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Reasoning and Problem Solving in the Common Core Era

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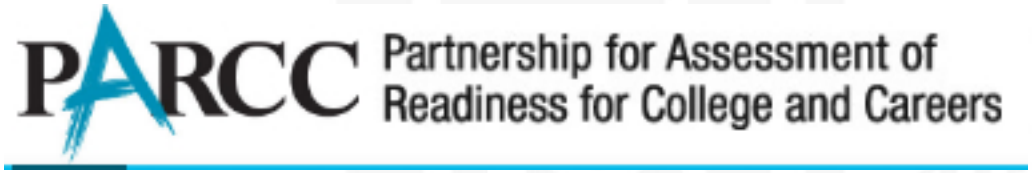
Common Core (CCSSM)



What is the Common Core?

State education chiefs and governors in 48 states came together to develop the Common Core, a set of clear college- and career-ready standards for kindergarten through 12th grade in English language arts/literacy and mathematics.

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Comparing PARCC and SB

- Both developing test to assess Common Core
- Each is a consortium of member states
- SB uses computer adaptive technology (correct answers lead to higher order thinking tasks)
- PARCC uses computerized assessment (not adaptive), includes PBA & EOY (concepts)
- PARCC Performance-Based Assessment (PBA) – test students' ability to *complete multi-step, real-world application problems in math.*

Common Core *Mathematical Practices*

- **Practice #1:** Make sense of problems and persevere in solving them (e.g., analyze givens, constraints, & goals; consider analogous problems, try simpler forms, monitor and evaluate, ask "Does this make sense?").
- **Practice #3:** Construct viable arguments and critique the reasoning of others. (e.g., use definitions, previous results, make conjectures, construct and analyze arguments).
- **Practice #7**: Look for and make use of structure. (e.g., identify patterns, notice common attributes).
- **Practice #8:** Look for and express regularity in repeated reasoning (e.g., generalize from examples).

NCTM's Principles to Action (2014)

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NCTM *Mathematical Teaching Practices*

- **Practice #2:** Implement tasks that promote reasoning and problem solving. … solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
- **Practice #5:** Pose purposeful questions. …to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

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Conjecture? Justify? This isn't Geometry!

Michelle wanted to investigate the effect on the vertex of the graph of $f(x) = x^2 + 6x$ when $f(x)$ is replaced by $f(x + k)$.

Michelle graphed functions of the form $f(x + k)$ for $k = 1, 2, 3$ and 4. For each of the functions she graphed, the x-coordinate of the vertex was negative and different for each value of k, but the y-coordinate of the vertex was the same for each value of k. Michelle made three conjectures based on her results.

1. The x-coordinate of the vertex depends on the value of k.
2. The x-coordinate of the vertex is negative for all values of k.
3. The y-coordinate of the vertex is independent of the value of k.

Determine if each of Michelle's three conjectures are true. Justify each answer.

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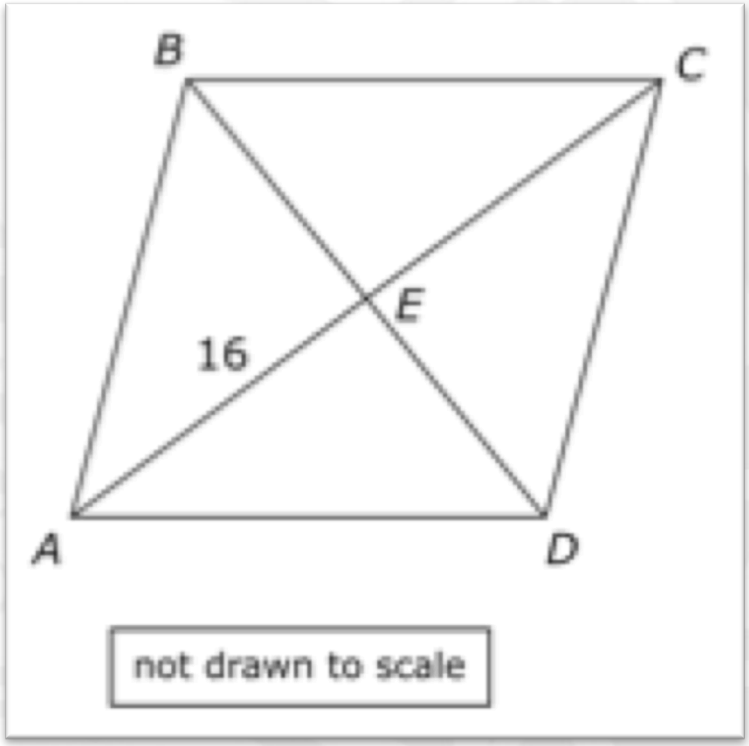
(PARCC)

Prove this is a….wait, what is it?

The figure shows parallel-ogram ABCD with AE=16.



- Let BE = $x^2 - 48$ and let DE = 2x. What are the lengths of $\overline{BE}$ and $\overline{DE}$?
- What conclusion can be made regarding the specific classification of parallelogram ABCD? Justify your answer.

(PARCC)

As you can see from the graph…wait…

Let $|x|+|y|=c$ where c is a real number.

Determine the number of points that would be on the graph of the equation for **each** given case:

Case 1: $c<0$

Case 2: $c=0$

Case 3: $c>0$

Justify your answers.

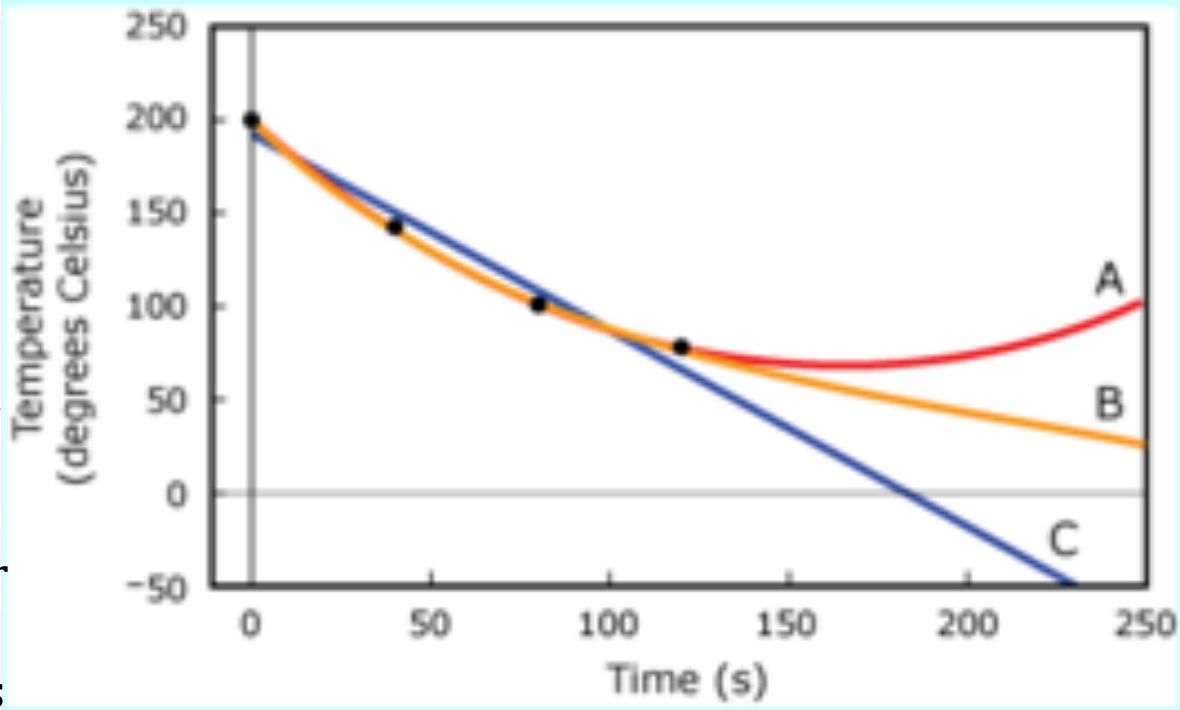
(PARCC)

Reasoning in a context

Time material is cooling (seconds)	0	40	80	120
Temperature (°C)	200	141	101	74

The figure shows the scientist's data (data points are plotted as large dots). Three possible models for the data are also shown: a linear model; a quadratic model; and an exponential model.

- Which model is linear? Which model is quadratic? Which model is exponential?
- Which model is the best fit for times $0 \le t \le 250$?
- Explain why the other models do not fit the data very well for the range of times $0 \le t \le 250$?



(PARCC)

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How many solutions?

Let $f(x) = ax^2$ where a>0 and let $g(x) = mx + b$ where m>0 and b<0. The equation $f(x) = g(x)$ has n distinct real solutions. Where are **all** the possible values of n? Justify your answers.

(PARCC)

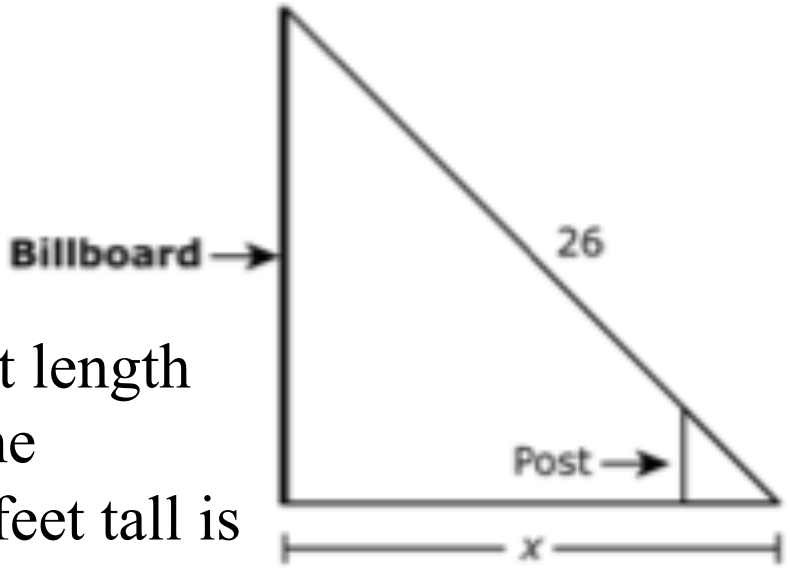
Prove the center is… wait, where is it?

A circle has its center at (6, 7) and goes through the point (1, 4). A second circle is tangent to the first circle at the point (1, 4) and has the same area.

What are the possible coordinates for the center of the second circle? Show your work or explain how you found your answer.

(Smarter Balanced)

What assumptions?



A billboard at ground level has a support length of 26 feet that extends from the top of the billboard to the ground. A post that is 5 feet tall is attached to the support and is 4 feet from where the base of the support is attached to the ground.

In the figure shown, the distance, in feet, from the base of the billboard to the base of the the support is labeled x.
Create an equation that can be used to determine x. Discuss any assumptions that should be made concerning the equation. Use your equation to find the value of x. Show your work or explain your answer.

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All that apply

Select **all** equations that have at least one integer solution.

- [ ] $\sqrt{4x} = 5$
- [ ] $\sqrt{3x} = 75$
- [ ] $\sqrt{x} = \frac{\sqrt{16}}{8}$
- [ ] $\sqrt{x} = x - 12$
- [ ] $\sqrt{10 - x} = x - 2$

(Smarter Balanced)

All that apply

The degree measure of an angle in a right triangle x, and $\sin x = \frac{1}{3}$. Which of the expressions are also equal to $\frac{1}{3}$. Select all that apply.

A. $\cos(x)$

B. $\cos(x - 45°)$

C. $\cos(45° - x)$

D. $\cos(60° - x)$

E. $\cos(90° - x)$

(PARCC)

All that apply

Suppose $\angle A$ is an angle such that $\cos A < \sin A$. Select all angle measures that are possible values of $\angle A$.

- ☐ 25°
- ☐ 35°
- ☐ 45°
- ☐ 55°
- ☐ 65°
- ☐ 75°

(Smarter Balanced)

Aspects of Reasoning & Prob. Solving

- Problem solving
  - What assumptions…?
  - Explore properties of unfamiliar equation…justify.
- Reasoning
  - Investigate the effect …(conjecture)
  - Draw conclusions…justify.
  - Which model is best…explain why?
  - What are all possible values/coordinates…justify?
- All that apply?

Review Textbook Examples (handout)

- Is there potential for reasoning or problem inherent in the example?
- How can you modify to promote conjecturing, generalizing, or justifying?
- How can you modify so that students must identify assumptions, select and use strategies, determine whether solutions make sense?
- How can you present challenging multiple choice questions?

Textbook Example 1

5. For which equation is the x-coordinate of the vertex at 4?

A $f(x) = x^2 - 8x + 15$

B $f(x) = -x^2 - 4x + 12$

C $f(x) = x^2 + 6x + 8$

D $f(x) = -x^2 - 2x + 2$

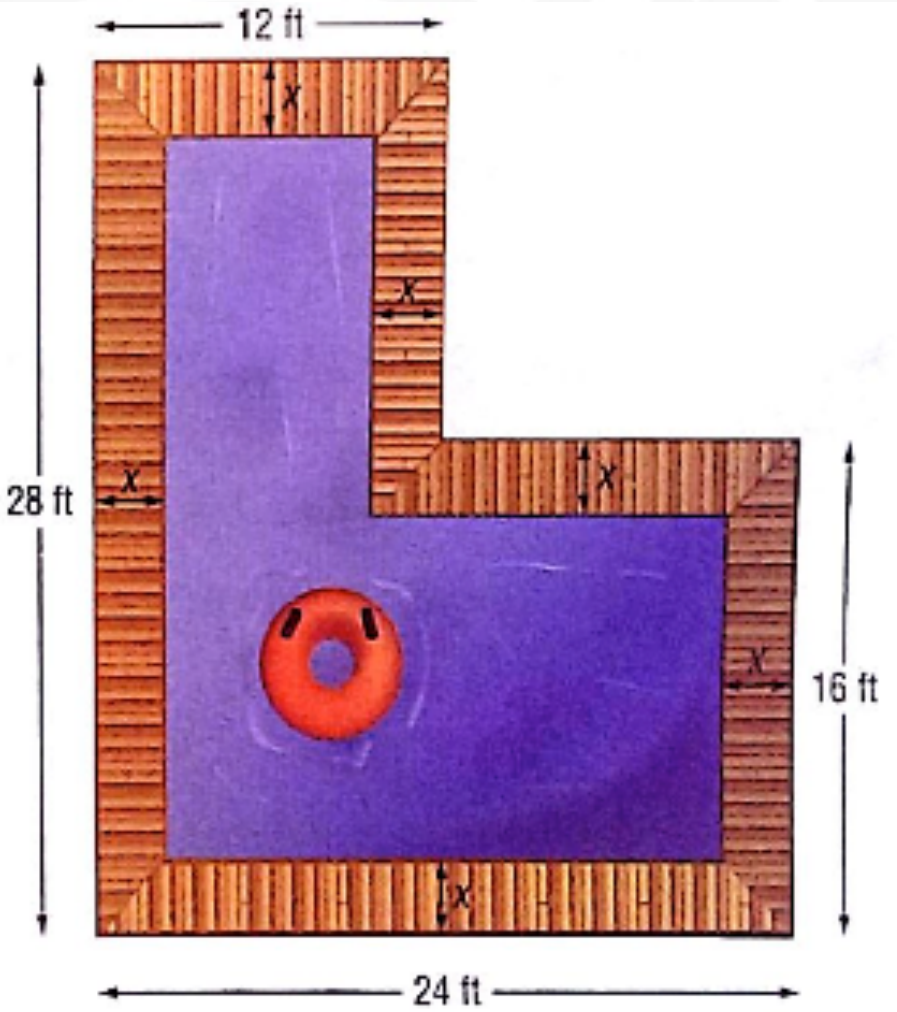
Textbook Example 2

Pool Design. Andrea wants to build a pool following the diagram at the right. The pool will be surrounded by a sidewalk of constant width.

a) If the total area of the pool itself is to be 336 ft^2, what is x?

b) If the value of x were doubled, what would be the new area of the pool?

c) If the value of x were halved, what would be the new area of the pool?



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Textbook Example 3

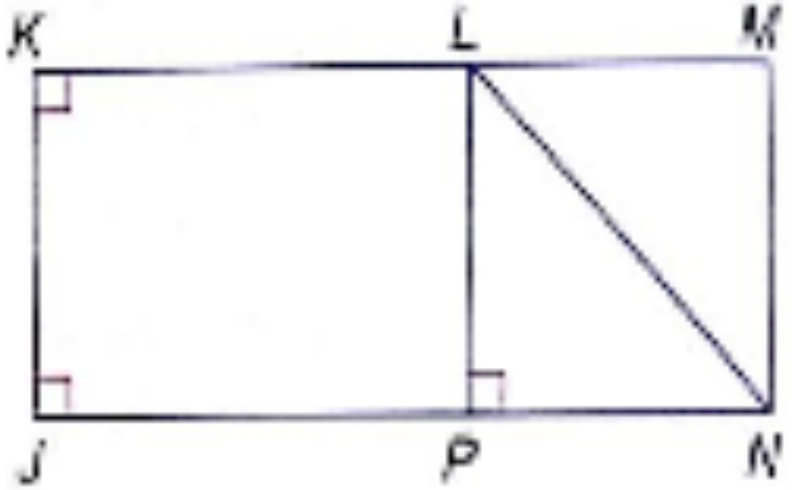
Determine whether each statement can be assumed from the figure. Explain. (Lesson 1-5)

54. $\angle KJN$ is a right angle.

55. $\angle PLN \cong \angle NLM$

56. $\angle PNL$ and $\angle MNL$ are complementary.

57. $\angle KLN$ and $\angle MLN$ are supplementary.



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Textbook Example 4

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither. Graph each line to verify your answer.

a) A(1, 5), B(4, 4), C(9, -10), D(-6, -5)

b) A(4, 2), B(-3, 1), C(6, 0), D(-10, 8)

c) A(8, 4), B(4, 3), C(4, -9), D(2, -1)

Textbook Example 5

Wildlife. A population of 100 deer is reintroduced to a wildlife preserve. Suppose the population does extremely well and the deer population doubles in two years. The number D of deer after t years is given by $D = 100 \cdot 2^{\frac{t}{2}}$.

a) How many deer will there be after 4½ years?

b) Make a table that charts the population of deer every year for the next 5 years.

c) Make a graph using your table.

d) Using your graph and table decide whether this is a reasonable trend over the long term. Explain.

Textbook Example 6

15. Sandy inherited $250,000 from her aunt in 1998. She invested the money and increased it as shown in the table below.

Year	Amount
1998	$250,000
2006	$329,202
2011	$390,989

a. Write an exponential function that could be used to predict the amount of money A after investing for t years.

b. If the money continues to grow at the same rate, in what year will it be worth $500,000?

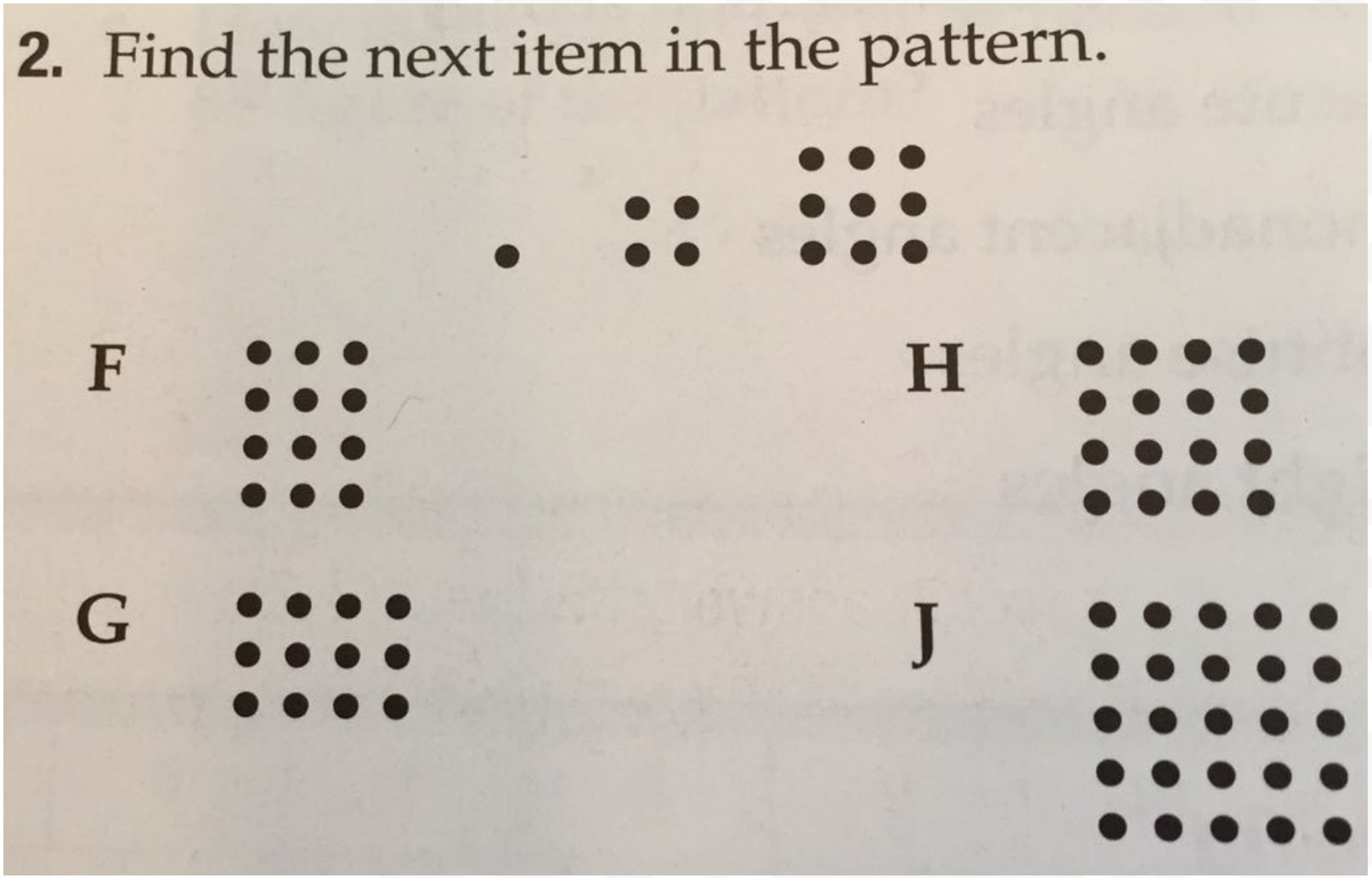
Textbook Example 7

For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

(a) A(-2, 5), B(-3, 1), C(6, 1), D(3, 5)

(b) Q(2, 5), R(-2, 1), S(-1, -6), T(9, 4)

Textbook Example 8



Textbook Example 9

14. The table at the right shows Ricardo's scores on the first 5 math quizzes this quarter. Each quiz is worth 100 points. There will be 1 more quiz this quarter.

Quiz	Score
1	86
2	79
3	80
4	85
5	77

a. In order to receive a B, Ricardo must have a quiz average of 82 or better. Write an inequality that can be solved to find the minimum score he must earn on Quiz 6.

b. Solve the inequality you wrote in part **a**.

c. What does the solution mean?

Textbook Example 10

Safety. A severe weather siren in a local city can be heard within a radius of 1.3 miles. If the mayor of the city wants a new siren that will cover double the area of the old siren, what should the radius of the new siren be? Round to the nearest tenth of a mile.

Your turn

- What strategies did you use for infusing reasoning and problem solving in textbook tasks during this activity?
- What other strategies have you used to incorporate reasoning and problem solving in daily teaching and learning activities?

References

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: The Council. http://www.nctm.org/PrinciplestoActions/

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. http://www.corestandards.org/Math/

Partnership for Assessment of Readiness for College and Careers (PARCC), http://parcc.pearson.com/

Smarter Balanced Assessment Consortium, http://www.smarterbalanced.org/

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Textbooks

Carter, J. A., Cuevas, G. J., Day, R., Malloy, C., Casey, R., & Holliday, B. (2012). *Algebra 2*. Columbus, OH: McGraw-Hill Education. (ISBN 978-0-07-895265-4)

Carter, J. A., Cuevas, G. J., Day, R., Malloy, C., & Cummins, J. (2012). *Geometry*. Columbus, OH: McGraw-Hill Education. (ISBN 978-0-07-895271-5)

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