

MIX THE "TRICKS"

Tina Cardone, Ashli Black

Do Now: Do Some Math!

- How would you solve the problems on your own?
- How might a student solve these problems?
- What are some possible misconceptions/errors?
- What tricks might cause students to make mistakes?

1. $(-4) - 7$

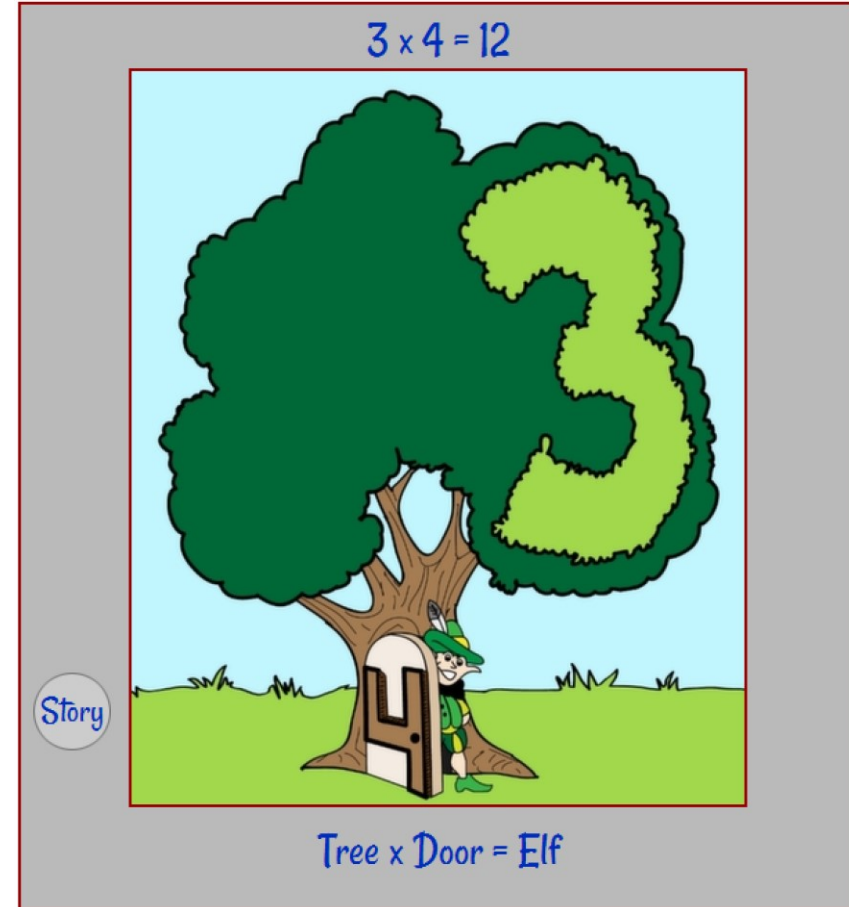
2. $100 / (-2)$

3. $\frac{8x}{30} = \frac{48}{18}$

Nix the Tricks

TINA CARDONE

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<http://www.multiplication.com/learn/learn-fact/3/x/4>



Is it a trick?

Method

Definition

Kids can explain:
Shortcut

“Because —
said so”:
Trick

New term:
Mnemonic

Based on
previous terms:
Trick

Math Makes Sense

After 3 hours, your phone battery is at 93%.

How long do you expect it will last before it dies?



Math Makes Sense



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- A few students used estimation or proportional reasoning.

Math Makes Sense



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How long do you expect it will last before it dies?

- A few students used estimation or proportional reasoning.
- Many students didn't calculate - they said the initial situation was unreasonable and made estimates based on their own experiences.

Math Makes Sense



After 3 hours, your phone battery is at 93%.

How long do you expect it will last before it dies?

- A few students used estimation or proportional reasoning.
- Many students didn't calculate - they said the initial situation was unreasonable and made estimates based on their own experiences.
- The final group, which was the largest, said - with complete confidence - it will last 31 hours, because $93 / 3 = 31$.

Math Mistakes

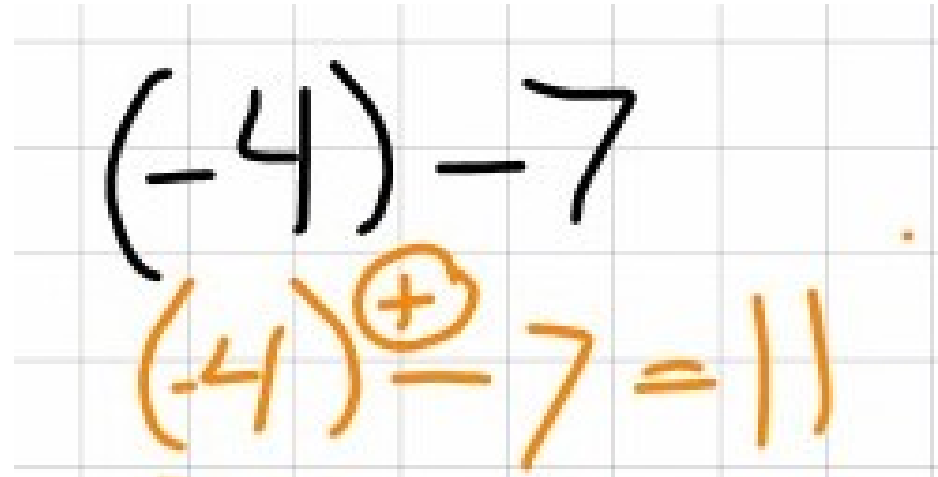
What mistakes might you expect?



1. $(-4) - 7$

Math Mistakes

Two negatives make a positive



The image shows a handwritten mathematical expression on a grid background. The top part shows the expression $(-4) - 7$ in black ink. Below it, the same expression is written in orange ink as $(-4) + 7 = 11$. A small orange circle with a plus sign is drawn around the minus sign in the second expression, highlighting the error of changing the sign.

<http://mathmistakes.org/?p=328>

Two negatives make a positive

$$2 - -5$$

$$2 + 5$$

or

$$2 - +5$$

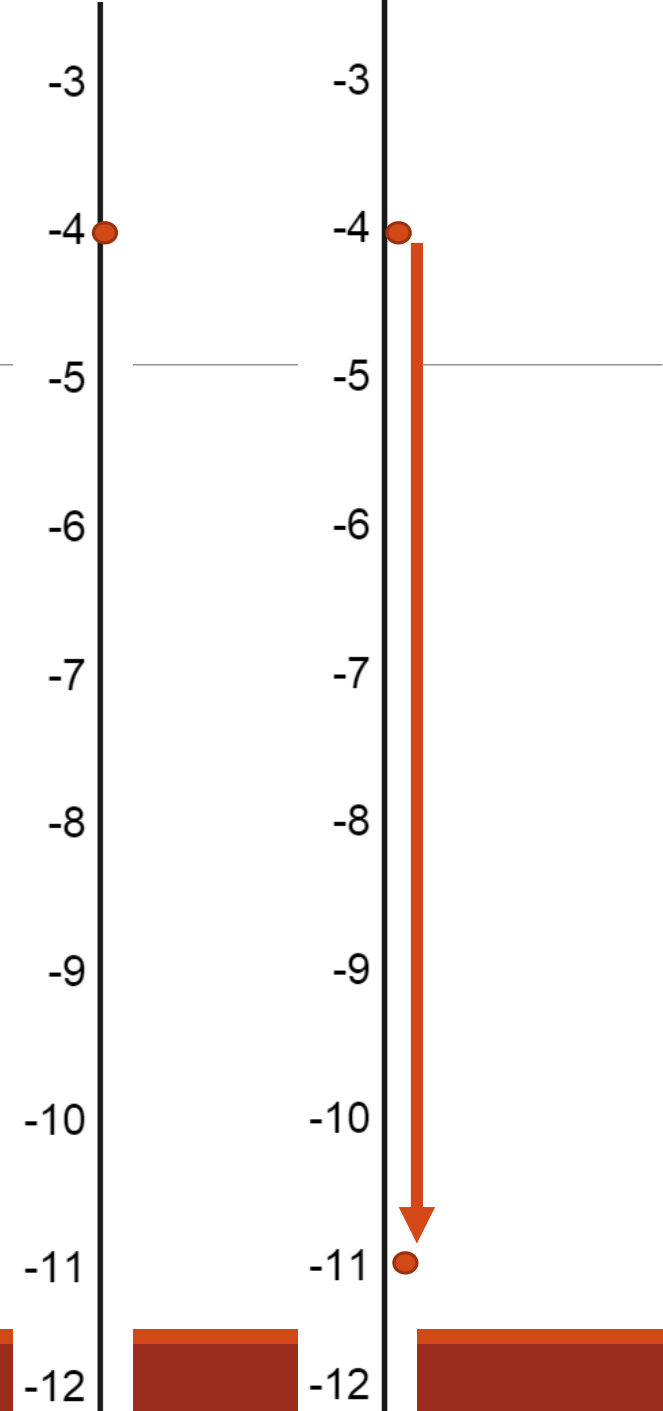
Fix:

Define subtraction as
adding the opposite.

Number Lines for Adding Integers

1. $(-4) - 7$

$(-4) + (-7)$



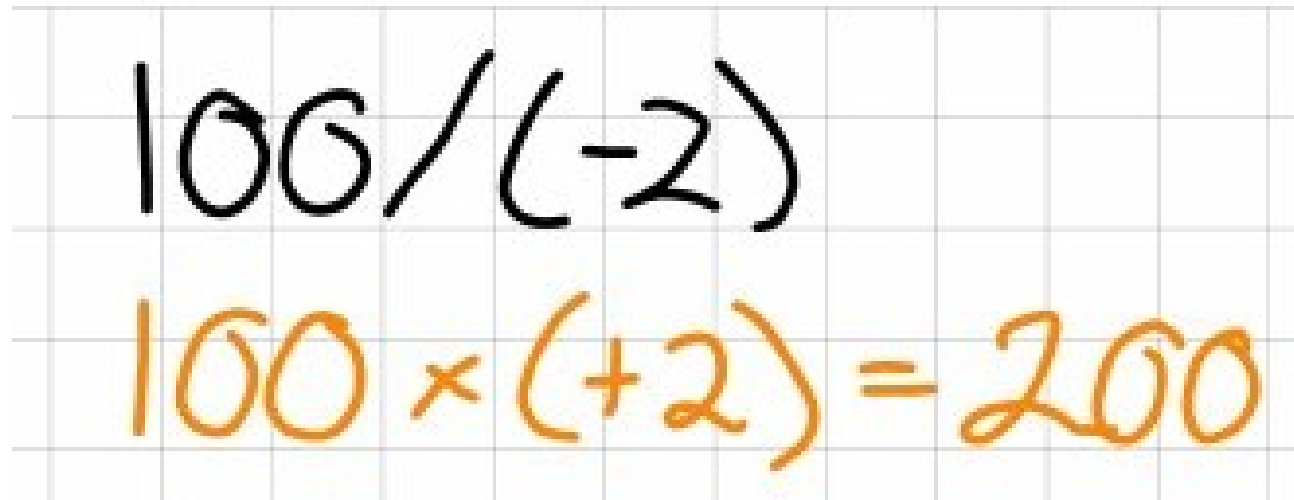
Math Mistakes

What mistakes might you expect on this problem?

$$2. 100 / (-2)$$

Math Mistakes

Same change flip, keep change change



The image shows a grid background with two lines of handwritten text. The first line is written in black ink and shows the expression $100 / (-2)$. The second line is written in orange ink and shows the expression $100 \times (+2) = 200$. This illustrates a common mistake where a student incorrectly changes the sign of the denominator when dividing by a negative number.

$$100 / (-2)$$
$$100 \times (+2) = 200$$

<http://mathmistakes.org/?p=328>

Same Change Flip

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4}$$

Fix:

Discover/prove the method of multiplying by the reciprocal.

Multiply by the Reciprocal

$$\frac{2}{3} \div \frac{2}{3} = 1 \quad \text{easy!}$$

$$\frac{2}{3} \div \frac{1}{3} = 2 \quad \text{makes sense}$$

$$\frac{4}{5} \div \frac{3}{5} = \frac{4}{3} \quad \text{not as obvious, but still dividing the numerators}$$

$$\frac{4}{5} \div \frac{1}{2} = ? \quad \text{no idea!}$$

Multiply by the Reciprocal

$$\frac{4}{5} \div \frac{1}{2} = ? \quad \text{no idea!}$$

$$\frac{8}{10} \div \frac{5}{10} = \frac{8}{5} \quad \text{makes sense}$$

Multiply by the Reciprocal

$$\begin{aligned} \color{blue}{\boxed{}} \div \frac{1}{2} &= \frac{\color{blue}{\boxed{}} \cdot 2}{\color{blue}{\boxed{}} \cdot 2} \div \frac{\color{green}{\boxed{1 \cdot 5}}}{2 \cdot 5} \\ &= \frac{\color{brown}{\boxed{4 \cdot 2}}}{\color{green}{\boxed{1 \cdot 5}}} = \frac{\color{blue}{\boxed{}} \cdot 2}{\color{blue}{\boxed{}} \cdot 1} \\ &= \color{blue}{\boxed{}} \cdot \frac{2}{1} \end{aligned}$$

Multiply by the Reciprocal

$$\frac{\text{blue}}{\text{yellow}} = \frac{\text{blue}}{\text{yellow}} \cdot 1 = \frac{\text{blue}}{\text{yellow}} \cdot \frac{2}{2} = \frac{\text{blue} \cdot 2}{\text{yellow} \cdot 1} = \frac{\text{blue} \cdot 2}{1}$$

Math Mistakes

What mistakes might you expect on this problem?

$$3. \quad \frac{8x}{30} = \frac{48}{18}$$

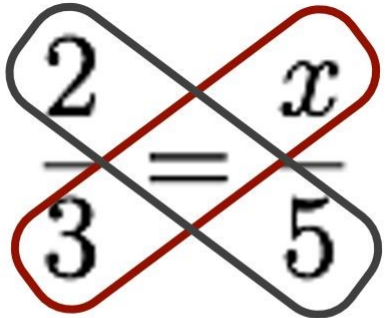
Math Mistakes

Cross multiply... cross something!

The image shows two handwritten equations on a grid background. The first equation is $\frac{8x}{30} = \frac{48}{18}$, which is circled in orange. The second equation is $\frac{26x}{26} = \frac{78}{26}$, which is crossed out with a large orange 'X'. Below the second equation, the solution $x=3$ is written in orange.

<http://mathmistakes.org/?p=1320>

Cross Multiply


$$\frac{2}{3} = \frac{x}{5}$$

$2 \cdot 5 = x \cdot 3$

Fix:

Solve proportions like any other equation – using inverse operations

Inverse Operations

$$30 \cdot \frac{8x}{30} = \frac{48 \cdot 30}{18}$$

$$\frac{1}{8} \cdot 8x = \frac{48 \cdot 30}{18} \cdot \frac{1}{8}$$

$$x = \frac{48 \cdot 30}{18 \cdot 8}$$

Nixing Those Tricks

Is HARD!

Nixing Those Tricks

Is HARD!

Hard for kids who have already learned them

Nixing Those Tricks

Is HARD!

Hard for kids who have already learned them

Hard for kids who are used to learning them

Nixing Those Tricks

Is HARD!

Hard for kids who have already learned them

Hard for kids who are used to learning them

Hard for teachers who have them in their vocabulary

Nixing Those Tricks

$$\sin^2(x) + \cos^2(x) = 1$$

Identity

$$\sin^2(x) = 1 - \cos^2(x)$$

Subtract $\cos^2(x)$

$$\sin^2(x) = [1 + \cos(x)][1 - \cos(x)]$$

Opposite of FOIL

$$\frac{\sin(x)}{1 - \cos(x)} = \frac{1 + \cos(x)}{\sin(x)}$$

Uncross multiply

$$\frac{\sin(x)}{1 - \cos(x)} = \frac{1 + \cos(x)}{\sin(x)}$$

Nixing Those Tricks

$\sin^2(x) + \cos^2(x) = 1$	Identity
$\sin^2(x) = 1 - \cos^2(x)$	Subtract $\cos^2(x)$
$\sin^2(x) = [1 + \cos(x)][1 - \cos(x)]$	Opposite of FOIL
$\frac{\sin(x)}{1 - \cos(x)} = \frac{1 + \cos(x)}{\sin(x)}$	Uncross multiply

Students have no idea how to undo a trick – because they don't know what they are doing!

Understanding something includes having an idea of the inverse.

Attend to Precision

Operation	Inverse
Add	Subtract
Multiply	Divide
Cross Multiply	?
FOIL	??

FOIL

$$(2x + 3)(x - 4)$$

$$\begin{array}{cccc} \textit{First} & & \textit{Inside} & \\ 2x^2 & - 8x & + 3x & - 12 \\ & \textit{Outside} & & \textit{Last} \end{array}$$

Fix:

Use the distributive property.

FOIL

$$\begin{aligned} & \blacksquare (x - 4) \\ & = (2x + 3)(x) + (2x + 3)(-4) \end{aligned}$$

	2x	3
x		
-4		

FOIL

$$\begin{aligned}(2x + 3)(x - 4) \\ &= (2x + 3)\blacksquare + (2x + 3)\blacksquare \\ &= 2x^2 + 3x - 8x - 12\end{aligned}$$

	2x	3
x	$2x^2$	$3x$
-4	$-8x$	-12

FOIL

$$\begin{aligned}(2x + 3)(x - 4) &= (2x + 3)(x) + (2x + 3)(-4) \\ &= 2x^2 \blacksquare - 12 \\ &= 2x^2 - 5x - 12\end{aligned}$$

	2x	3
x	$2x^2$	$3x$
-4	$-8x$	-12

Focus on Understanding

With Tricks Students:

- Are stuck if they forget

$$2 + (-5)$$

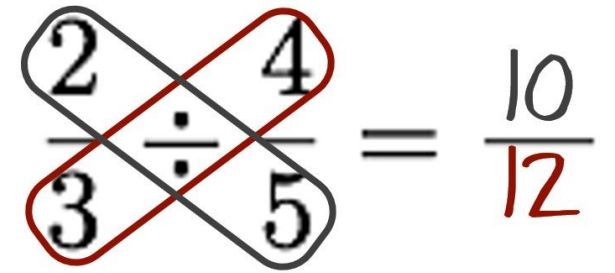
Keep Change Change

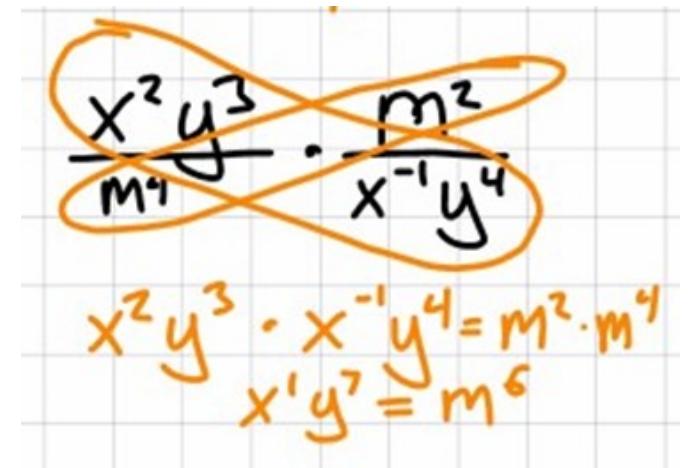
$$2 - (+5)$$

Focus on Understanding

With Tricks Students:

- Are stuck if they forget
- Generalize to dissimilar contexts


$$\frac{2}{3} \div \frac{4}{5} = \frac{10}{12}$$


$$\frac{x^2 y^3}{m^4} \cdot \frac{m^2}{x^{-1} y^4}$$
$$x^2 y^3 \cdot x^{-1} y^4 = m^2 \cdot m^4$$
$$x^1 y^7 = m^6$$

Focus on Understanding

With Tricks Students:

- Are stuck if they forget
- Generalize to dissimilar contexts
- Don't generalize to similar context

$$(2x + 3)(x - 4)$$

First *Inside*
 $2x^2 - 8x + 3x - 12$
Outside *Last*

$$(2x + 3)(4x^2 + x - 4)$$

Focus on Understanding

With Tricks Students:

- Are stuck if they forget
- **Generalize** to dissimilar contexts
- **Don't generalize** to similar context

"I nixed a trick!"



It's Not Too Late

Day 120:

I think I have successfully cured my pre-algebra students of using "cross-multiply." When we started proportions, I heard it with every other problem. Now, I don't think I've heard it in over a week. What I'm hearing instead is "don't we multiply 15 to both sides?"

<http://relearningtoteach.blogspot.com/2014/03/day-120-buffon-and-quiz.html>

"I nixed a trick!"



It's Not Too Late

Day 143:

After we completed the 8 word problems, I asked a simple question:

Me: "What did I not see in any of these problems? What tactic was not used?"

S: "Cross-multiplication."

Me: "Did anyone even think to use it?"

S: **silence**

YES!!! One trick nixed!

<http://relearningtoteach.blogspot.com/2014/04/day-143-pointed-questions-and-jelly.html>

Now What?

READY TO MAKE A CHANGE:

NixTheTricks.com

See other's ideas

Make a suggestion

Have a debate

NIX THE ~~TRICKS~~

[DOWNLOAD](#)[DRAFT SECTIONS](#)[REVIEWS](#)[HOMEPAGE](#)

Draft Sections

The full text currently includes 7 chapters with illustrations and examples. Is there a trick you hate to see that's missing? Is there a better way to teach a trick that's been nixed? What about things that could be taught better but don't necessarily fall under the heading "trick"?

Peer Review

Once someone submits a trick it goes to [this document](#) for peer review. Leave a comment on whether the proposed additions are truly tricks and the best way to fix them. All comments will be considered for the next edition of the book.

Define Vocab

Students may memorize more definitions of vocabulary terms without understanding than they do methods of problem solving. Ideally students have enough experience with a concept that they already understand the meaning before they have a word to describe that thing they have been talking about. [This document](#) is for creating succinct definitions of terms without losing meaning. And for disambiguation.

Proper Notation

In mathematics, the symbols we use are as much a part of the language as the vocabulary terms are. [This document](#) presents appropriate notation and considers when it is best to introduce the symbols.

? or !

This form is the [place](#) to submit any and all thoughts that don't fit in a comment on one of the documents above. Want to add a new trick, term or symbol to the draft pages? Have an example of student work that exemplifies the issues with using tricks? Want to volunteer to help with this project? Any commentary at all, goes to the submission form.

Download the Book

Buy the paperback on Amazon



Preview the Table of Contents

Email [this resource](#) to a colleague or administrator.

Are your students struggling with the very same issues as are described in this book? If you run across examples of errors that might be caused by students who memorized a trick rather than understanding the concept submit them to [Math Mistakes](#) and to [the book](#).

"The worst thing about mnemonics is not that they almost always fall apart, they don't encourage understanding, and never justify anything; it's that they kill curiosity and creativity - two important character traits that too many math teachers out there disregard."
-Andy Martinson