# 217 NCTM's Principles to Actions: Putting the Punch in Proportional Reasoning


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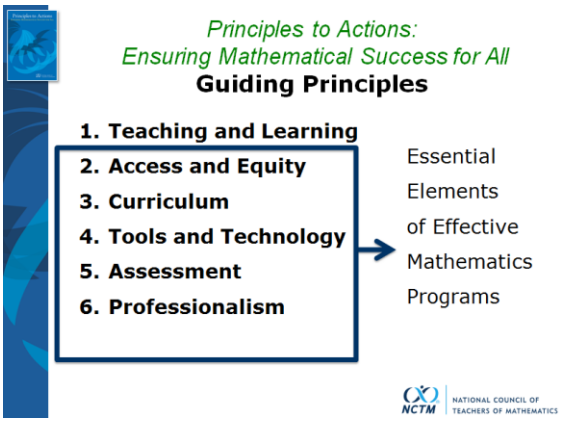


## Eight High-Leverage Instructional Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

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## High-Leverage Instructional Practices: 1 and 2

**(1)Establish goals to focus learning and,**

**(2) Implement tasks that promote reasoning and problem solving.**

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and that allow for multiple entry points and varied solution strategies.

## Eight High-Leverage Instructional Practices

- We will begin with examples of middle grades teachers applying practices 1 and 2 from my book,
- ***Bringing the Common Core Standards to Life: Exemplary Practices from Middle Schools (2014)*** **published by Taylor and Francis**

**Tasks that Provide Multiple Entry Points and Use of Varied Tools:**

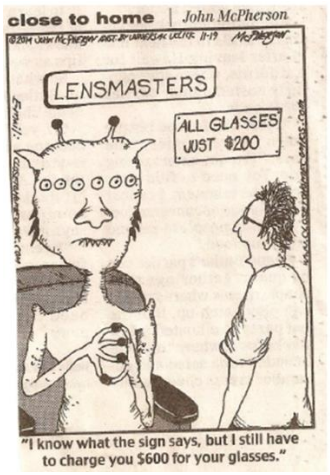
*Given a staircase with 5 steps composed of 15 blocks, create a presentation for three ways to find the number of blocks required for a staircase with 100 steps. Use whatever tools will help you: calculators, computer spreadsheet, or manipulatives such as rainbow cubes.*

Laura Mullen says "I use this problem on the first day of school to provide me with some personal information about my students' learning styles and cooperation gifts."

Tasks that Encourage Varied Approaches and Strategies

"Letting students know that their ideas and learning preferences are valued, increases the likelihood that lower achieving students will feel safe enough to share their own ideas, while encouraging higher achieving students to take greater risks"






***"It is estimated that more than half of the adult population cannot be viewed as proportional thinkers." (Lamon, 1999).***

## What do we want students to know, and be able to do?



## How are problems related?

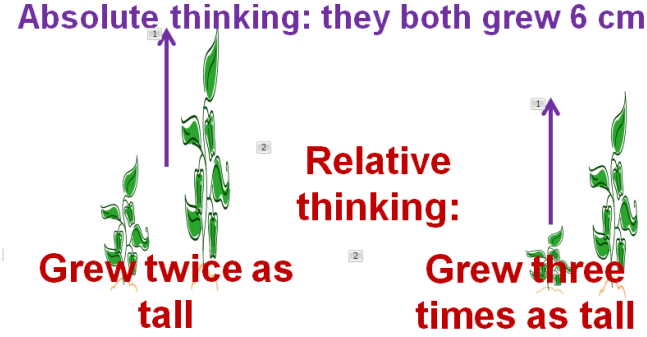

## How can we help students make connections?



**Partitioning** **Unitizing** **Relative thinking**
**Ratio sense** **Rational numbers**
**Quantities and change**

**If one plant grows from 6cm to 12cm, and another plant grows from 3cm to 9cm, which plant grew more?**

**Absolute thinking: they both grew 6 cm**



**Relative thinking:**

**Grew twice as tall**

**Grew three times as tall**

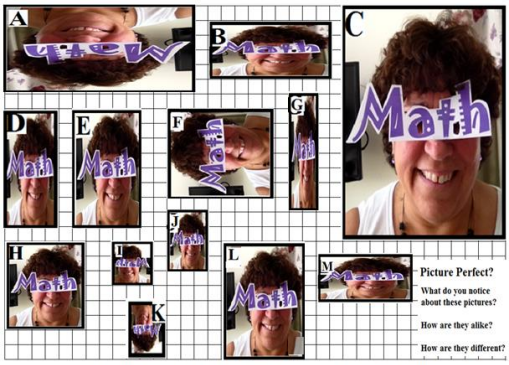
**A. If 6 boys can deliver papers in 3 hours, then 4 boys could do that route in 1 hour.**

**C. 8 oz. of lemon concentrate will make 12 cups of lemonade, so 20 oz. will make 30 cups of lemonade.**

## SENSE or NONSENSE?

**B. Jasmine spends $4 on 3 tickets. That means she would spend $9 on 8 tickets.**

**D. If it takes 40 minutes to bake a whole sheet of cookies, then it will take 20 minutes to bake half a sheet of cookies.**




(reorient each picture so it is facing up before counting the dimensions)

Consider picture L to be the "original" picture.

1) Which pictures have the same relationship between the length and width as picture L has?

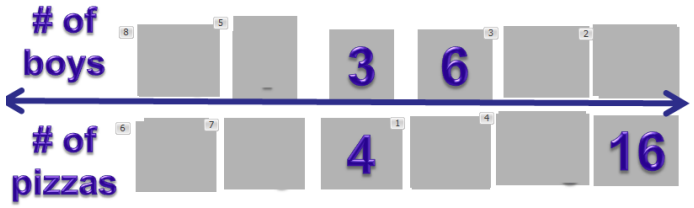
2) Which pictures are enlargements of picture L?

3) Which pictures are reductions of picture L?

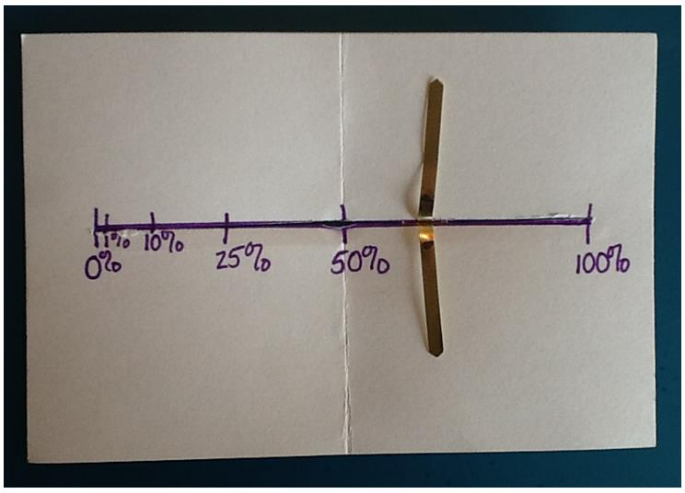
4) Which other pictures have the same relationships between height and width?

| | A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HEIGHT | | | | | | | | | | | | | |
| WIDTH | | | | | | | | | | | | | |

**3 boys will eat 4 pizzas at the party.**
**How much pizza will 1 boy eat?**
**How many pizzas will n boys eat?**




## Percent/Proportion slider





*"All ability to reason using proportional relationships is a complex process that develops over an extended period of time. It takes many varied physical experiences to develop an understanding of what a proportional relationship is and then more time to gain the ability to deal with it abstractly."*

*(Cordel, & Mason, 2000)*

## Index card Ratios

Fold your index card into four equal pieces and tear them apart. Write one of the following digits on each part.

| 2 | 3 | 8 | 12 |
|---|---|---|---|

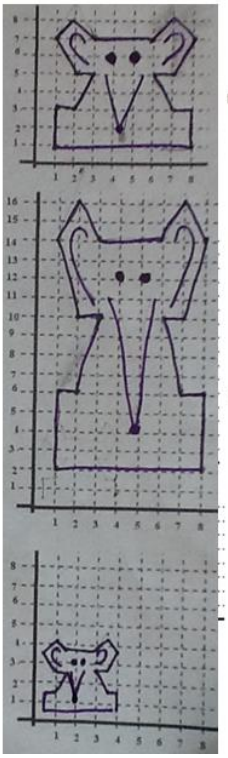
Arrange the cards to make as many different ratios as you can.
Collect data as you work.

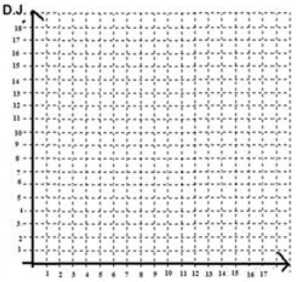
**Proportion match up –** Cut the cards apart. Match each to its solution.

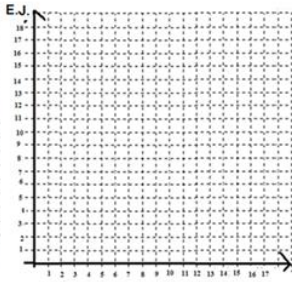
| | | | |
|---|---|---|---|
| | | 10 | $\frac{16}{88} = \frac{x}{55}$ |
| $\frac{4}{9} = \frac{12}{x}$ | 27 | 90 | $\frac{22}{30} = \frac{66}{m}$ |
| $\frac{c}{5} = \frac{6}{15}$ | 2 | 6 | $\frac{x}{18} = \frac{21}{63}$ |
| $\frac{6}{16} = \frac{9}{w}$ | 24 | 5 | $\frac{20}{w} = \frac{12}{3}$ |
| $\frac{x}{14} = \frac{28}{56}$ | 7 | 15 | $\frac{x}{10} = \frac{6}{4}$ |
| $\frac{20}{b} = \frac{15}{9}$ | 12 | 100 | $\frac{n}{12} = \frac{125}{15}$ |

**Marilyn Dibble's Similar Mouse Family** -- Complete the table by following the rule below each mouse name. For example, for B.J. the rule is (x, 2x) so the x coordinate stays the same, but the y coordinate is multiplied by 2. Plot the points from the coordinates of each mouse, connecting each point to the previous one as you plot them. Finally connect the last point with the first point, add 2 eyes, a smile and a curved line in each ear. Identify which ones are members of A.J.'s family. (Hint: A.J.'s family members are all similar.)

| A.J. | B.J. | C.J. | D.J. | E.J. |
|---|---|---|---|---|
| *(x, y)* | *(x, 2y)* | *$(\frac{1}{2}x, \frac{1}{2}y)$* | *(2x, 2y)* | *(2x, y)* |
| (1, 1) | (1, 2) | $(\frac{1}{2}, \frac{1}{2})$ | (2, 2) | (2, 1) |
| (1, 3) | | | | |
| (2, 3) | (2, 6) | (1, 1.5) | (4, 6) | (4, 3) |
| (3, 5) | | | | |
| (2, 5) | | | | |
| (1, 7) | | | | |
| (2, 8) | | | | |
| (3, 7) | | | | |
| (6, 7) | | | | |
| (7, 8) | | | | |
| (8, 7) | | | | |
| (7, 5) | | | | |
| (6, 5) | | | | |
| (7, 3) | | | | |
| (8, 3) | | | | |
| (8, 1) | | | | |







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