# Making Sense of Integer Operations:
## Contexts, Tasks, and Models

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## Introductory Context
(Gregg & Gregg, 2007)

- Mrs. Brady gives her children a monthly allowance based on whether or not they complete their assigned chores during the month. She gives them a +1 chip worth $1 for each chore completed and a -1 chip for each chore they fail to complete. She records these on a credit-debit board on the refrigerator. At the end of the month, she figures the amount each child should receive (or the amount they owe, if they have more debits than credits).

# Introductory Problems

- Find the value of each board.

| **Debbie's Board** | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Credits | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | | | |
| Debits | -1 | -1 | -1 | -1 | -1 | | | | | | | | |

| **Mike's Board** | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Credits | +1 | +1 | +1 | +1 | +1 | +1 | | | | | | | |
| Debits | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | | | | | |

| **Rachel's Board** | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Credits | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | | | | |
| Debits | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | | | | |

# Building Further in this Context

Write a number sentence for computing the result in each of the following situations:

- Sam's credit-debit board showed a net value of +4 on Monday morning. As a reward for getting an A on his spelling test, Mrs. Brady removed -2 in debits. What value does his board show after this change?
- Becca's board showed a value of -1 on Tuesday morning. At the end of the day, her board was worth +3. What change (or changes) did Mrs. Brady make on Tuesday?
- On Wednesday evening, Mrs. Brady added -3 in debits to Quigley's board, making its new value +8. What was its value before this change?

# Sam's Credit Board

- Sam's credit-debit board showed a net value of +4 on Monday morning. As a reward for getting an A on his spelling test, Mrs. Brady removed -2 in debits. What value does his board show after this change?

| Credits | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Debits | -1 | -1 | -1 | | | | | | | | | |

**+4 – (-2) = +6**

# Becca's Credit Board

- Becca's board showed a value of -1 on Tuesday morning. At the end of the day, her board was worth +3. What change (or changes) did Mrs. Brady make on Tuesday?

| **Credits** | **+1** | **+1** | **+1** | **+1** | +1 | +1 | +1 | +1 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Debits** | **-1** | **-1** | **-1** | **-1** | **-1** | | | | | | | |

**-1 + (+4) = +3**

| Credits | +1 | +1 | +1 | +1 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Debits | -1 | | | | | | | | | | | |

**-1 – (-4) = +3**

# Quigley's Credit Board

- On Wednesday evening, Mrs. Brady added -3 in debits to Quigley's board, making its new value +8. What was its value before this change?

| Credits | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Debits | -1 | -1 | -1 | -1 | | | | | | | | |

**+8 – (-3) = +11** **+11 + (-3) = +8**

# Types of Start-Change-Result (SCR) Problems

- Result unknown
- Start unknown
- Change unknown

# Task Types

Classify each Start-Change-Result problem according to what is unknown.

- Sam's credit-debit board showed a net value of +4 on Monday morning. As a reward for getting an A on his spelling test, Mrs. Brady removed -2 in debits. What value does his board show now?
- **Result Unknown**
- Becca's board showed a value of -1 on Tuesday morning. At the end of the day, her board was worth +4. What change (or changes) did Mrs. Brady make on Tuesday?
- **Change Unknown**
- On Wednesday evening, Mrs. Brady added -3 in debits to Quigley's board, making its new value +8. What was its value before this change?
- **Start Unknown**

# Models for Addition & Subtraction

- Chip/Counting Models
- Number-Line Models

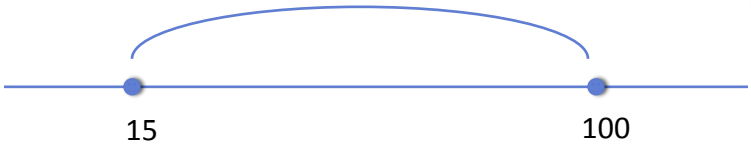
# Comparing Temperatures on the Number Line

| State | Record Low | Record High |
|---|---|---|
| Hawaii | 15°F | 100°F |
| Missouri | -40°F | 118°F |

- Use a number line model to find the difference in temperatures within each state. (How many degrees from the record low to the record high?)

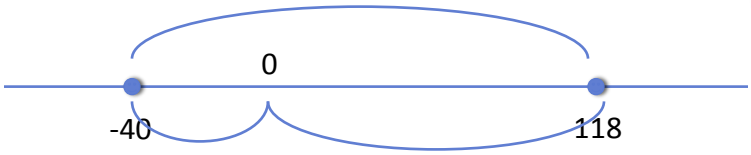
Hawaii _____ Missouri _____

# For Hawaii




100 – 15 = 85

# For Missouri




118 – (-40) = 118 + 40 = 158

# Combining Multiple Changes

At the beginning of the month, Cara had $123.62 in her savings account. During the month, she made the following deposits and withdrawals (negative amounts indicate withdrawals):
-20, +50, -40, +50, -20

a) What was the **net change** in her balance during the month?

b) What was her balance at the end of the month?

# Solutions

At the beginning of the month, Cara had $123.62 in her savings account. During the month, she made the following deposits and withdrawals (negative amounts indicate withdrawals):
-20, +50, -40, +50, -20

a) What was the net change in her balance during the month?

(-20) + (+50) + (-40) + (+50) + (-20) =

[(-20) + (-40) + (-20)] + [(+50) + (+50)] =

-80 + 100 =

+20

b) What was her balance at the end of the month?

123.62 + (+20) = 143.62

# Contexts for Integer Operations

- Money
- Temperature
- Some sports (football and golf, for example)
- Others?

# Positive and Negative Numbers in Sports

Football

- Gains in yardage count as positive, losses as negative.
- Results of 5 successive plays might be +2, -5, +10, -3, +15.

Golf

- Scores on each hole are measured against par. Negative scores are under par; positive scores are over par. (Lower scores are better in golf.) You can combine scores on two or more holes or scores on two or more rounds. (A round is 9 holes.)

# Write Your Own Problem

- Choose a context (money, temperature, football, golf, or other). Work with one or more partners. Your challenge is to write a problem where subtracting a negative number makes sense.

# A Constant Rate Problem

Water is flowing into a tank at a rate of 2 gallons per minute.

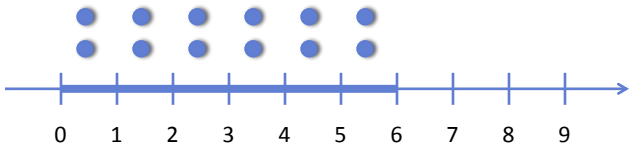
a) Compared to the amount in the tank now, how much would be in the tank after 6 minutes have passed?

# Number-Line with Chips

Water is flowing into a tank at a rate of 2 gallons per minute.

a) Compared to the amount in the tank now, how much would be in the tank after 6 minutes have passed?

6 x 2 = 12

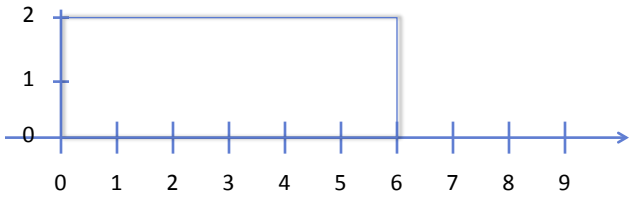


# Coordinate Graph with (Signed) Area Shown

Water is flowing into a tank at a rate of 2 gallons per minute.

a) Compared to the amount in the tank now, how much would be in the tank after 6 minutes have passed?

6 x 2 = 12




# A Constant Rate Situation

b) Compared to the amount in the tank now, how much was in the tank 15 minutes ago?

c) When would there be 18 more gallons in the tank than there is now?

d) When would there be 24 gallons less in the tank than there is now?

# A Constant Rate Situation

b) Compared to the amount in the tank now, how much was in the tank 15 minutes ago?

**-15 x 2 = 15 x (-2) = -30**

c) When would there be 18 more gallons in the tank than there is now?

**18/2 = 9**

d) When would there be 24 gallons less in the tank than there is now?

**-24/2 = -12**

# Net Change at Constant Rate

- (Net Change) = Rate x Time
- Time = (Net Change)/Rate

# Negative Rate

Emily got a huge bag of M&Ms for Christmas. She wants them to last a long time, so she only eats 8 each day.

a) Compared to her M&M supply now, how much will she have 7 days from now?

b) Compared her current M&M supply, how much did she have 30 days ago?

c) How long would it take her M&M supply to drop by 40?

d) When did she have 200 more M&Ms than she has now?

# Negative Rate

Emily got a huge bag of M&Ms for Christmas. She wants them to last a long time, so she only eats 8 each day.

a) Compared to her M&M supply now, how much will she have 7 days from now?

**7 x (-8) = -56**

b) Compared her current M&M supply, how much did she have 30 days ago?

**-30 x (-8) = +240**

c) How long would it take her M&M supply to drop by 40?

**(-40)/(-8) = +5**

d) When did she have 200 more M&Ms than she has now?

**200/(-8) = -25**

# Write Your Own Problem 2

(if time permits)

Working with one or more neighbors, create a context in which a quantity is decreasing at a constant rate.

a) Write a problem where multiplying a negative number by a negative number makes sense.

b) Write a problem where dividing a negative number by a negative number makes sense.

# Questions or Comments?

# Thank you!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