



The Cycloid: From Geometry to Calculus

NCTM National Conference, April 2015

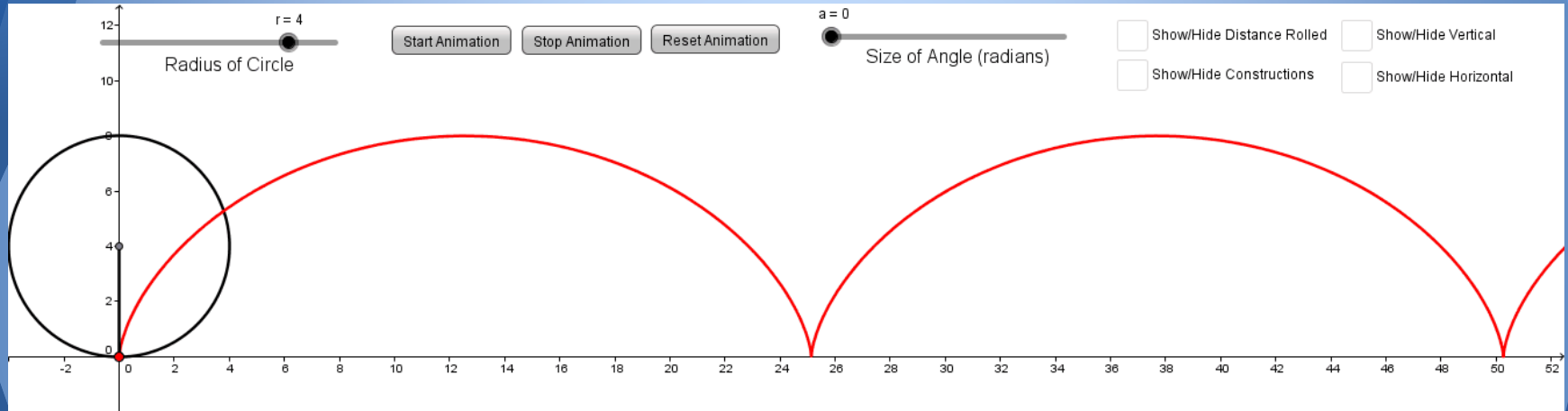
Maria Hernandez, NC School of Science and Math
Taylor Gibson, NC School of Science and Math

Goals for Our Session

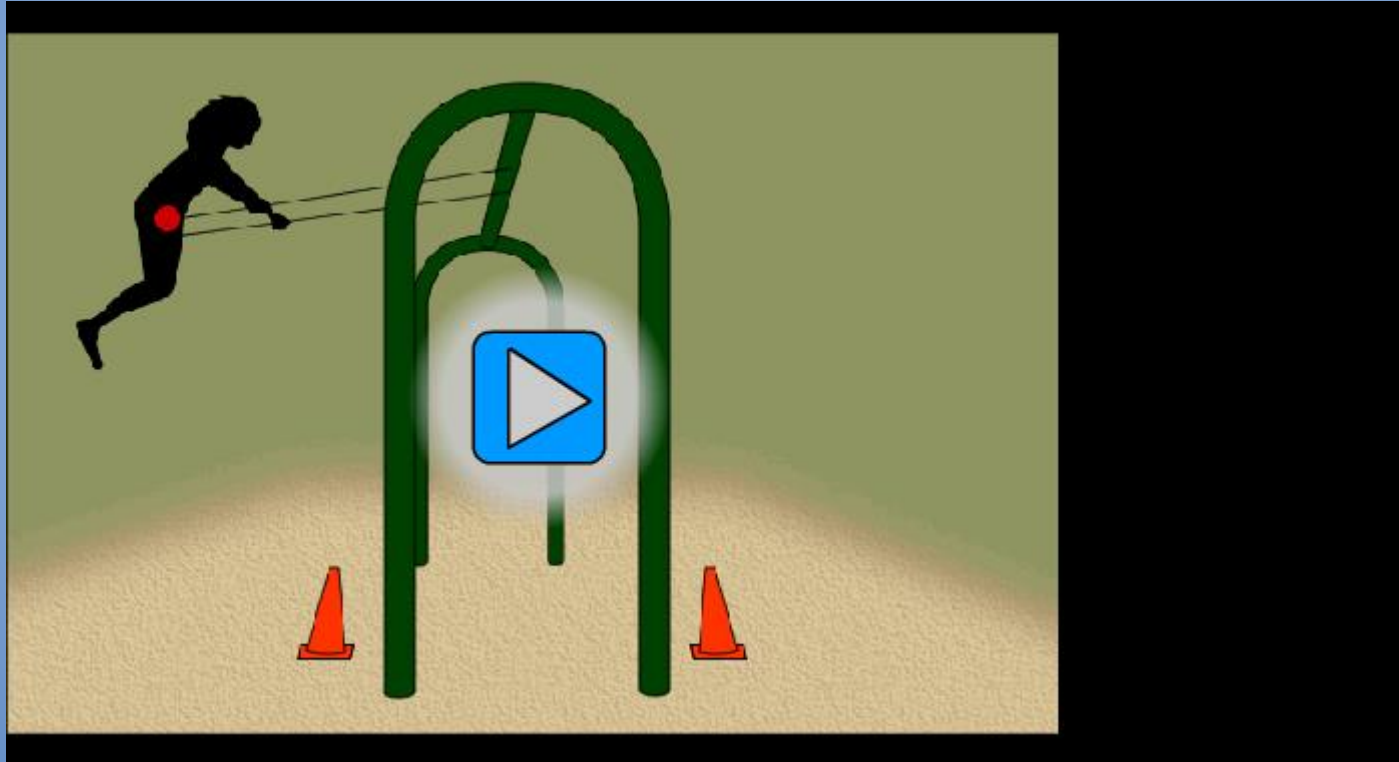
- Introduce the project and parametric equations
- Demo data collection via LoggerPro
- Create models for the collected data
- Evaluate models
- Explore the geometric representations
- Share resources to extend the project



The Cycloid



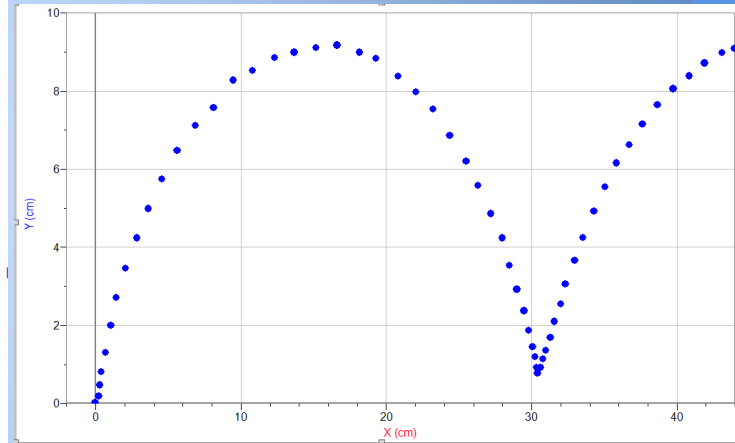
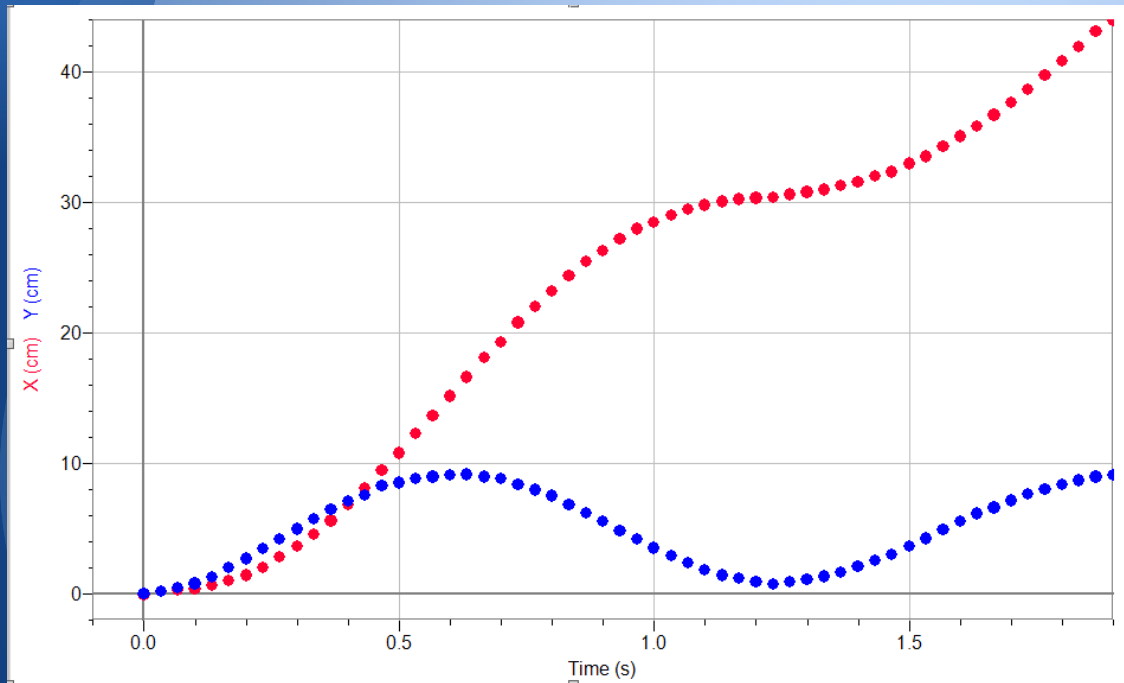
Intro to Parametric Models



Creating Models From Data

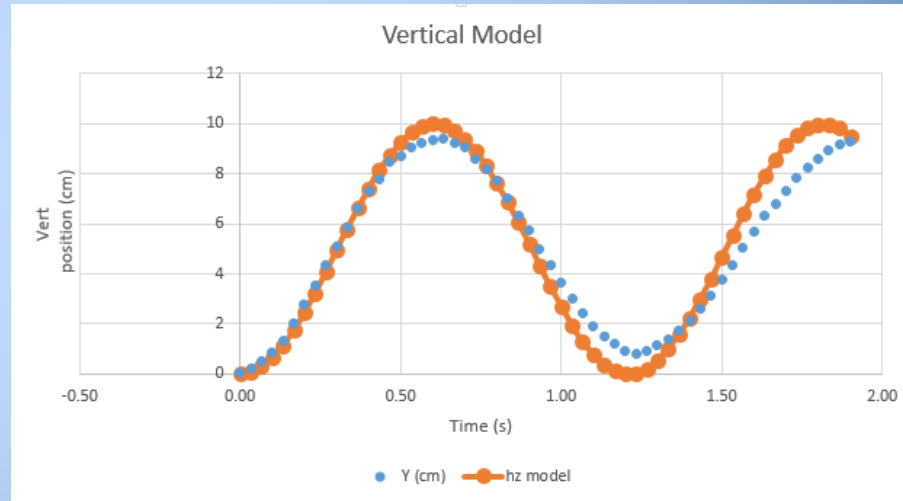
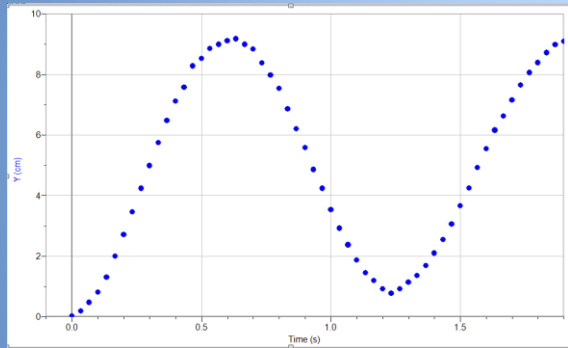
Logger Pro is a tool to capture data from a video.
Our data will be shared in other forms for your use.

Data Collected from Video



Vertical Model

Below we see the data (time, vertical position of dot)

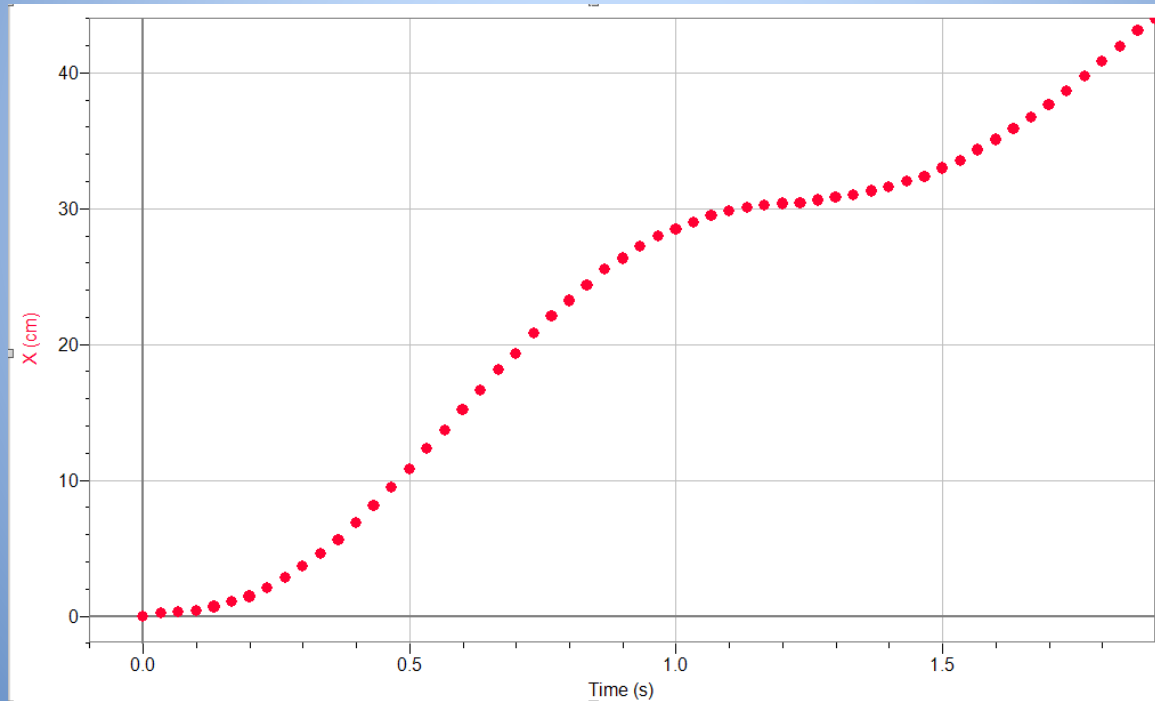


$$v(t) = -5 \cos\left(\frac{2\pi}{1.21}t\right) + 5$$

What is the contextual meaning of the constants?

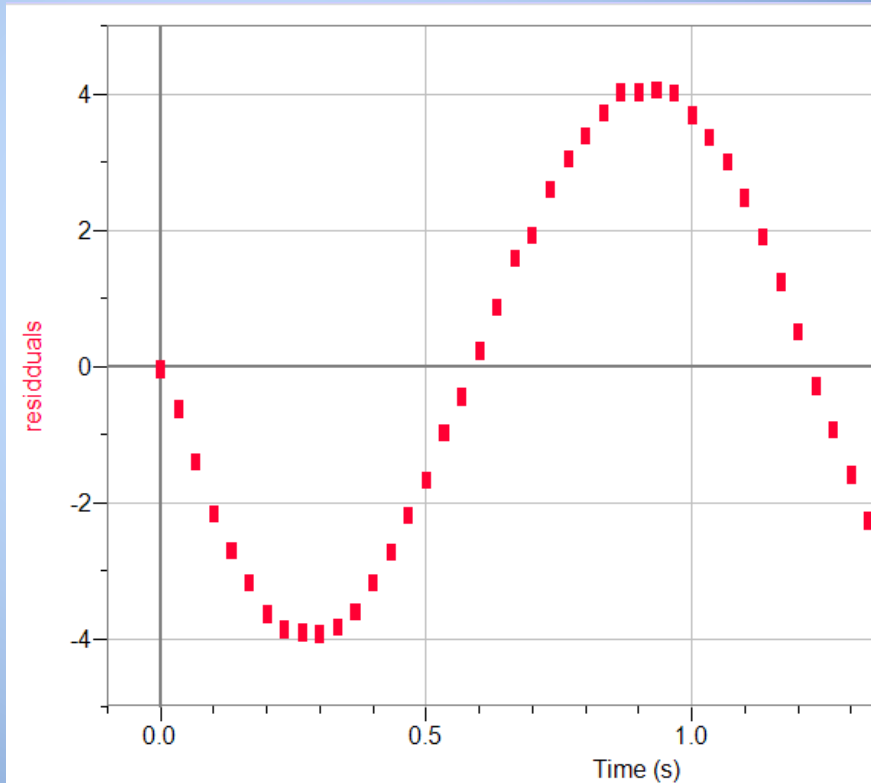
Horizontal Model

Below we see the data (time, horizontal position of dot)

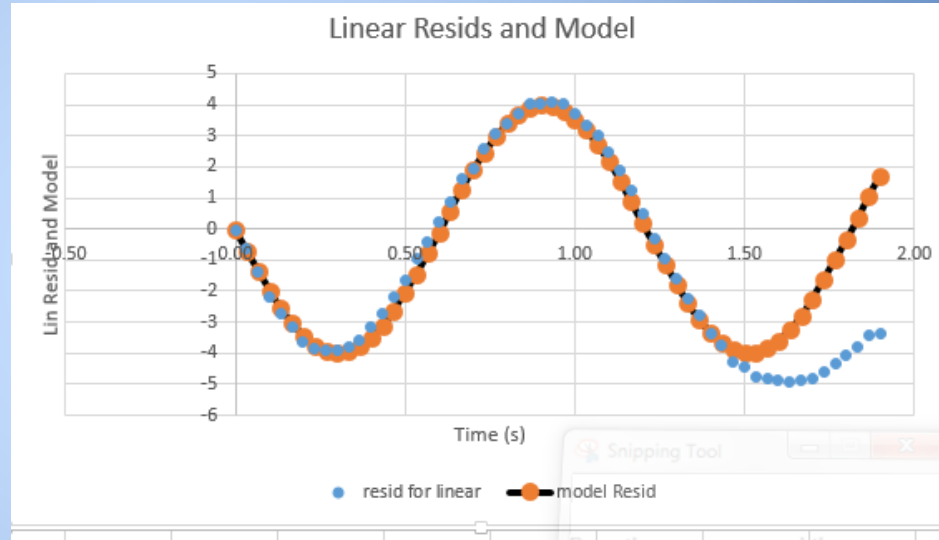
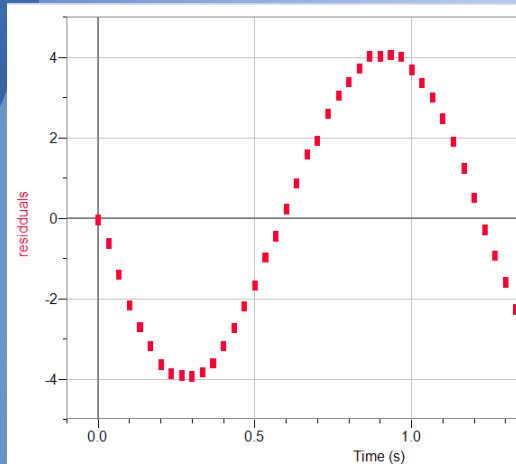


Linear Model and Residuals for Linear Model

$$x(t) = 25.5t$$



Residuals for Linear Model and Model for Residuals



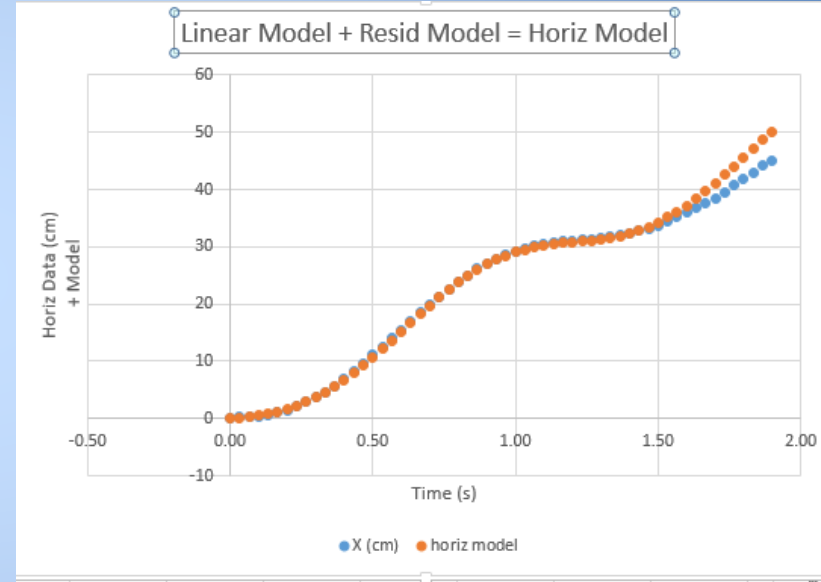
$$R(t) = -4 \sin\left(\frac{2\pi}{1.21} t\right) + 4$$

Putting the *Model* Together

What are some possible choices?

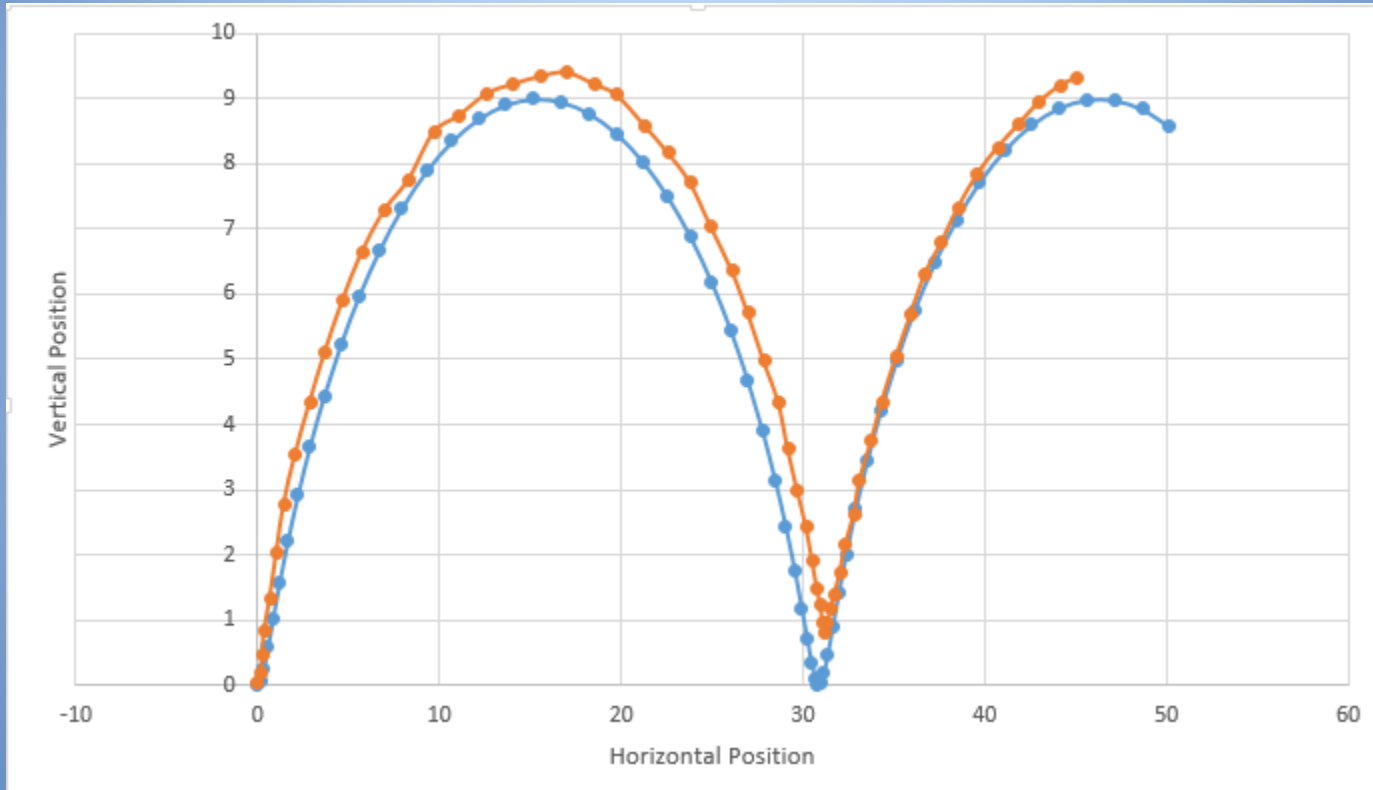
Summing the Models

$$h(t) = 25.5t - 4 \sin\left(\frac{2\pi}{1.21}t\right) + 4$$



What do the constants mean in context?

Final Model for Position of Dot

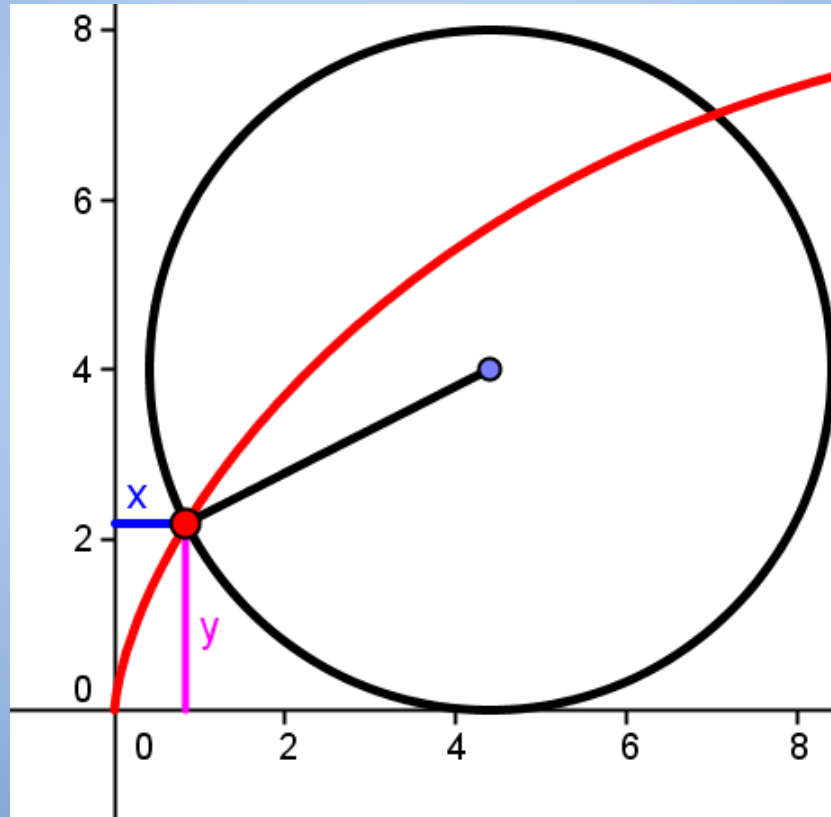


Evaluating the Models

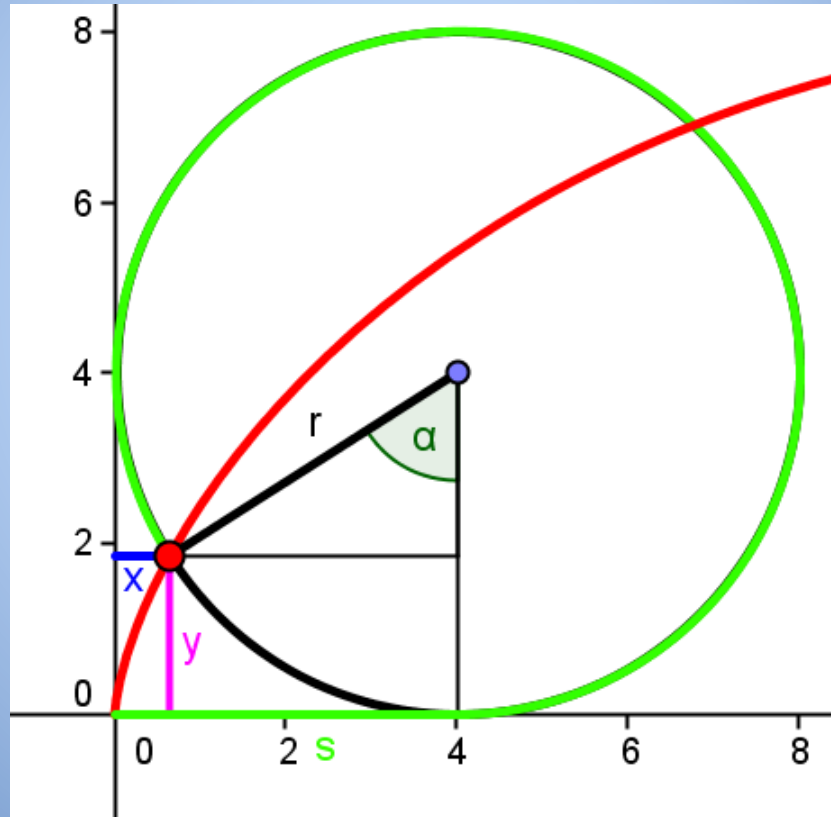
- What are the strengths and weaknesses of our models?
- How can evaluating our models help students make connections between the mathematics and the physical phenomenon?

Modeling With Geometry

Modeling with Geometry

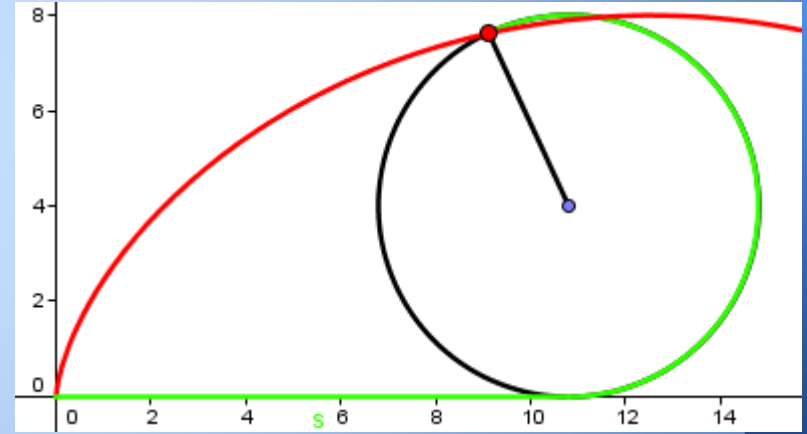
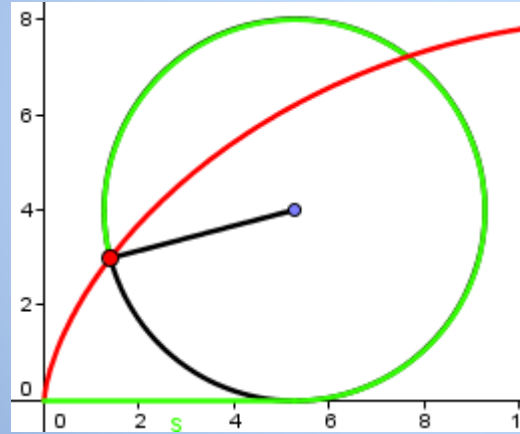
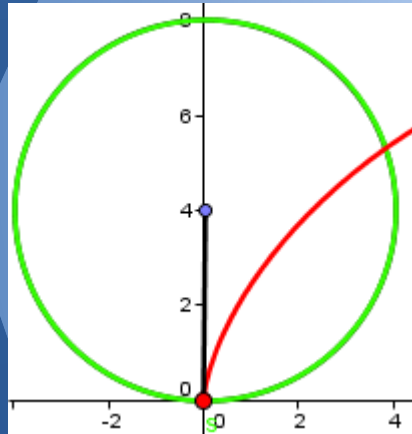


Modeling with Geometry

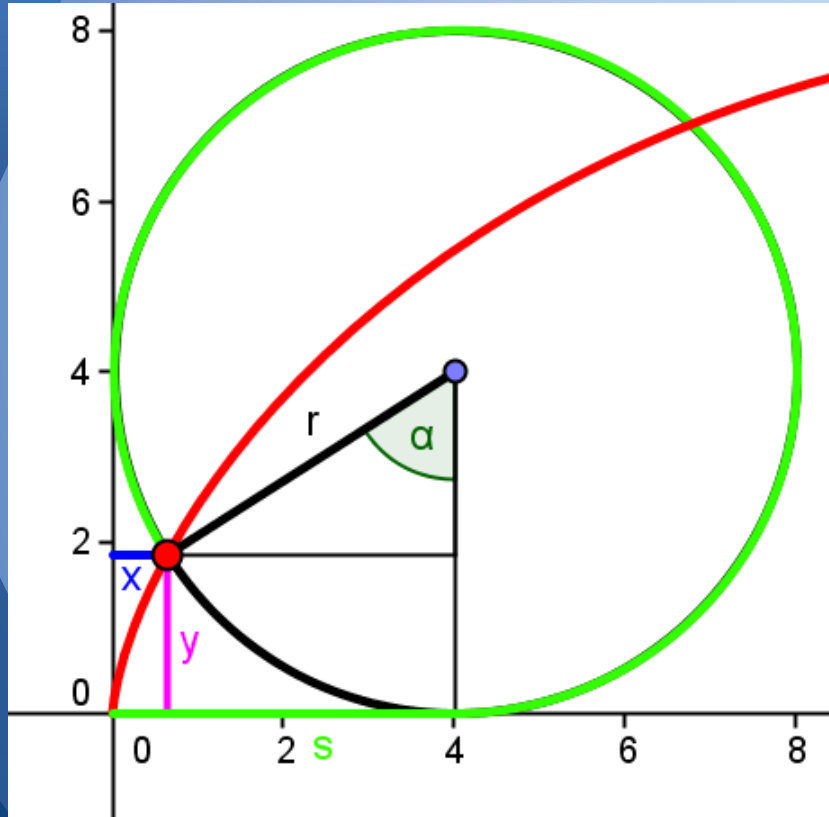


Geometric Model

Help me determine s , the distance rolled



Modeling with Geometry



Final Geometric Model

$$x(\alpha) = r\alpha - r \sin \alpha$$

$$y(\alpha) = r - r \cos \alpha$$

Student Reflections

"One of my favorite aspects of this project was that it was (with your input and guidance when we were stuck) a **self-discovery**." - Thomas

"I think the coolest part of the cycloid project was that **it made you think differently than normal**; My first thought would have never been a combination of a cosine and linear function. I didn't even know that was a thing until this project." - Vishwa

"Hopefully, in the future, we'll be able to do a few **more independent modeling investigations**." - Jules

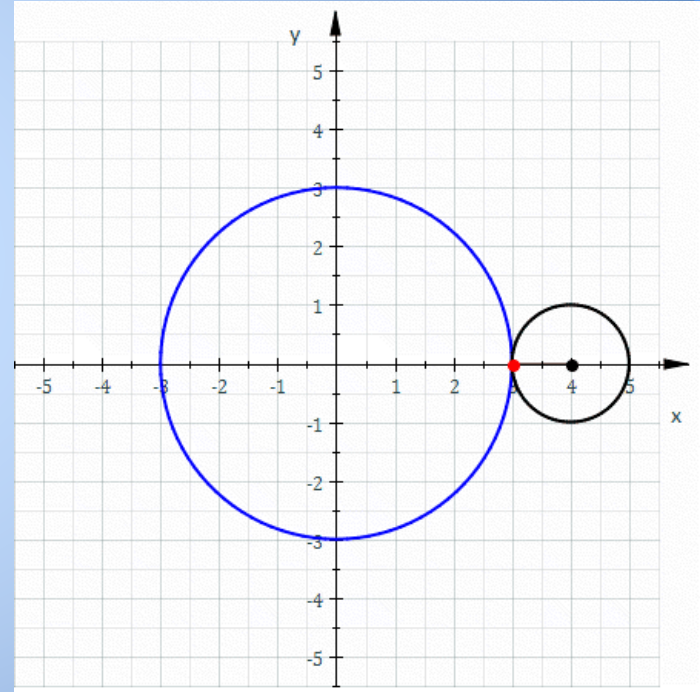
"I felt like I really wanted to learn this answer in this lab because **it was something new that I got to discover by myself**. Overall, the cycloid lab was very interesting and **a very fun and inventive way to introduce the class into combining equations**." - Matthew

Questions to Consider

- Modeling Cycle - How does the project fit into a focus on math modeling?
- What's powerful about it for students?
- What's powerful about it for teachers?

Calculus and Geometry Extensions

- Length of path for an arch
 - 4 · diameter
- Speed of the dot (vectors)
- Area under an arch
 - $3 \cdot (\text{Area of circle})$
- Tangent line through dot
 - Passes through top of circle
- Epicycloids/Hypocycloids



References and Resources

- NCSSM Swing Lab Materials
<http://www.dlt.ncssm.edu/stem/content/swing-lab-documents>
- Cycloid Materials from This Presentation
<http://www.ncssm.edu/courses/math/cycloid/>
- Inspirations Video by Cristobal Vila
<https://www.youtube.com/watch?v=oVthC6neqVc>

Conferences

1. Bridges Conference: Mathematics, Music, Art, Architecture and Culture
Baltimore, MD July 29 - Aug 1, 2015



2. Anya Greer Math, Science and Technology Conference
Phillips Exeter Academy, June 21 - 26, 2015



3. MAA/PREP Teaching Mathematical Modeling as
Creating Mathematical Discovery
Lincoln, NB July 20 - 25



NCSSM Teaching Contemporary Mathematics Conference

The NC School of Science and Mathematics
Durham, NC, January 29 - 30, 2016

<http://www.ncssm.edu/courses/math/tcm/TCM2015/>



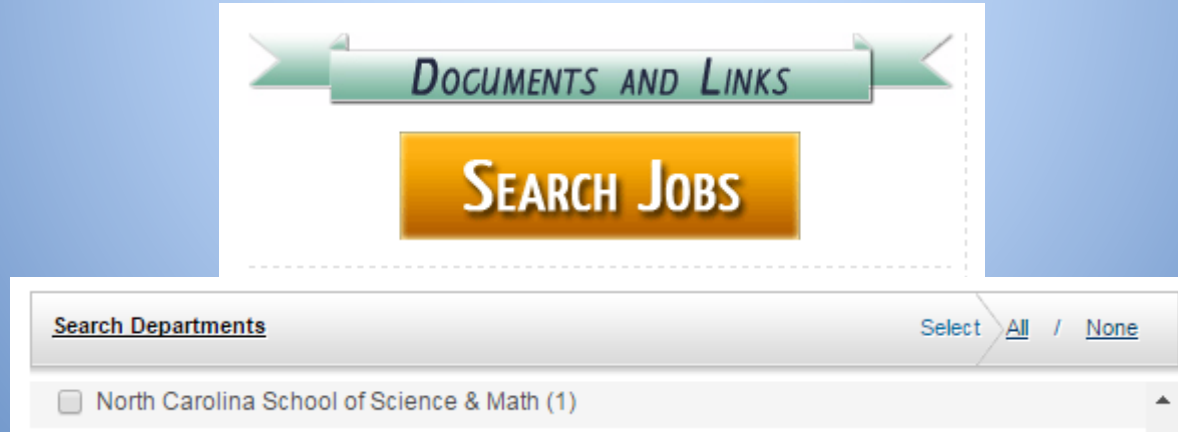
NCSSM

Questions?

Open Position @ NCSSM

<http://www.oshr.nc.gov/jobs/>

- Search Job Positions
- Departments: NCSSM



Thank You!

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