is only ¼ full of blue marbles and bag Z only 1/5 full of blue marbles. I choose bag Y because 1/3 is biggest.

E

Bag x is ¼ blue and bag y is 1/3 blue.
Better chance with bag y.

Bag y has 1 blue to 2 reds.
Bag z has 1 blue to 4 reds.

Better chance with bag y.

F

The x bag has 75 red and 25 blue.
There are 50 extra marbles that are red.

The z bag has 100 red and 25 blue. There are 75 extra red than blue.

The y bag has 40 red and 20 blue. There are 20 extra red than blue.

So I picked bag y because there are less extra reds that you might get.

G

Notice in the first bag there are 75 red and 25 blue, that is a 1:3 chance.

In the second bag there are 40 red and 20 blue, so that is a 1:2 chance.

In the third bag there are 100 red and 25 blue. That is a 1:4 chance.

That shows that in bag Y you would have a better chance to pick a blue marble.

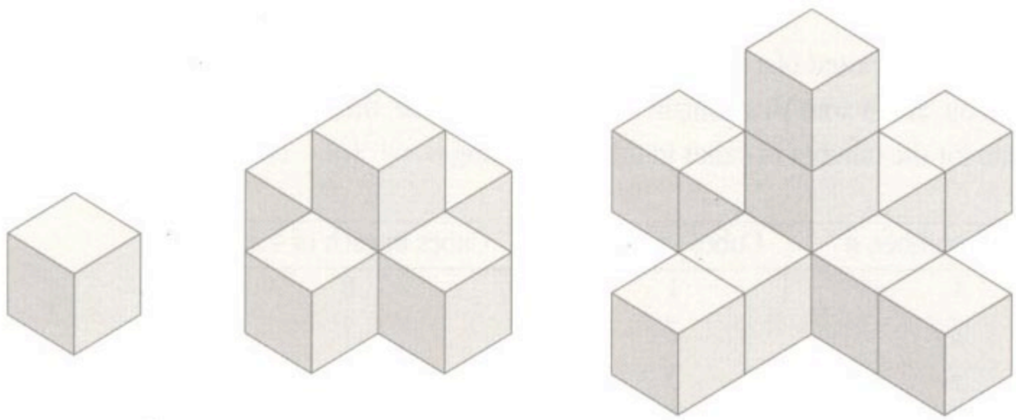
H

Bag X has 75 red and 25 blues and bag Z has 100 red and 25 blues.

In bags X and Z the blues are the same, so then you would have to look at the red to see which one is the least between them. Bag X has 75 red and 75 is less than 100. So bag X is better than Z because there are the same blues but less reds.

Bag X is better than bag Y because bag Y has less marbles than X and fewer blue ones. So X is best.

Counting Cubes Task



Building 1 Building 2 Building 3

1. Describe a pattern you see in the cube buildings.

2. Use your pattern to write an expression for the number of cubes in the nth building.

3. Use your expression to find the number of cubes in the 5th building. Check your results by constructing the 5th building and counting the cubes.

4. Look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the nth building.

Adapted from "Counting Cubes", Lappan, Fey, Fitzgerald, Friel, & Phillips (2004). *Connected MathematicsTM, Say it with symbols: Algebraic reasoning* [Teacher's Edition]. Glenview, IL: Pearson Prentice Hall.