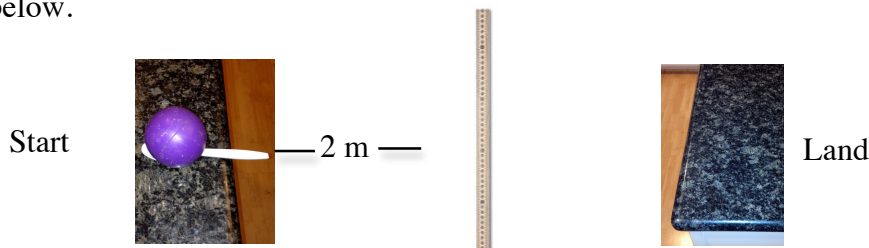


Spoon Launcher

Setting up the Scenario

Your team will be using a plastic spoon to launch a ball over an object. You will be measuring the time the ball just clears the top of your object and the time the ball hits the table. You will graph the data comparing the time to the height of the ball. See diagram below.



Prediction

What type of graph do you think the data will produce? Describe any features of the graph using the words in the box to the right.

Linear	Maximum	Range
Quadratic	Minimum	symmetry
Non-Linear	Increasing	positive
Exponential	Decreasing	negative
y-intercept	Domain	rate of change

The Experiment

Set up your team so that the following conditions are in place.

- ◇ The spoon must rest on the edge of the table or a thick book with the handle hanging off.
- ◇ The ball must be placed on the spoon.
- ◇ You must select an object at least 2 meters high (or use 2 meter sticks) that is at least 2 meters away from the spoon that the ball will clear when launched.
- ◇ The ball must be able to land on a table or thick book of the same height as where it started.

Select roles of the team members.

- ◇ Launcher-hits the handle portion of the spoon to launch the ball so that it JUST clears the given height.
- ◇ Holder of the Meter sticks (or object to clear)- holds meter sticks and verifies if the ball just cleared the top.
- ◇ Timer 1 and Timer 2- look at time when ball clears height and time when it hits the table.

Conduct the Experiment

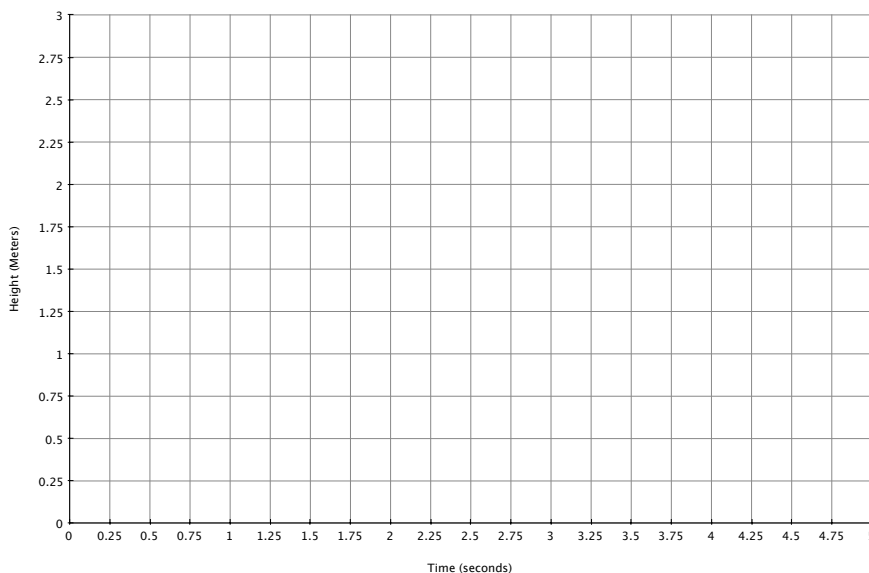
With the holder and timers in place, the launcher will make attempts to get the right launch so that the ball will just clear the height. Record the time of the attempt that is most accurate (the ball is closest to the height without touching it).

Data Table

Beginning of Experiment		When Ball Cleared Height		When Ball Landed	
Time	Height	Time	Height	Time	Height

Graph the data

Graph the three data points from your table on the graph below.



Analysis- Answer the questions to help you determine what type of function best represents this data.

1. Do the data represent a function? Why or why not?
2. Are the data continuous (does it make sense to connect the points)?
3. Is the rate of change consistent?
4. Given your three data points and the knowledge of the experiment, what type of graph do you think best represents this data? Sketch this curve.
5. Explain what is happening with the graph using some of the words from the box to the right.

Beginning	Maximum	Range
End	Minimum	symmetry
Spoon	Increasing	positive
Die	Decreasing	negative
Domain	rate of change	

Angry Birds Launch Video

Watch the video of the 3 launches from Angry Birds. Using the sketches below for each bird, record a point where the bird would likely hit the ground and then sketch the curve representing the path the bird took.

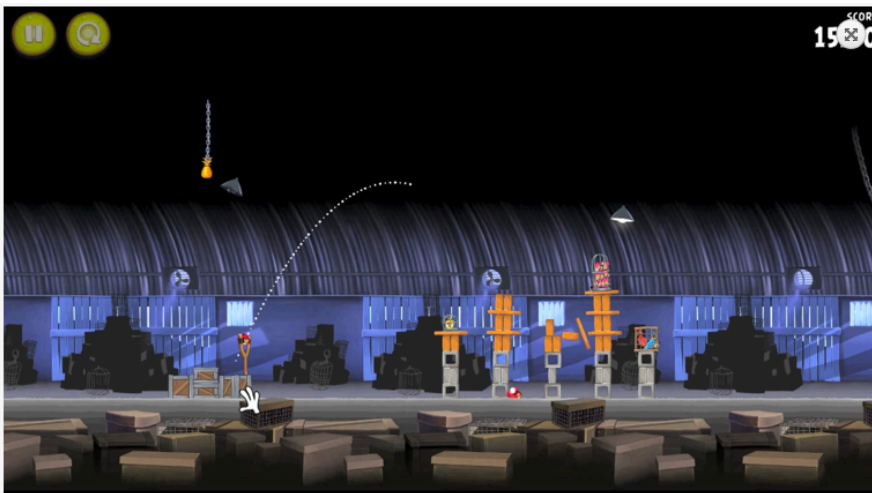
Bird 1



Bird 2

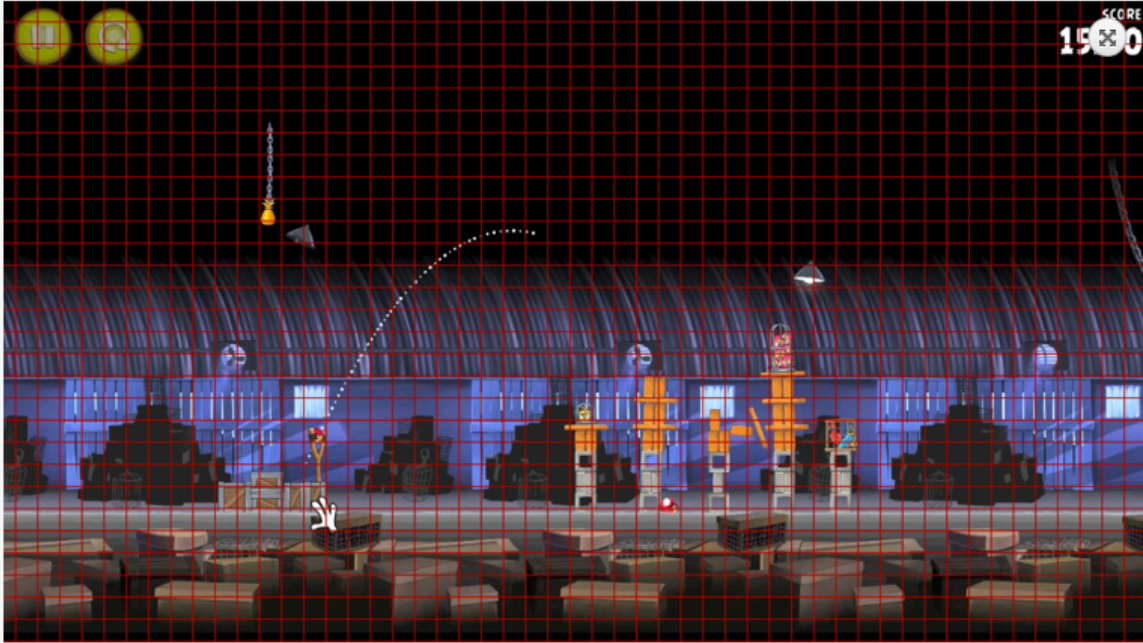


Bird 3



Angry Bird #3 on Coordinate Axes

Using part of the flight path for Angry Bird #3, sketch the likely continued path of this bird using the coordinate grid placed on top of the picture. Note- you will need to decide where to draw the x- and y-axis.



1. Compare your sketch to the person sitting next to you. How are the graphs similar and how are they different?

2. We call the graph of this type of function a *parabola*. Describe what a parabola is in your own words.

3. Describe what type of data you think makes a parabola when graphed.

Teacher Directions

Materials:

Plastic Spoon- 1 per group

Meter Sticks- 2 per group

Small Ball (raquet or large bouncy ball- NOT ping pong)- 1 per group

Timers on phones- 2 per group

Space for launch (approximately 4-8 meters in length and 3-5 meters in height)- per group

Objective:

In this hook lesson, students will launch a ball over a given height and graph the data comparing time and height to begin to gain an understanding of parabolas.

Directions:

Pass out the activity sheet and have a student read the **setting up the scenario** aloud.

Model the set up of the experiment (and launch a ball if you want!) for the class.

Direct the class attention to the **prediction** section. Give each students 2 minutes to silently think about and write down their ideas. Note: it is not expected that students know how to use all of the words, but encourage them to use as many as they can. After the 2 minutes, have students share their predictions with an elbow partner and call on a few students to share with the class.

The Experiment

Note- if you are unable to let students do this experiment on their own, a class demonstration is the next best option.

You will need to decide beforehand where to conduct the experiment (note the space needed) as well as if students can launch from a table to another table/bench (such as at the lunch area) or in the hall from a thick textbook onto a row of textbooks.

Once you are confident the students understand the set up, have someone from each group of 4 come get the supplies- 1 spoon, 2 meter sticks, and 1 ball. Encourage students to use timers on their phone. Have the group assign roles (or you can assign based upon a characteristic). Tell the class they will have 5 minutes to complete the experiment.

Before they begin, question the class to ensure they understand what they need to do and record- 1) launch the ball so that is JUST barely clears the 2 meter mark; 2) Make note of the *time* the ball passes the 2 meter mark; and 3) make note of the *time* the ball hits the table/book (original starting height). Have the students record this information in their data table (for groups who are unsure of what to put for “beginning of experiment”, question them to see that before they launched, time was 0 and height was 0.

Graph the data

Have students complete their graph of the 3 points. Note that most graphs will have a domain of 0 to between 1 and 2 seconds. Have the groups answer the four analysis questions together while you circulate to ask questions. Here are some guiding questions. For #1, is height dependent upon time? Can the ball be at two different

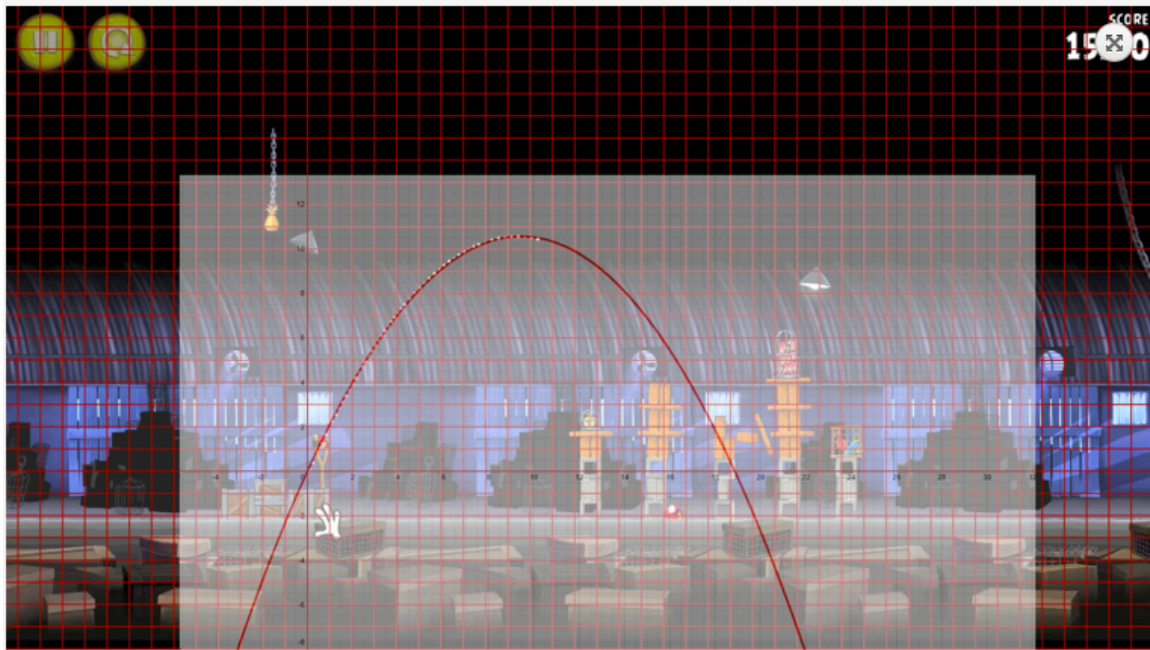
heights at the same time? For #2, would the point (for example) .5 seconds, 1.25 meters make sense in the context of the problem? For #3, is the graph growing by the same amount every 10 seconds? After a few minutes to discuss, come back as a class to go over the answers for questions 1-4 and then give the class 2 minutes to complete question #5 on their own.

Angry Birds Launch (lesson idea, video and pictures from Robert Kaplinsky)

Explain to the class that you will now look at a similar experiment where they data is more accurate. Explain that they will watch the video of three angry birds being launched and for each bird, they need to sketch, on the appropriate figure, where the bird landed. Pause the video after each bird is launched. Once all three are done, give the class 2 minutes to sketch their curves to represent the flight path (of height over time). The link to the video is pasted below (if it does not work, search You Tube- Using angry birds to find the equation of a quadratic function from a graph).

<http://www.youtube.com/watch?v=H8YBiJF62Us>

Direct the class attention to **Angry Bird #3 on Coordinate Axes**. Have the students, individually, use the beginning of the curve to a) decide where to draw the x- and y-axes and 2) sketch the path so that the bird hits where they predicted above. Have the students complete question 1 by comparing their graph to the person next to them. Then put up your sketch of the path (shown below).



Finally, give the class 2 minutes to complete the final two questions. Collect these two answers as well as analysis question #5 from the previous section as your formative assessment and as a guide for what students will need support with throughout this unit.

Rolling Hot Wheels

Scenario: You will be dropping a hot wheel down a track that is at an incline. Do you predict the relationship between time (seconds) and height of the car on the track (in cm) will represent a function? If, what type of function and why?

Setting up the Experiment

Set up your hot wheel tracks so that one end of the track is clamped at a height between 25 and 60 cm. Connect the track pieces and place textbooks (or other objects) under the connections of each set of tracks so that a consistent incline is supported throughout. Attach enough tracks until the final track is flat on the ground. See picture below.



Conducting the Experiment

Before beginning the experiment, measure the height of the ramp at the starting point and then at the location of each “connector” and record this in the table below.

Your group needs to assign the following roles:

- ◇ **Starter**
- ◇ **Timer 1**
- ◇ **Timer 2**
- ◇ **Recorder**

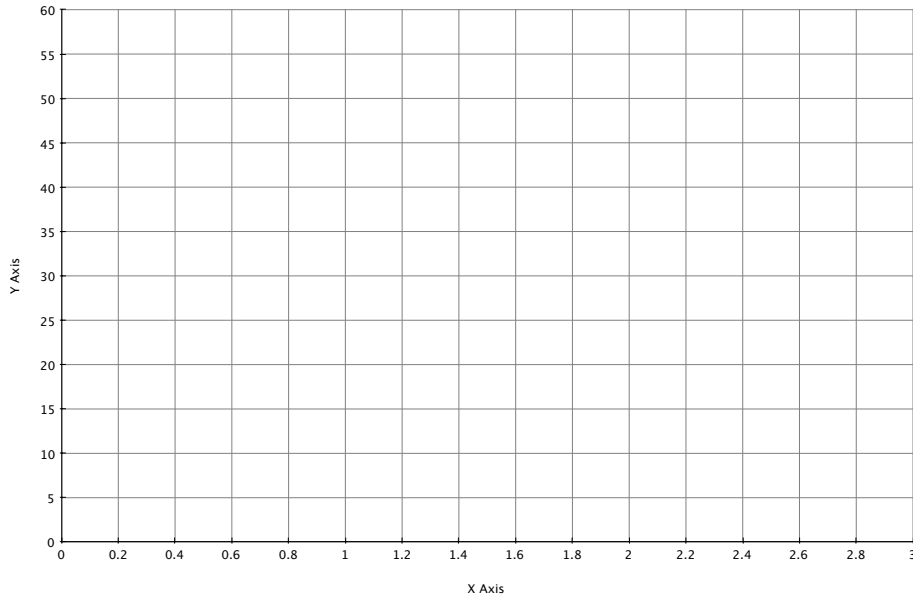
The starter will release the car from the top of the track after saying, “on your mark, get set, go”. Upon hearing “go”, both timers begin their stopwatch. The timers need to be standing where the first two tracks connect. You will need to repeat the rolling of the car *multiple* times for each distance until both timers agree on a consistent time. Repeat the experiment, measuring the time it takes the car to get to each of the connectors and record the times below.

	Height at beginning	Height at 1 st connector	Height at 2 nd connector	Height at 3 rd connector	Height at 4 th connector	Height at 5 th connector
Time						

Organizing the Data: Take the data you collected and record it in the t-chart below

Time (seconds)	Height of Ramp (CM)

Graph the Data:



Analyze the Data

1. Looking at the t-chart and the graph, what type of function do you believe the data represent? Why?
2. What is the domain of the data graphed?
3. What is the range of the data graphed?
4. How do you think the graph of the data would change if you raised or lowered the starting height of the ramp?
5. Trial #2: Repeat the experiment with a starting height (still between 25 and 60 cm) that is at least 10 cm different than your original height. Graph the data on the same axes above. How did your prediction match the results?

Teacher Directions

Materials:

Hot Wheel Cars (1 per group)
Hot Wheel Track with connectors (approximately 6-10 per group)
Hot Wheel Clamp for track (1 per group)
Timers (easiest to use phones)- 2 per group
Rulers (1 per group)
Textbooks (or other stackable objects)- approximately 15 per group

Objective:

Students will collect data representing time (in seconds) vs. height of a car being rolled down a ramp to understand the effect of gravity over time as a quadratic function. Students will graph the data and determine which type of function the data and scenario represents. Students will compare the graphs of two quadratic functions.

Directions:

Model the set up of the hot wheel track and explain to the class that you will release the car and measure the height of the car (cm) over time (seconds). Pass out the activity sheet and have the students complete the prediction question. Have the groups determine which roles each person will have. Have groups come get the necessary materials and give them 20 minutes to collect data. Encourage MULTIPLE trials for each connector to get the most accurate times as possible. Optional- have students use cell phone cameras to video and go back to frames to see time (if they have this capability).

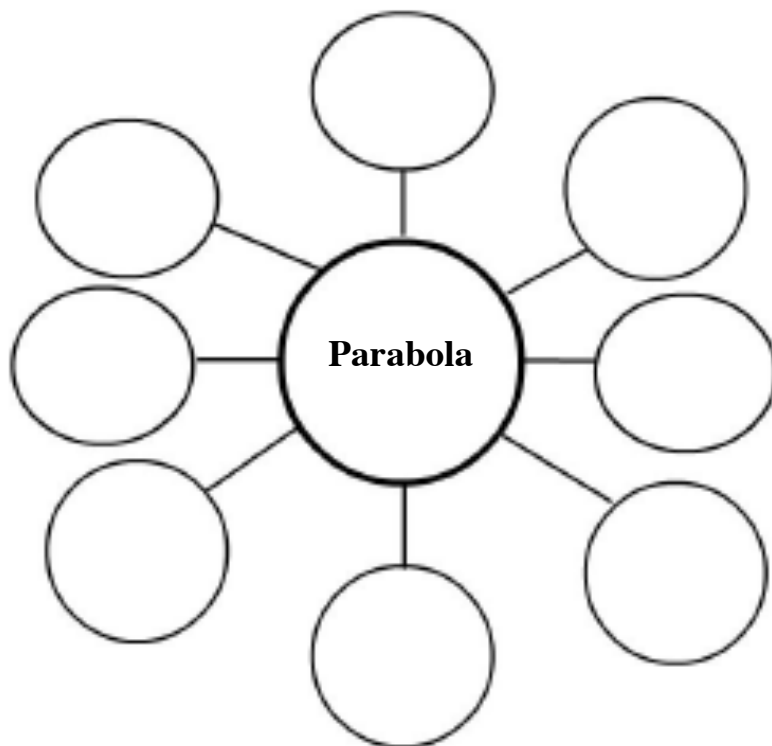
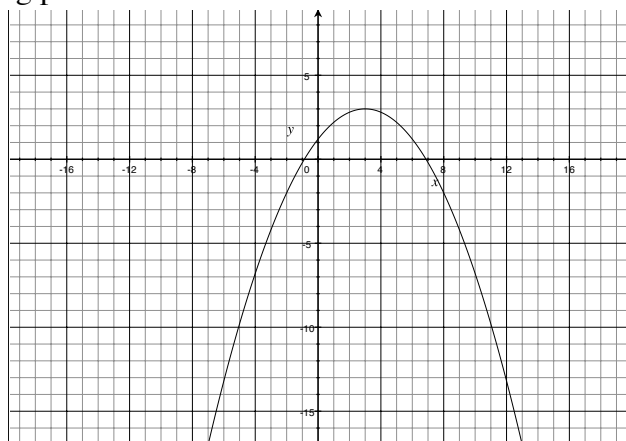
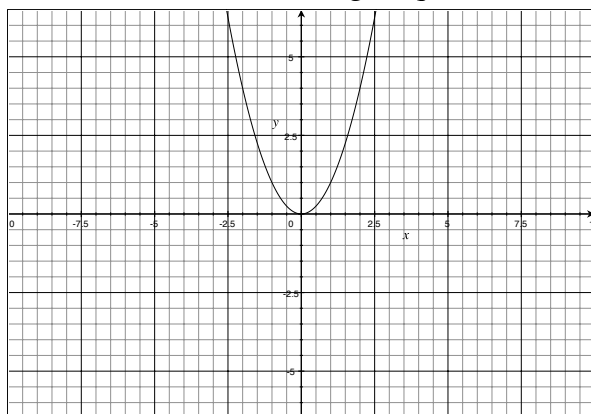
Once data is collected, have students work independently to complete the t-chart, graph and questions 1-4. Have groups discuss their answers and then use random selection to have students come share their graphs and explain what type of function they believe the data represent. Encourage the class to consider the effects of gravity ($-9.8m/s^2$) in considering what type of function the data represent.

Ask each group to report their prediction for question #5 and then have the students repeat the experiment from a different starting height, graphing the data on the same axes. Have groups report out on what change they made and how this affected their graph.

What is a Parabola?

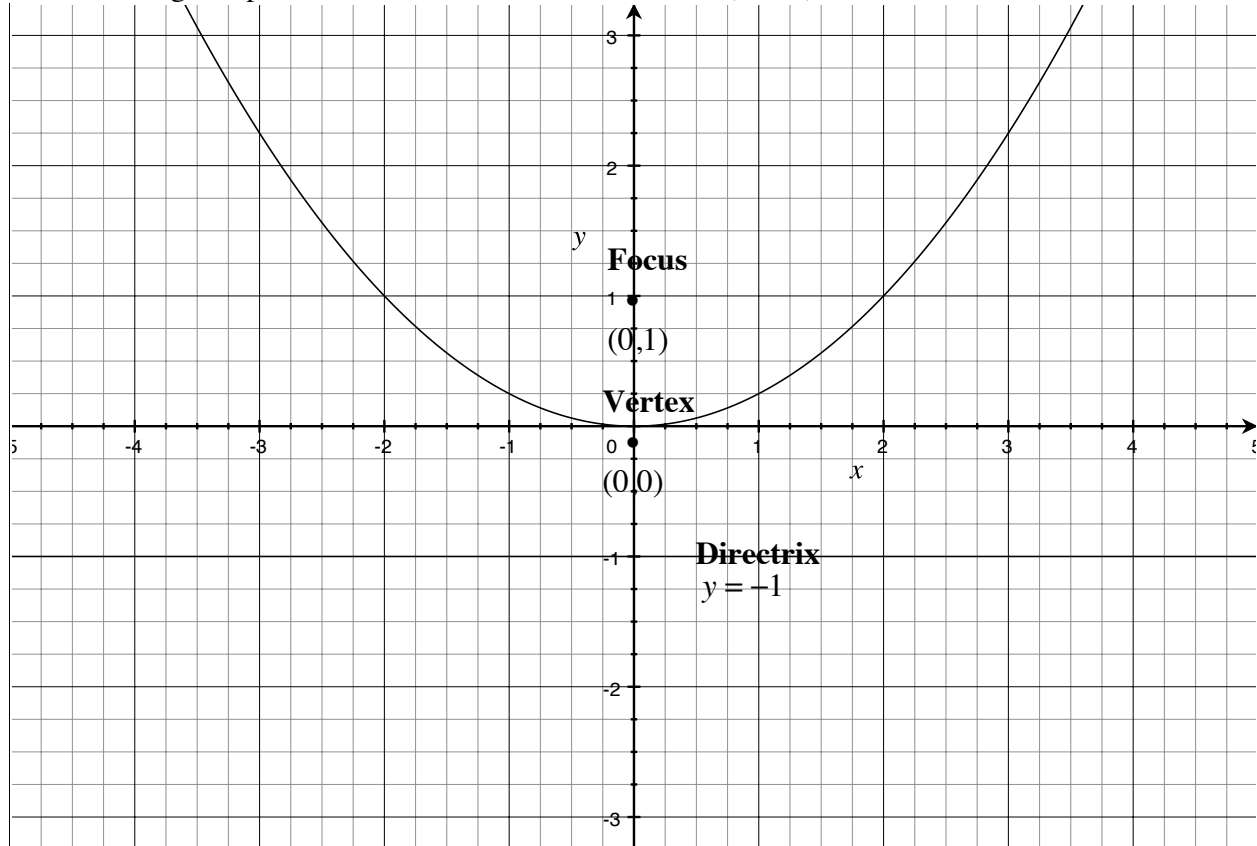
Opening Brainstorm

The last few days you have been studying patterns and data that result in the graph of a **parabola** (quadratic functions). Below are two examples of parabolas. Using what you see and what you know, list ALL the features of a parabola on the bubble map below. Consider its appearance (how it looks), rate of change, equations, interesting points or features.



Defining a Parabola

The following is a parabola with the focus, the directrix (a line), and the vertex marked.



Mark three points on the parabola and fill out the following chart. Use your ruler to find the distance from the focus to the point and from the point to the directrix (using a segment perpendicular to the directrix (measure in millimeters)).

Point	Distance to focus	Distance to directrix (line from point, perpendicular to directrix)
A		
B		
C		
D		

1. What do you notice about the distance to the focus and the distance to the directrix in each of the cases in the table above?

2. We can define the parabola to be a “locus of points” - a set of points that follow a rule. Using what you discovered above, how would you define a **parabola**?

3. What is the **focus** of a parabola?

4. What is the **directrix** of a parabola?

Creating your own parabola

- ◇ You need to work in a group of 3
- ◇ Each group needs a piece of chalk, a rope at least 4 feet in length and an index card.
- ◇ Assign group roles: P- marks the points of the **parabola**; F- marks and stands on the **focus**; D- draws and stands on the **directrix**.
- ◇ Once outside, begin by have person F and D pull the rope tight and mark a point at the end of each side of the rope to represent the **focus** and a point on the **directrix**.
- ◇ Next, have person D draw a line of approximately 10-15 feet lying on the **directrix**.
- ◇ Now, figure out how to use the rope and chalk (now in person P's hands) to draw enough points on the **parabola** so that you can accurately sketch it. Person D must ensure the rope creates a 90° angle from the directrix to the point on the parabola by using the index card to ensure the line segment is perpendicular. Note: Recall what distance you need from each point on the **parabola** to both the **focus** and the **directrix**.

Sample Set up outside

• Focus



Summary Questions

1. How did your team create the parabola?

2. What is the definition of a parabola?

3. Draw a picture to illustrate the definition.

4. How could you find the vertex of the parabola you drew outside?

5. What would happen to the parabola if the focus moved closer to the directrix?

6. What would happen to the parabola if the focus moved further from the directrix?

Teacher Directions

Materials:

Chalk- 1 piece per group of 3

Rope- 1 piece at least 4 feet in length (up to a maximum of 12 feet) per group of 3

Index Cards- 1 per group of 3

Rulers- 1 per person

Objective:

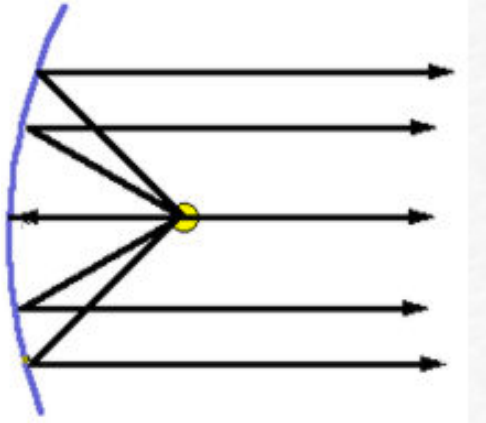
Students will share prior knowledge of parabolas and then investigate the geometric properties of a parabola by measuring the distance from points along the parabola to the focus and directed distances to the directrix to discover all points along the parabola are equidistant from the focus and directrix. To further this conceptual understanding, students will work in teams of three using rope and chalk to build a parabola from a given focus and directrix. Note: Students will derive the equation of a parabola given the focus and directrix later in the year with coordinate geometry; this lesson is meant as conceptual understanding of what makes a curve a parabola and how we define a parabola.

Directions:

Begin by asking students if they have ever seen parabolas used in the real world. Elicit a few responses (honoring those that are and saying “we’ll learn more about these today” for those inaccurate suggestions). Then show the picture of a car’s headlight below.



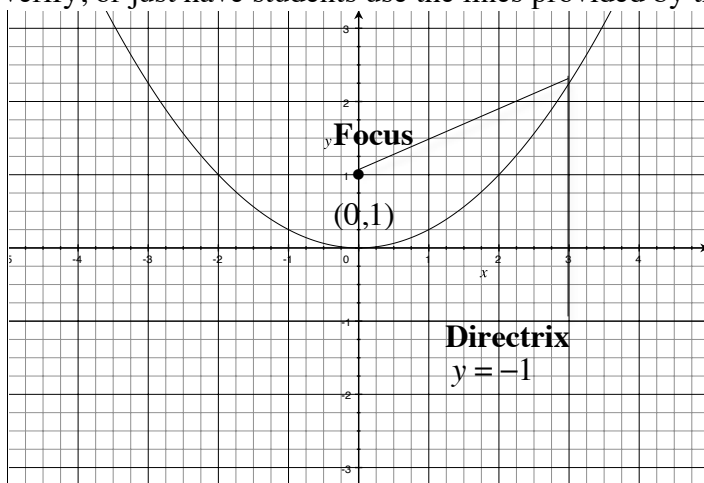
Ask the class if they know how it is possible for the light to reflect out of the headlight in parallel lines (beams). Explain that headlights use parabolic reflectors and show the next picture.



Tell the students that today they will come to understand some special features that make a “curve” a parabola.

Pass out the activity sheet. Have a student read the directions aloud and then give the students 3 minutes to complete the bubble map. Select students at random to share and add the features to the class bubble map you create up front. Make sure to get the following main features from the students (or guide them to notice these): it is symmetrical, it has a vertex (or maximum or minimum), it is a curve.

Next explain to the class that they will now investigate what makes some curves parabolas. Direct the class’ attention to page 2 and the picture at the top of the page. Pass out rulers to each student. Explain and explicitly show them the point called the vertex, the point called the focus and the line called the directrix. Let them know they will be investigating those in this lesson. Choose a point on the parabola to model what distance to calculate for the students. List this point in the first row of the table. From the point you choose, draw a line directly to the focus and use your ruler to measure (to the nearest mm) this distance and record this in the table. Now show the class how you will draw and measure a line segment from the point on the parabola to the directrix such that it forms a right angle or is perpendicular (an index card can be used to verify, or just have students use the lines provided by the graph).



Now have the students choose three other points on the parabola and repeat the same steps. Give them 5 minutes to do this and then use roundtable to call on students from each group to share a data point and record this on a class chart. Once you have about 10 rows complete, have the students think silently and then record their thoughts to question #1. Allow them to share ideas with a partner and then select students to share. Once students understand that the distance from the any point to the focus and to the directrix are equal, regardless of the location of the point on the parabola, ask students to answer #2-#4 individually.

The next part involves taking the class outside to build parabolas. An alternative to this is doing this part on large paper using string instead of rope and pencils instead of chalk. Have a volunteer read through the steps of **creating your own parabola**. Question students to make sure they understand the directions. Once outside in groups of 3, pass out a piece of chalk, a rope and an index card to each group. Give them a maximum of 10 minutes to create their parabola. Make sure to circulate and verify that the rope forms a 90° angle from the directrix to the point on the parabola and that the rope goes directly from the point on the parabola to the focus (likely on a diagonal). After the 10 minutes have elapsed, have students come look at one example while that group models how they created their parabola.

Go back inside and complete the lesson by having individuals answer the summary questions (you may wish to collect this as a ticket out the door). Note that questions 5 & 6 are extension questions and you should not expect all students to understand this from this short lab (but it will be addressed later).

Packing in Seats on an Airplane



Background: An airline is investigating ways to increase their profits. They have noticed a trend where airlines sell seats with “more leg room” for higher prices. Using this idea, they want to investigate what would happen if they added more seats to each flight but *dropped* the price of every seat.

Predict:

1. Do you think an airline could make more money by adding additional seats to each flight even if that means dropping the price per seat? Why or why not? What type of addition in number of seats and reduction in price do you think would be ideal?

Investigate:

One airplane has 140 seats and the airline charges \$450 per seat for a certain flight.

2. How much money will the airline collect for a full flight? Show your math below.

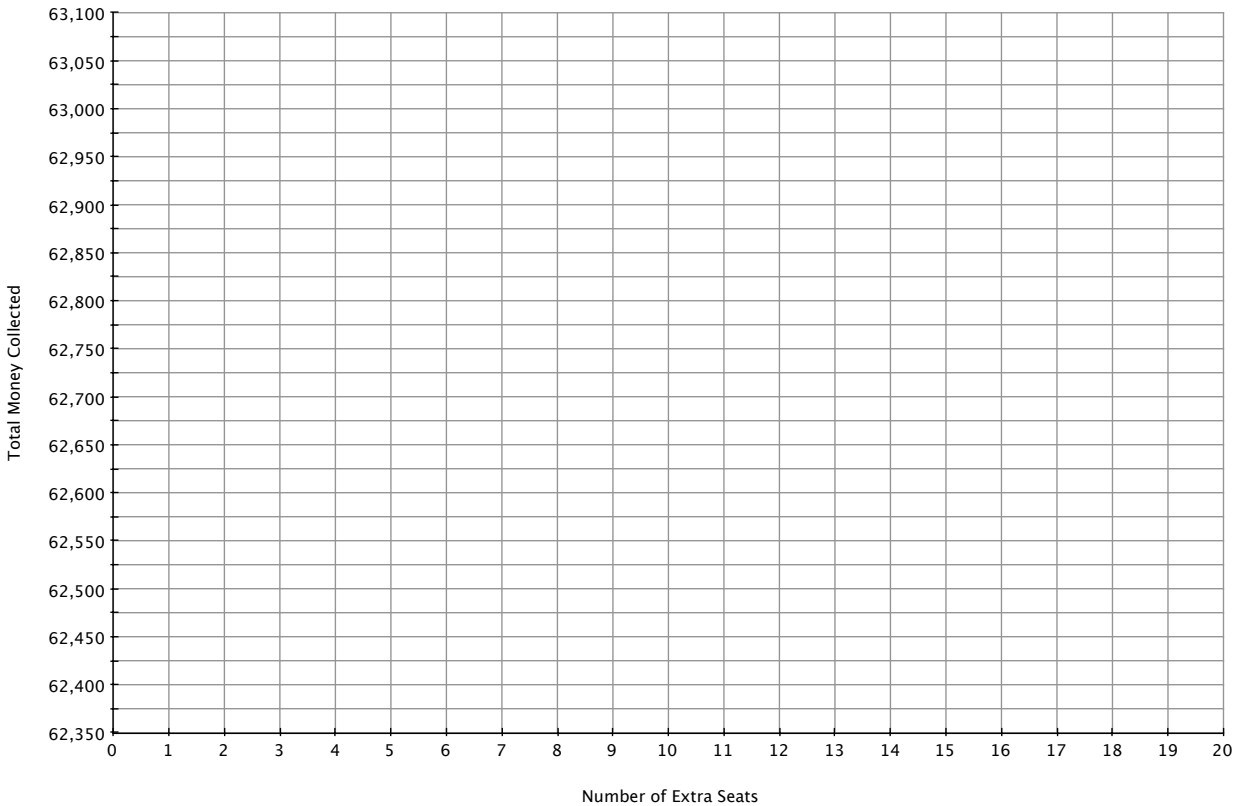
3. The airline will reduce the cost of each ticket by \$3 for every additional seat they add. Using the table below, calculate the amount of money the airline will collect for each full flight.

Number of Additional Seats	Total Number of Seats	Cost per Ticket	Total Money Collected
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
20			
x			

4. How many additional seats should the airlines add to maximize profits?

Graph

5. Using the graph below, plot the points from the table representing number of additional seats and total money collected.

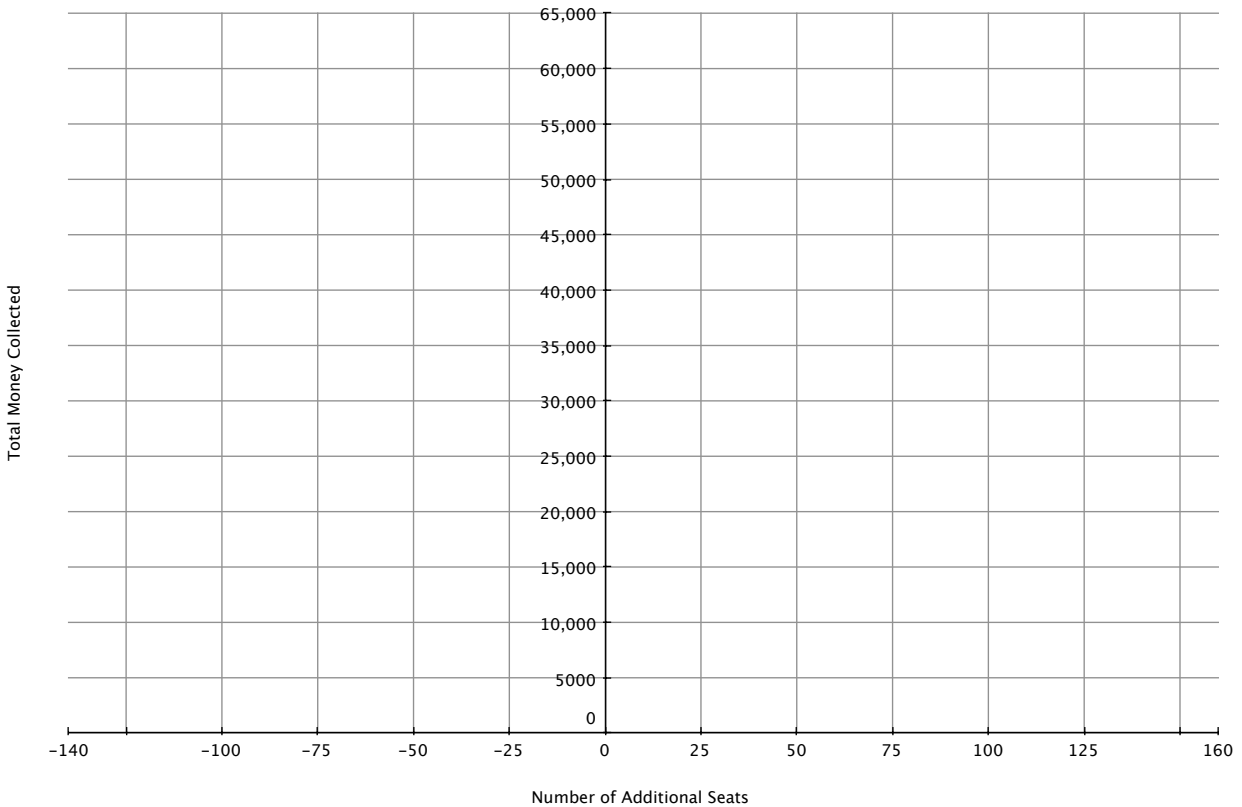


6. What type of function is represented by the graph?

7. Where does the answer you wrote in #4 show up in the graph? What is the meaning of this point? What do we call this point?

8. Should the points on the graph be connected? Why or why not? (Is the graph continuous?)

9. Sketching the curve represented by this data can be helpful to find the x-intercepts. Find the x-intercepts using the graph below on the next page (it will be helpful to find the total money collected for 50 and 100 additional seats to make the graph) and one other method. What do the x-intercepts mean in the context of this problem?



10. What values of x make sense for this set of data (what is the domain)? Why?
11. What values of $f(x)$ make sense for this set of data (what is the range)? Why?
12. Over what interval(s) is the graph increasing? What does this represent?
13. Over what interval(s) is the graph decreasing? What does this represent?
14. How would the graph of the function change if the airlines only reduced the price by \$2 per seat for each additional seat added?
15. How would the graph of this function change if the airlines began with more seats? What if it began with a higher price for the flight?



Teacher Directions

Materials:

- ◇ Calculators

Show the students the picture below of an airline who has seats with different amounts of leg room.



Ask a student to read the **background** section aloud. Have a second student read the **predict** section aloud. Once you are confident the students understand what is happening in the scenario (question students to make sure and draw a diagram if it helps), give students 2 minutes to silently think about and write down their prediction. Have students share their ideas with an elbow partner and then randomly select students to share their ideas. Note that without numbers, there is not a “right” answer, so take and encourage all good thinking.

Direct the students’ attention to the **investigate** section. Give the class 1 minute to read and independently complete #2. Then ask the students to complete the first row of the table and stop to have a student explain the numbers they wrote in the first row and why. When you are sure the students are able to correctly calculate, have them complete the rest of the table and answer #4. Once they have finished #4, have them discuss their answer with a partner. Set the timer for 10 minutes for students to complete problems 4-8 independently and then have them share ideas with their partner or group. Select students to share their graph and answers to the questions. The big idea of these questions is for students to see this as a quadratic or non-linear function and that the maximum or vertex is the solution they saw in the table.

Direct the students’ attention to #9 and the graph on the beginning of page 3. Make sure students go back to the table to fill in a row for 50 and 100 additional seats or they will not be able to extend the parabola to find the intercepts. Encourage students to also find another method to determine the intercepts (such as calculating when the cost of a seat would be \$0).

Finally, have students complete questions 10-15 independently and then discuss with their group before having a class discussion.

Name: _____ Date : _____ Period: _____

Multiplying Linear Functions

1) What do you expect to see if the function values of two functions whose graphs are two lines are multiplied together?

2) Move the lines around. Write down at-least three interesting observations you have made.

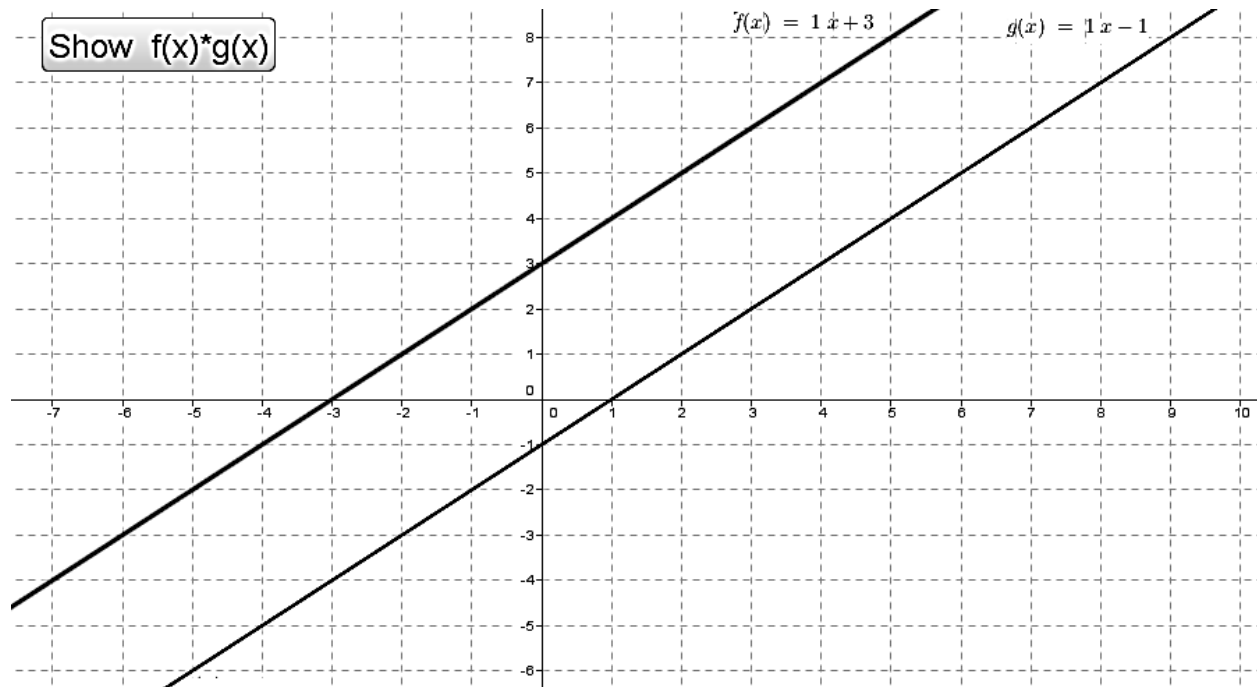
1. _____

2. _____

3. _____

3) Write down any interesting observations your classmates have made.

4. Without using Geogebra, graph the parabola created by multiplying $f(x)$ and $g(x)$.

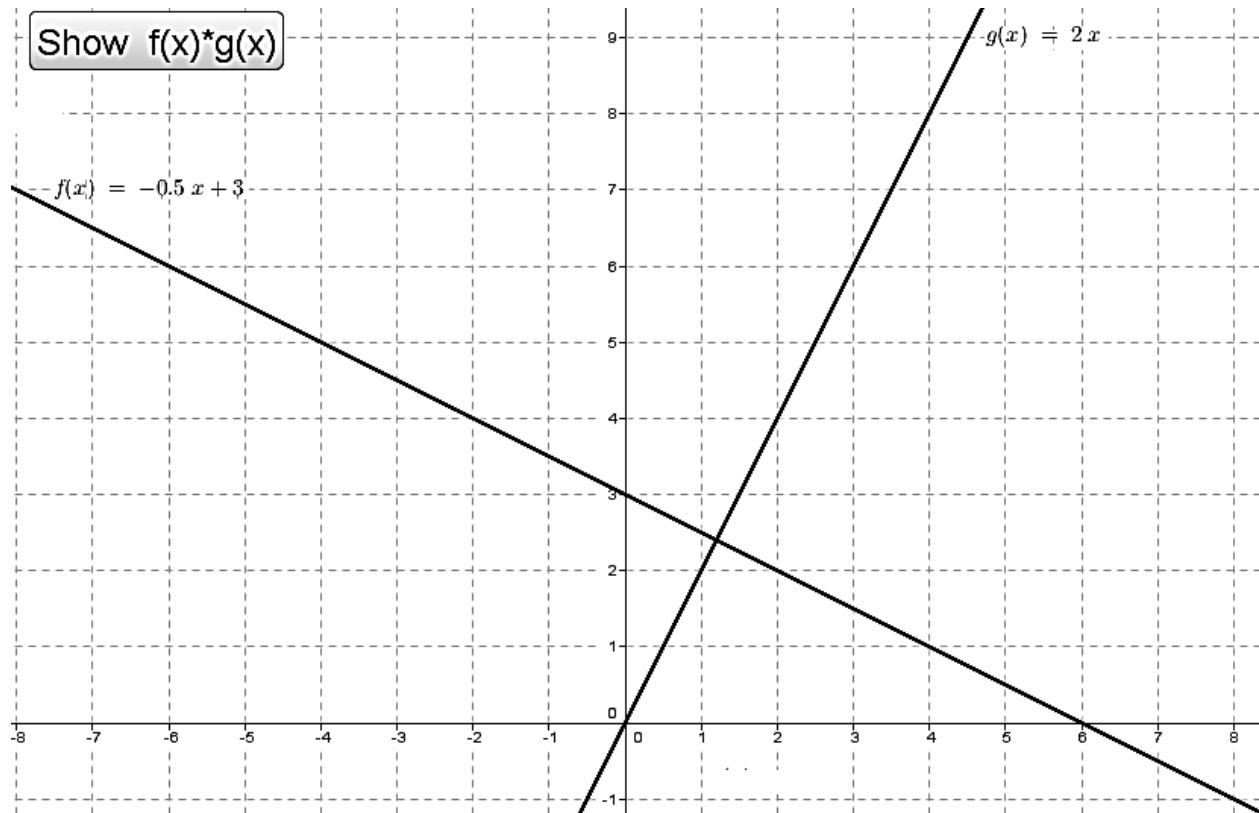


5. Identify the vertex of the parabola. Draw in the axis of symmetry of the parabola.

Name: _____ Date : _____ Period: _____

6. Use Geogebra to determine when a parabola will open up or down. Explain your findings below.

7. Without using Geogebra, graph the parabola created by multiplying $f(x)$ and $g(x)$.



8. Use the graph to explain why the parabola will open downward.

9. Challenge: Find the vertex of the parabola in problem 6.

Name: _____ Date : _____ Period: _____

Teacher Directions:

Objective: Build parabolas from linear functions. Develop vocabulary related to key features of a parabola.

Materials:

Teacher Demonstrates: Geogebra/Computer/Projector

Students Investigate: Laptop/Computer that can access Geogebra- 1 per pair

Color Pencils or Markers: 1 marker or color pencil per student

<http://www.geogebra.org/m433453>

Directions:

This activity can be done with teacher modeling to whole class or students working in pairs on Laptop/Computer.

Open the geogebra page and have to the two lines displayed: $f(x) = x + 1$ and $g(x) = x - 1$.

Ask questions to review linear functions.

- What can you tell me about the two lines? How do you know this?
- Can you determine which line is $f(x)$ and which is $g(x)$?
- What will happen to the graph if we change the definition of $f(x)$ to $f(x) = 2x + 1$?
- What will happen to the graph if we change the definition of $g(x)$ to $g(x) = x - 3$?
- How can you find the x and y intercept of one of the lines?

Have students read Question 1 and Think-Pair-Share?

Students may state that the graph will be a parabola. But some will still state it will be a line. Try to have all different types of answers shared with the class. Once the class has finished the discussion, click on the button: Show $f(x) \cdot g(x)$. This will verify that the product of the two given linear functions has a graph which is a parabola. **** Remind the students that the graph is called a parabola. ****

Question 2: Give students time to manipulate the linear functions and determine patterns.

Observations will vary. The goal will be to have student share that the x-intercepts of the linear functions are the x-intercepts of the parabola. Many interesting topics of discussion are possible based on this investigation. For example, the product of what two linear functions will produce a function whose graph is a parabola with only 1 x-intercept. Why? Or the product of two linear functions will not always produce a parabola? Give examples. Can we tell if the product of two linear functions will produce a wide or narrow parabola? Explain. *** We will explore parabolas opening up and down in problem 6 so for now acknowledge that some parabolas open upwards and other open downwards.

Question 4: Let students struggle with creating the graph of the parabola. If students need help remind them that the parabola will be the product of the $f(x)$ and $g(x)$. Before moving on have students label the vertex and axis of symmetry. Ask students, what transformation do they see? Show how each point on the graph is a reflection across the axis of symmetry.

Name: _____ Date : _____ Period: _____

Have students work on 6, 7, 8 on their own.

Question 6: Answers will vary. No need to go over this with the class. Go over question 8 instead.

Question 8: Have students share answers with to the class. Looking for an answer similar stating the x-intercepts create 3 regions. The y-values in the leftmost region have 1 positive and 1 negative y-value. The middle region will have 2 positive y-values. The y-values in the rightmost regions have 1 positive and 1 negative y-value. Therefore the graph will start off negative, pass through the intercept, stay positive then pass through the other intercept and end negative.

Question 9: Challenge. Vertex at (3,9)