Moving to Action: Effective Teaching Practices in the Elementary Grades

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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Equation Strings Task

Solve the set of addition problems. Each time you solve a problem, try to use the previous problem to solve the next problem.

7 + 3 = _____

17 + 3 = _____

27 + 3 = _____

37 + 3 = _____

37 + 5 = _____

After you have solved all of the problems, describe some patterns that you notice in the sequence of equations.

Show how you might represent student reasoning with a drawing or on a number line so that students could visually see the relationships among the quantities.

Connections to the Common Core State Standards for Mathematics (CCSSM)
"The Case of Jennifer DiBrienza and the Equation Strings Task"

Standards for Mathematical Content	
Domain: Number and Operations in Base Ten (NBT) Cluster: *Use place value understanding and properties of operations to add and subtract.*	
1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Source: National Governors Association Center for Best Practices and Council of Chief State School Officers. (2014). *Common core state standards for mathematics*. Washington, DC: Authors. Retrieved from http://www.corestandards.org/Math/Content/1/NBT/.

Standards for Mathematical Practice (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

SMP 7. Look for and make use of structure.

Mathematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (MP.8).

SMP 8. Look for and express regularity in repeated reasoning.

Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities.... Mathematically proficient students formulate conjectures about what they notice.... As students practice articulating their observations, they learn to communicate with greater precision (MP.6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (MP.3).

Source: Illustrative Mathematics. (2014, February 12). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Tucson, AZ. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf (p. 18-19)

Equation Strings Lesson

Teacher: Jennifer DiBrienza
New York Community School District 2
Grade: 1

Teacher: Seven plus three. So many thumbs went up right away. Let's see, Mathew.

Student: Ten.

Teacher: How'd you know Mathew?

Student: I just knew it.

Teacher: You just knew it? How many kids just knew seven plus three? Great. Keeping that in mind, the next one, you ready? *(Teacher records 17 + 3 on the board.)* Seventeen plus three. Give a minute of think time. Tasmine.

Student: Twenty.

Teacher: Twenty, how'd you get that answer?

Student: I put seventeen in my head and put three on there. *(Teacher draws an open-number line and writes 17 at the start of the number line. Makes three jumps on the number line and writes 18, 19, 20 respectively.)*

Teacher: Okay, you said you put seventeen in your head and then what?

Student: I counted three more.

Teacher: So you counted on from seventeen, you counted one, two, three more? So what's that? 18?

Student: Nineteen.

Teacher: Nineteen.

Student: Twenty.

Teacher: Twenty. Okay, great. Who did it Tasmine's way? Okay, who tried a different way? Destiny?

Student: I know, like, it's on the top 7 plus 3 equals 10 plus – and then I knew just 17 plus three was 20.

Teacher: So you – this helped you figure this one out? Okay, how did it help you figure it out?

Student: Cause it was like 10 plus 10 equals 20.

Teacher: It was like 10 plus 10 equals 20. So you already knew you had ten and you just knew you had to add 10 more on? Anybody else want to try and explain that? Danelle?

Student: She broke up the 17 into the 10 plus a 7 and she gave the 3 to the 7 so that equal 10 so 10 plus 10 equal 20.

Teacher: Okay, I'm going to put a new one up. Keep Destiny's strategy in mind. What's 27 plus 3? *(Writes 27 + 3 = __on the board.)* Aneesa.

Student: I have 27 in my head and 27, 20 … 27, 28, 29, 30.

Teacher: Okay, so you counted on from 27, you started at 27 and you went 28, 29, 30. *(Draws an open-number line and writes 27 at the start of the line. Marks three jumps, writing 28, 29, 30 at each of the respective marks. Records 27 + 3 = 30.)* Okay. What did you notice?

Student: It's going by all zeros.

Teacher: Oh, they all end in a zero?

Student: Up to the one thing.

Teacher: So you notice they all end in zeros. What else did you notice?

Student: And threes, 1, 2, 3 and 7.

Teacher: Whoa, 10, 20, 30?

Student: And tens and threes.

Teacher: And threes.

Student: And sevens.

Teacher: And sevens.

Student: *[Inaudible comment]*

Teacher: Raphael? So people notice that there's sevens in every column here, there's threes going down here and it's going 10, 20, 30. Why do you think that's happening? Let's look over here on our hundreds board. We started out with 7 right?

Student: Yeah.

Teacher: Jamie.

Student: Yes.

Teacher: We start out with seven and we added three right? One, two, three. And then when we did 17 you said we added 10 more on right? *(Teacher points to the numbers on the hundreds chart.)* Let's see, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Look, we just moved down one right?

Student: And 27's

Teacher: Twenty-seven's right below it. How many more do you think if I start at 17, how many more do you think I have to add to get to 27? *(Points to 17 on the hundreds chart and then moves down to 27.)* I'm going to move on to the next one. If 37 plus 3 more is 40, what's 37 plus 5? *(Records 37 + 5 = __.)* I want to see lots of hands up, 37 plus 5. I'm going to give everyone a chance to think about it.

Student: I had 37 in my head.

Teacher: You had 37 in your head. *(Records an open-number line on the board and writes 37 at the start of the number line.)*

Student: Then I counted five more.

Teacher: You counted five more, 38, 39, 40, 41, 42 like that? *(Starts at 37, marks a jump, records 38, marks a jump and records 39, marks a jump and records 40, and continues until 42 is reached.)* Okay, did anybody do it differently? I want to know what you noticed, did anyone do it differently?

Student: I ah… broke up the 5 into 3 plus 2.

Student: I think 3, the 27.

Teacher: Okay to the 37.

Student: Yes.

Teacher: Okay.

Student: And that equals 40.

Teacher: You just knew that one right?

Student: Yeah.

Teacher: Okay, and what did you have left?

Student: And I had left the two.

Teacher: And that left two. So, instead of just making jumps of one you started at 37 and you knew the three more made 40 so you knew you had to add 2 more and that would give you 42. *(Draws an open number line and records 37. Marks a jump of 3 and records a sum of 40. Then makes a jump of 2 and* records a *sum of 42.)* Yeah? Is that how you were thinking about it?

[End of Audio]

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

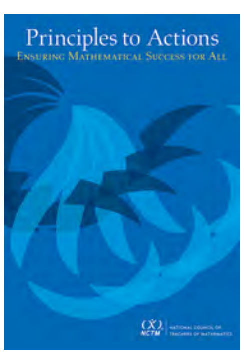
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*



National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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Fluency

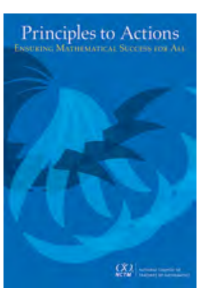
Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.

Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems. This sequence is beneficial whether students are building toward fluency with single- and multi-digit computation with whole numbers or fluency with, for example, fraction operations, proportional relationships, measurement formulas, or algebraic procedures.

Conceptual Understanding and Procedural Fluency

When procedures are connected with the underlying concepts, students have better retention of the procedures and are more able to apply them in new situations (Fuson, Kalchman, and Bransford, 2005). Martin (2009, p. 165) describes some of the reasons that fluency depends on and extends from conceptual understanding:

> *To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.*



National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author. (p. 42)