INVESTIGATION 3.1:
A Moving Melody

Did You Hear...

The oldest known piece of written music was discovered in the early 1950s during the excavation of an ancient city in present-day Syria. An excerpt from this music is shown below. The original writing was in *cuneiform*, which used wedge-shaped clay impressions created by the edge of a blunt reed.

The cuneiform writing above the double horizontal lines were the words to this "hymn," while the writing below the lines were instructions for playing the music. The translation of these instructions into modern musical notation, achieved by Professor Anne Draffkorn Kilmer of the University of California, shows not only a melody line, but a harmony line as well. Before this discovery, it was believed that the ancient Greeks were the first to use harmony in their music – but that was just over 2000 years ago, and the tablets on which this music was written are more than 3400 years old!

"The Oldest Song in the World." blog.wmfu.org. WFMU's Beware of the Blog, n.d. Web. 18 Mar. 2014. <http://blog.wfmu.org/freeform/2006/12/the_oldest_song.html>

A MOVING MELODY (CONTINUED)

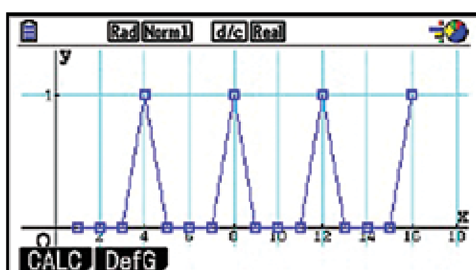
In 1975, Denys Parsons developed a system for recording the motion of pitches.[1] In his "Parsons Code for Melodic Contours," an asterisk (*) represented the first note, the letter U indicated to move up to get to the next pitch, D indicated to move down to the next pitch, and R indicated to repeat the same pitch. For example, listen to the first few measures of *Tiger Rag*, the fight song for Clemson University (and some other schools). [Insert audio clip of *Tiger Rag*.] The Parsons Code for this song would be represented as:

*** R R U D R R U D R R U D R R U**

It is important to realize that the letters U and D do *not* indicate how *much* to move up or down, only the direction. Also, there is no indication of rhythm.

We can use the Parsons Code to create a mathematical relation, where each *x*-value represents a note's position in the melody and each y-value represents its pitch relative to the previous note. Let's call the first note (the asterisk in the Parsons Code) Note 1 and Pitch 0. For each successive note, increase the y-value by 1 if the pitch ascends, leave the *y*-value alone if the pitch remains the same, and decrease the *y*-value by 1 if the pitch descends. The first few notes in *Tiger Rag* that define the relation follow: {(1, 0), (2, 0), (3, 0), (4, 1), (5, 0), (6, 0), (7, 0), (8, 1), (9, 0), (10, 0), (11, 0), (12, 1), (13, 0), (14, 0), (15, 0), (16, 1)}. Below is an XY-Line graph of these ordered pairs. (Though the data are discrete, we have selected an XY-Line graph to better show the changes.)

[1] "The Parsons Code." musipedia.org. Musipedia Melodic Search Engine, n.d. Web. 18 Mar. 2014. <http://www.musipedia.org/pc.0.html?&L=0>



A MOVING MELODY (CONTINUED)

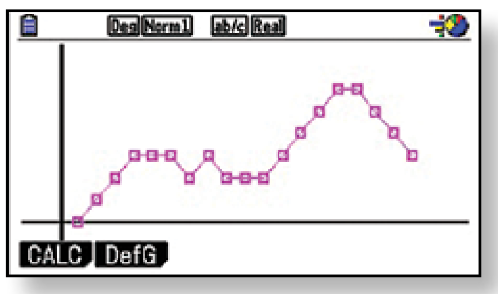
A Hum a few measures of *Twinkle, Twinkle, Little Star*. Then write the Parsons Code that represents the first 14 notes of this melody.

B Construct a relation of ordered pairs from your Parsons Code as described above. Explain why some negative values are necessary.

C Construct an XY-Line graph for the Parsons Code you've just created. Discuss what the graph reveals.

D Think about two of your favorite songs. Construct the Parsons Code for at least the first 16 notes of each melody. Convert each 16-character code into a mathematical relation, and then construct two XY-Line graphs on the same set of axes. Comment on the similarities and differences you find in the graphs.

E Consider the XY-Line graph below. Convert the graph into a set of ordered pairs, and then translate these into a Parsons Code.



F Determine the probability that two melody fragments, each of which contains eight notes, would have the same Parsons Code. Discuss the implications of your findings.

SAMPLE SOLUTION 3.1: **A Moving Melody**

A Hum a few measures of *Twinkle, Twinkle, Little Star*. Then write the Parsons Code that represents the first 14 notes of this melody.

We assume that everyone is familiar with *Twinkle, Twinkle, Little Star*, though we *have* found that some students did not realize it has the same melody as the Alphabet Song. In any case, before writing the Parsons Code version, students should sing or hum the melody a couple of times, or even just play the melody on a keyboard or other melodic instrument. Recall that the first note is designated by *, and then each pitch is designated by U if the melody goes higher, D if it goes lower, or R if it remains the same. The Parsons Code for the first 14 notes is * R U R U R D D R D R D R D, which covers the lyrics "Twinkle, twinkle, little star; how I wonder what you are."

B Construct a relation of ordered pairs from your Parsons Code as described above. Explain why some negative values are necessary.

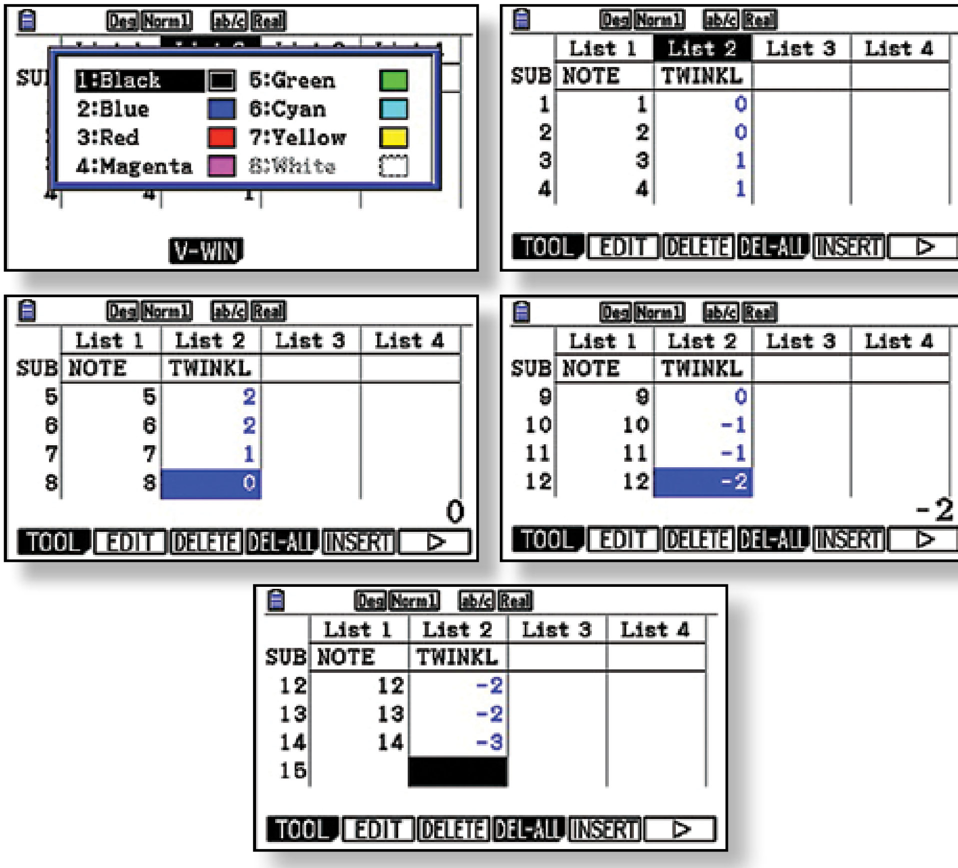
We begin with the ordered pair (1, 0), as done in the example in the Introduction. Again, each *x*-value represents the note's position in the melody, and each *y*-value represents whether the note is higher than, lower than, or the same as the previous pitch:

{(1, 0), (2, 0), (3, 1), (4, 1), (5, 2), (6, 2), (7,1), (8, 0), (9, 0), (10, -1), (11, -1), (12, -2), (13, -2), (14, -3)}.

Negative values are necessary because in this melody, there are more occurrences of the pitch moving lower than of the pitch moving higher. Note that this can help students develop an intuitive understanding of operations with negatives, such as why -1 – 1 = -2.

We'll now enter these values into the calculator. From the Main Menu, select Statistics. In **List 1,** enter the *x*-values (pressing EXE after each entry), and in **List 2**, enter the *y*-values. See below:

A MOVING MELODY (CONTINUED)

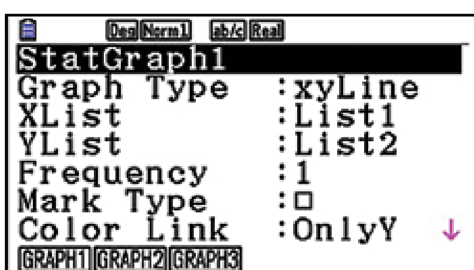


To color **List 2** blue as shown, highlight the header for **List 2**, press SHIFT 5 (Format) and then 2 (Blue). If desired, you can also type in a SUB-title for each list. Press SHIFT ALPHA to enter Alpha-lock, and then type in the desired subtitle.

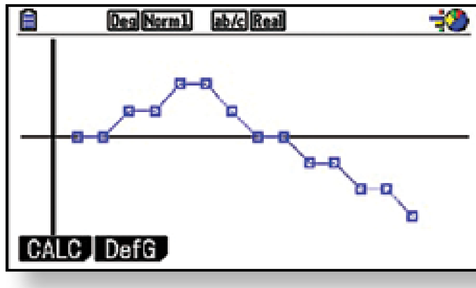
C Construct an XY-Line graph for the Parsons Code you've just created. Discuss what the graph reveals.

Continuing from the last screenshot above, press SHIFT EXIT (QUIT) to return to the home Statistics screen. Then:

- Select F1 (GRAPH) and F6 (SET). Set up **StatGraph1** as shown below:



- Press EXIT and F1 (GRAPH1).

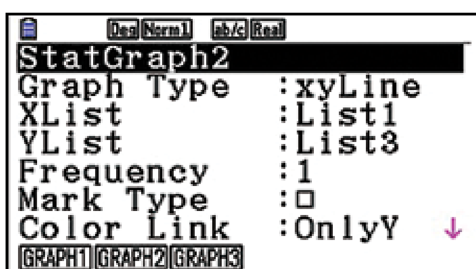


The graph reveals a melody that is mostly descending. We can tell this because our final y-values are all negative, and increasingly so. Contrast this with the example melody given in the introduction (*Tiger Rag*), which required no negative y-values.

D Think about two of your favorite songs. Construct the Parsons Code for at least the first 16 notes of each melody. Convert each 16-character code into a mathematical relation, and then construct two XY-Line graphs on the same set of axes. Comment on the similarities and differences you find in the graphs.

Answers, of course, will vary. We encourage you to have students share and discuss their songs, the codes they have created, the conversion to ordered pairs, and the combined XY-Line graph.

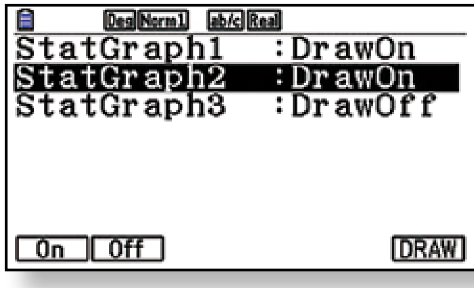
We will assume that the note positions remain in **List 1**, and the relative pitch numbers from the code from the first song are in **List 2**, while the pitch numbers of the code from the other song are in **List 3**. **List 2** is blue, and **List 3** we've made red. To view both graphs together, from the Statistics home screen, select F1 (GRAPH) and F6 (SET). **StatGraph1** has already been set up properly, so we only need to set up **StatGraph2** as shown:



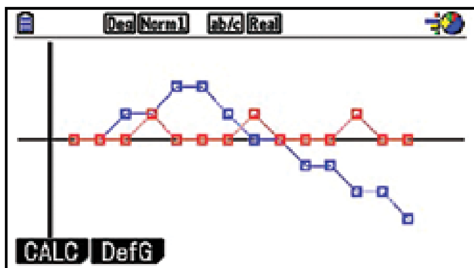
- Next, press EXIT, and F4 (SELECT).

A MOVING MELODY (CONTINUED)

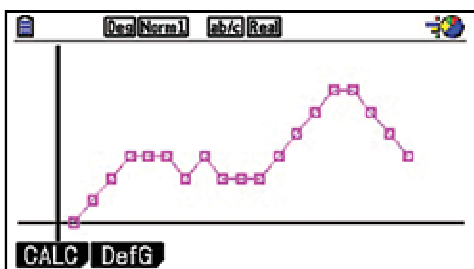
- Make sure **StatGraph1** and **StatGraph2** are both turned On as shown:



- Now, simply select F6 (DRAW) and both graphs should be displayed. The example below shows the graphs of *Twinkle, Twinkle* in blue and *Tiger Rag* (just the first 14 notes) in red. (Students' graphs will be longer, as they will involve 16 notes.)



E Consider the XY-Line graph below. Convert the graph into a set of ordered pairs, and then translate these into a Parsons Code.



This gives students experience with reading coordinates on a graph and interpreting the values. The ordered pairs are shown below.

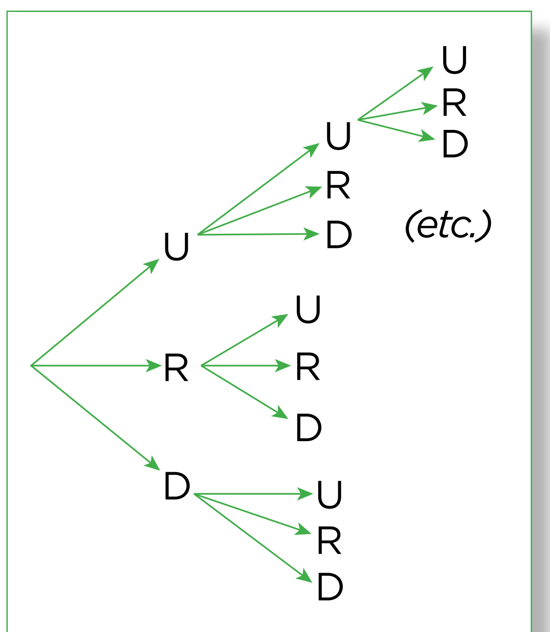
{(1, 0), (2, 1), (3, 2), (4, 3), (5, 3), (6, 3), (7,2), (8, 3), (9, 2), (10,2), (11, 2), (12, 3), (13, 4), (14,5), (15, 6), (16, 6), (17, 5), (18, 4), (19,3}.

A MOVING MELODY (CONTINUED)

For those who are interested, the melody is the first two phrases from *You Are My Sunshine*. Here's a clip of this song if it is not familiar to you: [Insert audio clip of "You Are My Sunshine."]

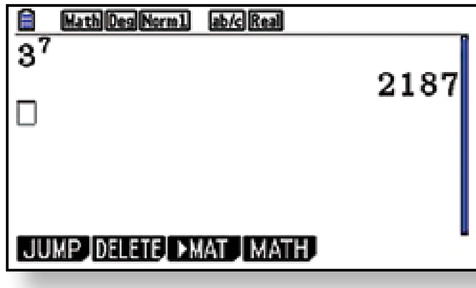
F Determine the probability that two melody fragments, each of which contains eight notes, would have the same Parsons Code. Discuss the implications of your findings.

Every Parsons Code begins with an asterisk (*). We then have 7 additional notes that we are considering. For each of these, there are three choices: U (if the pitch goes up), D (if the pitch goes down), and R (if the pitch remains the same). Teachers may want to use this opportunity to develop the Multiplication Principle associated with probability. For example, there are 3 ways to get from the first note to the second note; to get from the first note to the third note, there are 3 × 3, or 9 ways. Why? From *each* of the 3 options for the second note, there are 3 options for the third note. Similarly, to get to the fourth note, each of the 9 options to get to the third note has 3 options, making a total of 27 options.

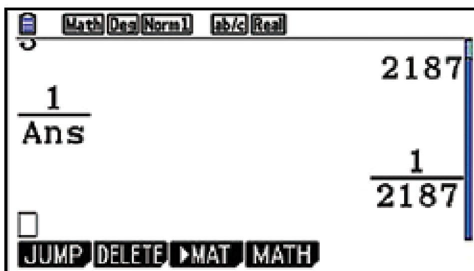


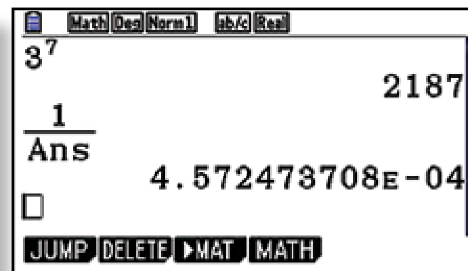
This suggests that if we have 7 notes' worth of possible changes, then there will be 3^7, or 2187 possible Parsons Codes.

A MOVING MELODY (CONTINUED)

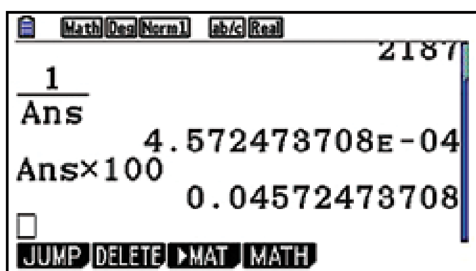


The probability that a second melodic fragment of 8 notes would match this would be 1 out of 2187, which is shown below. To see this probability expressed as a decimal, press the F↔D (Form to Decimal) key:





To convert to a percent, simply multiply by 100 and press F↔D:



This tells us that two melodic fragments of 8 notes have less than a 1% (less than *half* of one percent, for that matter) probability of matching. So it's *extremely* unlikely that two such Parsons Codes will match. Also, keep in mind that the Parsons Codes do not consider how *much* the melody goes up or down, *or* the rhythm involved! Consequently, it should be easy to make two songs sound different. This can even be used as a legal argument when some-one is accused of stealing another person's copyrighted melody.

INVESTIGATION 3.2: **How Low Did You Go? How High Did You Fly?**

Did You Hear...

Quick, think – what musical instrument has the greatest range? In other words, if we looked at the interval between the lowest and highest possible notes that can be produced, for which instrument would this interval be the *widest*?

Was the first instrument of which you thought a string instrument, like the violin? Violins and cellos have about a 4-octave range, but did you consider the *harp*? It has a range of more than 6 octaves!

What about the piano? It has a range of more than 7 octaves, but the *pipe organ* has a whopping 9-octave range, capable even of playing notes lower (16 Hz) than the typical human ear can hear (20 Hz).

But, in fact, the answer to this question isn't even man-made. The average human voice has about a 2-octave range, but amazing things happen when it is trained and stretched to the bounds of its capability. The Guinness World Record for the widest human vocal range is 10 octaves (0.7973 Hz – 807.3 Hz), achieved by Tim Storms in Missouri in 2008!

"Frequency Range of Instruments." hometheatershack.com. Home Theater Forum and Systems, n.d. Web. 22 Mar. 2014. <http://www.hometheatershack.co forums/diy-subwoofers-general-discussion/27171-frequency-range-instruments.html>
"Greatest vocal range, male." Guinness World Records, n.d. Web. 22 Mar. 2014. <http://www.guinnessworldrecords.com/records-3000/greatest-vocal-range-male/>

INVESTIGATION 3.2: **How Low Did You Go? How High Did You Fly?**

In the previous investigation, we explored whether melodies were going up, down, or staying the same. However, as students should have realized, we ignored how *much* they went up or went down when they moved from one pitch to another. In this investigation, we will take the amount of movement into account, measuring from the tonic note of the scale (the first note of the scale but not necessarily the first note of the melody) in half-steps. We will assign the value of 0 to one particular tonic note found in the melody, with 1 indicating one half-step above the tonic and -1 representing one half-step below the tonic. For the purposes of this investigation, we are ignoring rhythms.

A Hum a few measures of *Twinkle, Twinkle, Little Star.* Then play the melody, find a copy of the music, or determine for yourself what notes are being played. This melody starts on the tonic pitch ("base" note) of the musical scale, so label the first note as 0. Now label the other notes in the melody based on how many half-steps each is above the tonic. (e.g. If C is 0, then we'll consider C# as 1, D as 2, D# as 3, and so on.) Using this system, write down the numerical label for each of the first 14 notes of the melody.

B Construct an XY-Line graph for the table you've just created. Discuss what the graph reveals about the *Twinkle, Twinkle* melody.

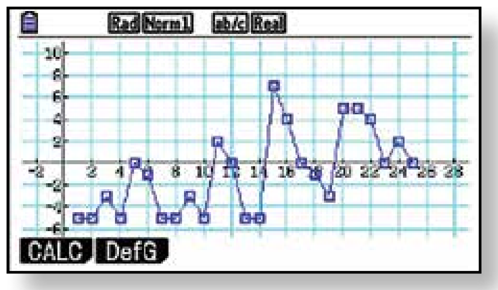
C Discuss the meaning of points that lie on, above, and below the x-axis. Why are there no points on or to the left of the y-axis?

D Discuss how the table and graph you have created can be used to transpose a song from one key to another. For example, what could you do mathematically if you wanted to play *Twinkle, Twinkle, Little Star* in the key of D instead of the key of C?

E Investigate the first-order differences for the first 14 notes of *Twinkle, Twinkle, Little Star.* What do these differences indicate about the melody line? What is the effect on the first-order differences when a song is transposed? How might these differences be helpful to someone learning to play the song on a keyboard?

F Think about one of your favorite songs. Construct a table of half-step values for at least the first 16 notes of the melody, including a column that shows the first-order differences. Then, create graphs, share them with others, and see if they can determine which song you selected.

G Consider the XY-Line graph below. Convert the graph into a table of values and then see if you can determine what this well-known melody is. Is there any indication of what the key signature might be? Explain.



H Explore some of your favorite melodies and determine if they end on the tonic pitch of their scale. In other words, if a song is written in the key of C, does it end on a C? Use your results to formulate a conjecture about the probability that any particular song will end on the tonic. Test your conjecture and report your results.

SAMPLE SOLUTION 3.2: **How Low Did You Go? How High Did You Fly?**

Keep in mind that throughout this investigation, we are ignoring rhythms.

A Hum a few measures of *Twinkle, Twinkle, Little Star.* Then play the melody, find a copy of the music, or determine for yourself what notes are being played. This melody starts on the tonic pitch ("base" note) of the musical scale, so label the first note as 0. Now label the other notes in the melody based on how many half-steps each is above the tonic. (e.g., if C is 0, then we'll consider C# as 1, D as 2, D# as 3, and so on.) Using this system, write down the numerical label for each of the first 14 notes of the melody.

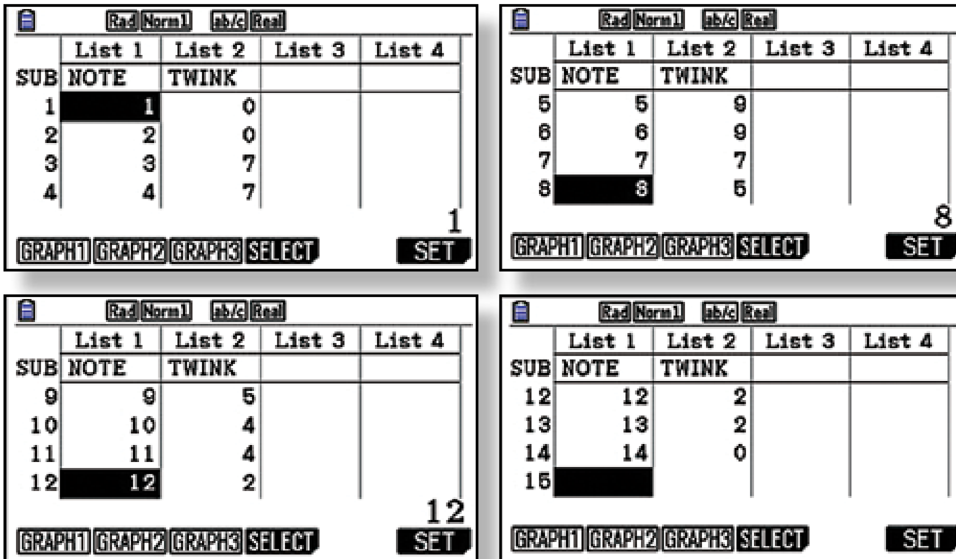
Some students may find it easier to create a list identifying the specific notes and their corresponding values, while others may wish to create a table as they go through the melody, or even notate the melody musically and write a number next to each note. In any case, we eventually want to end up with a table of values, so guide students to pair the numbers 1 through 14 (each note's *position* in the melody) with their half-step labels.

The original melody of *Twinkle, Twinkle,* as notated by Mozart, begins on the note C. Because the melody only contains notes at C or higher, we will assign no negative half-step values. Using the key of C, the half-step distances from C are as follows:

C (0), C (0), G (7), G (7), A (9), A (9), G (7),

F (5), F (5), E (4), E (4), D (2), D (2), C (0).

On the PRIZM, enter the notes' **positions** (1 through 14) into **List 1**, and the half-step values into **List 2**. These lists are shown below in four sections:



SUB	List 1 NOTE	List 2 TWINK	List 3	List 4
1	1	0		
2	2	0		
3	3	7		
4	4	7		

SUB	List 1 NOTE	List 2 TWINK	List 3	List 4
5	5	9		
6	6	9		
7	7	7		
8	8	5		

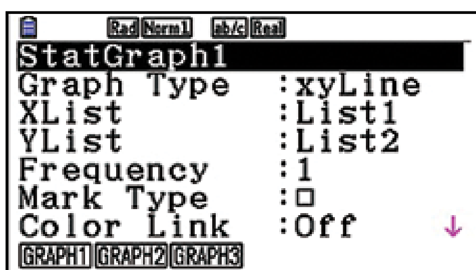
SUB	List 1 NOTE	List 2 TWINK	List 3	List 4
9	9	5		
10	10	4		
11	11	4		
12	12	2		

SUB	List 1 NOTE	List 2 TWINK	List 3	List 4
12	12	2		
13	13	2		
14	14	0		
15				

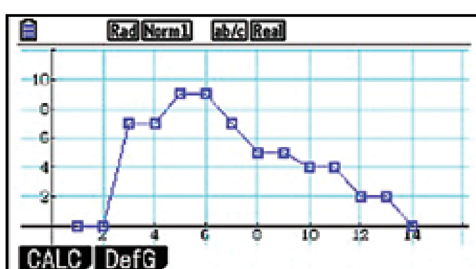
B Construct an XY-Line graph for the table you've just created. Discuss what the graph reveals about the *Twinkle, Twinkle* melody.

From the home screen in Statistics mode:

- Select F1 (GRAPH) and F6 (SET). The key components of the settings are shown below:



- When finished with the Settings, press EXIT, and then F1 (GRAPH1).



HOW LOW DID YOU GO? HOW HIGH DID YOU FLY? (CONTINUED)

We notice that the graph is either on the *x*-axis or in Quadrant I. It rises steeply initially, and then descends more slowly (just as the melody does), and finally returns to the *x*-axis, the tonic of the scale, on the 14^{th} note.

Some may argue that a scatterplot would be more appropriate here than an xy-line graph. While it is true that we are dealing with a discrete domain, so that "in-between" values make no sense, we find that an xy-line graph more clearly conveys the sense of the ***shape*** *or* ***contour*** *of a melody, an important concept musically. This can lead to a very worthwhile classroom discussion about domain and the information conveyed by different types of graphs.*

C Discuss the meaning of points that lie on, above, and below the *x*-axis. Why are there no points on or to the left of the *y*-axis?

Points on the *x*-axis represent notes at the tonic of the scale. In our *Twinkle, Twinkle* example, points on the *x*-axis represent the tonic note C, which is most likely Middle C on a piano keyboard. Points above the *x*-axis represent pitches that are above the tonic, while any points below the *x*-axis would represent pitches that are below this tonic. Teachers, if they so desire, can use this context to help students develop a better understanding of negative numbers.

We labeled the first note of the song as Note #1. In other words, our *x*-values started with 1. Consequently there are no points on the *y*-axis, as we did not have a Note #0. Similarly, we don't have any points left of the *y*-axis, because negative note *positions* would be nonsense. Our *x*-values are really counting a quantity of notes, which means that only natural-number values make sense. This gives teachers a context to help students develop a better understanding of number sets, and their role in sometimes restricting the domain of a function.

D Discuss how the table and graph you have created can be used to transpose a song from one key to another. For example, what could you do mathematically if you wanted to play *Twinkle, Twinkle, Little Star* in the key of D instead of the key of C?

Transposing in music is analogous to a translation (sometimes called a "slide") in mathematics. We simply need to move every one of the notes the same distance and in the same direction.

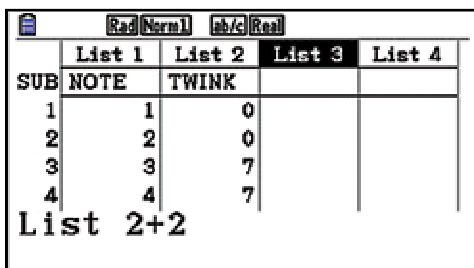
Students may attack this task in different ways. Some students may want to use the same table of values, but re-label the notes. So instead of thinking of C as 0, C# as 1, D as 2, D# as 3, E as 4, and so on, they may wish to think of D (or whatever key they choose to transpose to) as 0, D# as 1, E as 2, F as 3, F# as 4, and so.

With this approach, our original value assignments, which were C (0), C (0), G (7), G (7), A (9), A (9), G (7), F (5), F (5), E (4), E (4), D (2), D (2), C (0), become instead:

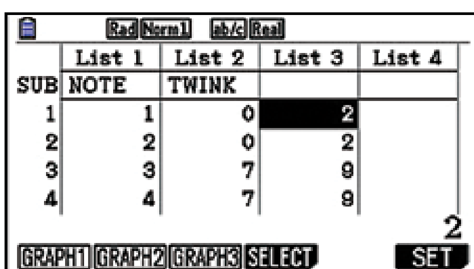
D (0), D (0), A (7), A (7), B (9), B (9), A (7), G (5), G (5), F# (4), F# (4), E (2), E (2), D (0). The *y*-values remain exactly the same! If we keep to the meaning of a *y*-value of 0 indicating the tonic of the scale, then this is the appropriate way to approach the problem.

A second approach is to add 2 to all of the **List 2** values. (We are adding 2 because D is 2 half-steps higher than C; if we were transposing to G, then we would add 7 to or subtract 5 from each of the values in **List 2**.) With this approach, we can still think of C as 0, C# as 1, D as 2, D# as 3, E as 4, and so on, but the values in **List 2** must change. Note, though, that, with this approach, a *y*-value of 0 does not represent the tonic of the key signature, but refers to middle C.

- With our original values in **List 2**, highlight the header for **List 3**. Then press [SHIFT] [1] (List), [2] to identify **List 2**, the addition symbol, and [2] (or a different value for a different transposition):



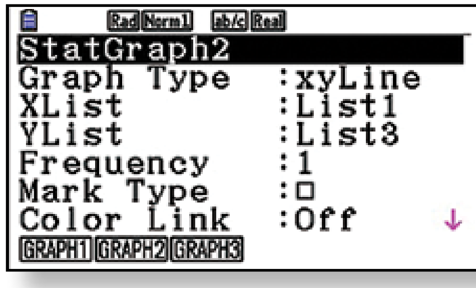
- Press [EXE]:



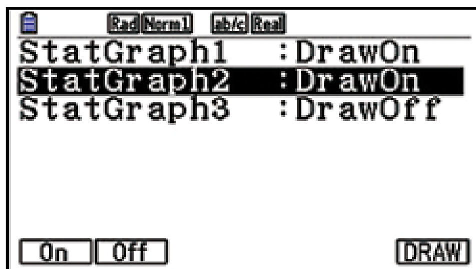
List 3 now contains the transposed values. Let's take a look at the graphs.

HOW LOW DID YOU GO? HOW HIGH DID YOU FLY? (CONTINUED)

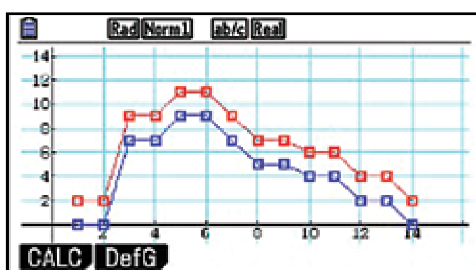
- From the screen above, select F6 (SET). Set up **StatGraph2** as shown below (not shown: we have set Graph2's Color as Red).



- Press EXIT, followed by F4 (SELECT). Set both **StatGraph1** and **StatGraph2** to **DrawOn**.



- Select F6 (DRAW). Students should see that the graphs (melodies) have the same shape, but the transposed graph (shown in red) is always 2 units higher than the original (shown in blue).



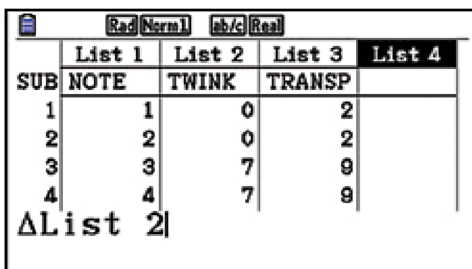
Teachers may find it helpful to use the transposition feature of an electronic piano keyboard (such as the Casio Privia PX-150) to relate to the vertical translation displayed on the graph. We shifted our notes upward by 2 half-steps each; this corresponds to a keyboard transposition value of +2.

E Investigate the first-order differences for the first 14 notes of *Twinkle, Twinkle, Little Star.* What do these differences indicate about the melody line? What is the effect on the first-order differences when a song is transposed? How might these differences be helpful to someone learning to play the song on a keyboard?

The first-order differences show the differences in the *y*-values (**List 2** for the original melody; **List 3** for the transposed melody). In other words, they show how many half-steps there are from one note to the next, with a positive value indicating ascending notes and a negative value indicating descending notes.

We will calculate the first-order differences for both **List 2** and **List 3**. From the graph shown at the end of part D, press SHIFT EXIT (QUIT) to return to the Statistics home screen. Then:

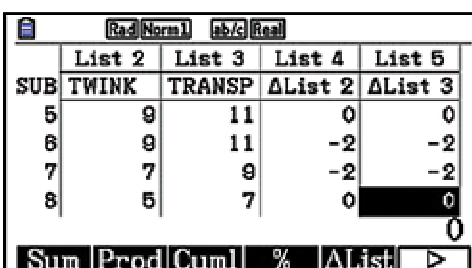
- Highlight **List 4's** title. Press OPTN, F1 (LIST), F6 twice for more choices, F5 (ΔList) and 2 to indicate we want the changes to **List 2**. Press EXE to generate the first-order differences.



	List 1	List 2	List 3	List 4
SUB	NOTE	TWINK	TRANSP	
1	1	0	2	
2	2	0	2	
3	3	7	9	
4	4	7	9	

ΔList 2

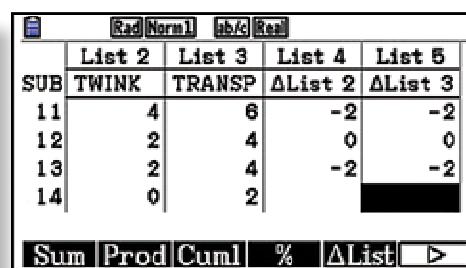
- Highlight **List 5's** title, select F5 (ΔList), type in 3 to indicate we want the changes to **List 3**, and press EXE. See below (we have added labels in the SUB field to identify what's in each list).



	List 2	List 3	List 4	List 5
SUB	TWINK	TRANSP	ΔList 2	ΔList 3
5	9	11	0	0
6	9	11	-2	-2
7	7	9	-2	-2
8	5	7	0	0

0

Sum | Prod | Cuml | % | ΔList | ▷



	List 2	List 3	List 4	List 5
SUB	TWINK	TRANSP	ΔList 2	ΔList 3
11	4	6	-2	-2
12	2	4	0	0
13	2	4	-2	-2
14	0	2		

Sum | Prod | Cuml | % | ΔList | ▷

HOW LOW DID YOU GO? HOW HIGH DID YOU FLY? (CONTINUED)

The transposition has no effect on the first-order differences. In other words, **List 4** and **List 5** are identical. This should make some sense, as the intervals between the notes remains the same no matter what key we are playing in. Students should also realize that the number of differences is one less than the number of values; between 14 notes there exist only 13 differences.

These first-order differences (which are sowing seeds for topics covered in advanced algebra and calculus) can make it easy for someone learning to play the song on a keyboard. The learner can start on any note whatsoever (for calculus teachers, this is similar to the +C that we include when we calculate an indefinite integral). Once we choose a starting note, we then:

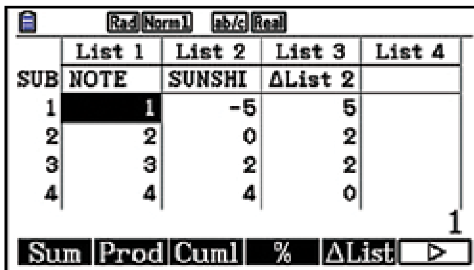
- Repeat the note (the first difference is 0);
- Go up 7 half-steps;
- Repeat the note;
- Go up 2 half-steps;
- Repeat the note;
- Go down 2 half-steps;
- Go down 2 half-steps;
- Repeat the note;
- Go down 1 half-step;
- Repeat the note;
- Go down 2 half-steps;
- Repeat the note;
- Go down 2 half-steps.

Again, this pattern would be identical no matter what note we start on.

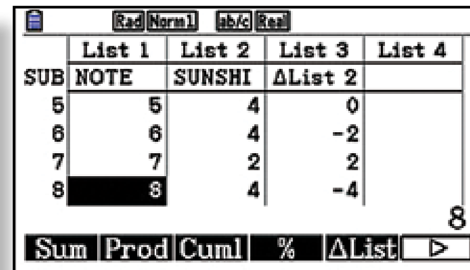
F Think about one of your favorite songs. Construct a table of half-step values for at least the first 16 notes of the melody, including a column that shows the first-order differences. Then, create graphs, share them with others, and see if they can determine which song you selected.

Answers, of course, will vary. Songs with a wide tessitura (note range) will result in graphs with a great vertical distance between the low and high pitches in the song, while those with a narrow tessitura will result in graphs with little vertical distance between low and high pitches. The domain of the graph will depend solely on how many notes of the song the student has included.

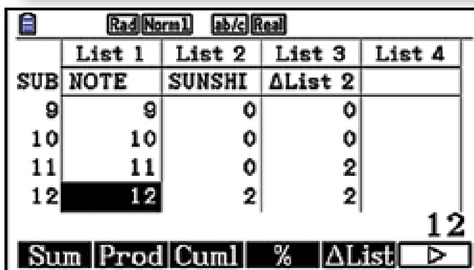
Below is the table of values for the first 19 notes of *You Are My Sunshine*, with a graph that follows. Also shown are the first-order differences. We encourage teachers to have students play each other's songs. For some songs, the lack of any information about the rhythm may make the identification difficult, which may lead to some rich class discussion.



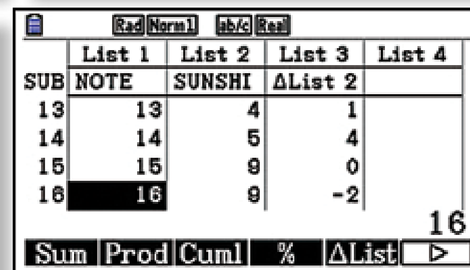
SUB	List 1 NOTE	List 2 SUNSHI	List 3 ΔList 2	List 4
1	1	-5	5	
2	2	0	2	
3	3	2	2	
4	4	4	0	



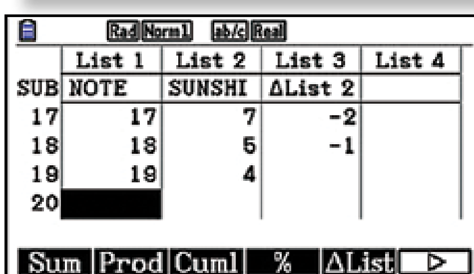
SUB	List 1 NOTE	List 2 SUNSHI	List 3 ΔList 2	List 4
5	5	4	0	
6	6	4	-2	
7	7	2	2	
8	8	4	-4	



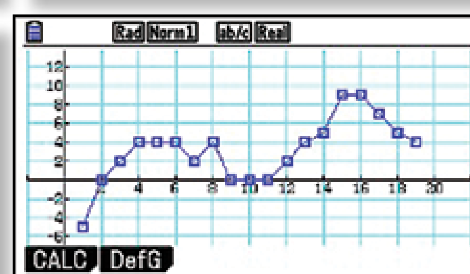
SUB	List 1 NOTE	List 2 SUNSHI	List 3 ΔList 2	List 4
9	9	0	0	
10	10	0	0	
11	11	0	2	
12	12	2	2	



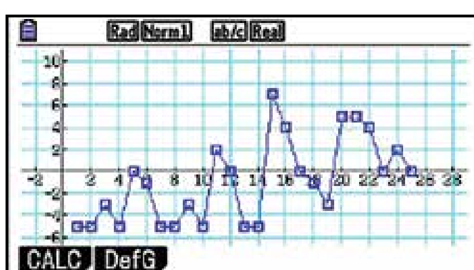
SUB	List 1 NOTE	List 2 SUNSHI	List 3 ΔList 2	List 4
13	13	4	1	
14	14	5	4	
15	15	9	0	
16	16	9	-2	



SUB	List 1 NOTE	List 2 SUNSHI	List 3 ΔList 2	List 4
17	17	7	-2	
18	18	5	-1	
19	19	4		
20				



G Consider the XY-Line graph below. Convert the graph into a table of values and then see if you can determine what this well-known melody is. Is there any indication of what the key signature might be? Explain.



HOW LOW DID YOU GO? HOW HIGH DID YOU FLY? (CONTINUED)

The table of values, including the first-order differences, is shown below.

SUB	List 1	List 2	List 3	List 4
	NOTE		ΔList 2	
1	1	-5	0	
2	2	-5	2	
3	3	-3	-2	
4	4	-5	5	

1
GRAPH CALC TEST INTR DIST ▷

SUB	List 1	List 2	List 3	List 4
	NOTE		ΔList 2	
5	5	0	-1	
6	6	-1	-4	
7	7	-5	0	
8	8	-5	2	

8
GRAPH CALC TEST INTR DIST ▷

SUB	List 1	List 2	List 3	List 4
	NOTE		ΔList 2	
9	9	-3	-2	
10	10	-5	7	
11	11	2	-2	
12	12	0	-5	

12
GRAPH CALC TEST INTR DIST ▷

SUB	List 1	List 2	List 3	List 4
	NOTE	SUNSHI	ΔList 2	
13	13	4	1	
14	14	5	4	
15	15	9	0	
16	16	9	-2	

16
Sum Prod Cuml % ΔList ▷

SUB	List 1	List 2	List 3	List 4
	NOTE		ΔList 2	
17	17	0	-1	
18	18	-1	-2	
19	19	-3	8	
20	20	5	0	

20
GRAPH CALC TEST INTR DIST ▷

SUB	List 1	List 2	List 3	List 4
	NOTE		ΔList 2	
21	21	5	-1	
22	22	4	-4	
23	23	0	2	
24	24	2	-2	

24
GRAPH CALC TEST INTR DIST ▷

SUB	List 1	List 2	List 3	List 4
	NOTE		ΔList 2	
23	23	0	2	
24	24	2	-2	
25	25	0		
26				

GRAPH CALC TEST INTR DIST ▷

The song is *Happy Birthday.* Because the half-step numbers and first-order differences are the same regardless of our starting pitch (and therefore our key), these data points and graph are of no benefit in determining the key signature. Students should, however, be able to determine that, whatever key *Happy Birthday* is played in, the tonic is 5 half-steps ABOVE the starting note, as the first note is 5 half-steps BELOW the tonic (as indicated by the "-5" half-step value).

H Explore some of your favorite melodies and determine if they end on the tonic pitch of their scale. In other words, if a song is written in the key of C, does it end on a C? Use your results to formulate a conjecture about the probability that any particular song will end on the tonic. Test your conjecture and report your results.

Results will vary. Although several modern-era songs do not end on the tonic, most still do. A very high percentage of older songs do end on the tonic, though some end on the third or fifth of the key in which the song was written. Teachers may wish to challenge their students to find songs that do not end on the tonic. Our purpose here is simply to familiarize students with the possibility of connecting ideas of probability with music.