Moving Through Arithmetic:
Overview of a Complex Numbers Unit

Welcome to our unit! We're excited to share this math with you.

This is stupid, we've heard kids say about imaginary numbers. *They're just made up? What's the point if they're just made up?*

This unit is an attempt to take the question seriously. The answer we sketch gives students experience with dilations, rotations, translations, number lines and planes, negative square roots and rules for plane transformations...all **before** they see complex numbers for the first time.

In short, the perspective in these materials is that all arithmetic can be interpreted as describing motion. The problem is that real arithmetic doesn't offer us a way to describe most rotations. This is the problem that complex numbers solve.

Thanks for taking a look, and enjoy!

Max and Michael

P.S. We're eager to talk more about the unit. Please, reach out to us by email or via twitter.

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Twitter: @mpershan and @maxmathforum

A Link to the Materials

http://bitly.com/complexnumbers2015

Very Very Short Sketch of the Unit

This unit is structured around 3 investigations. Each investigation is designed to stand on its own. [Pacing guide below.]

- **Investigation 1:** Arithmetic and Transformations (4 to 6 days)
- **Investigation 2:** Transformations and Complex Numbers (4 to 6 days)
- **Investigation 3:** Complex Numbers and Rotations (4 to 6 days)

Students use their skills with real arithmetic to learn complex arithmetic, and then students use those skills to answer a variety of “real” questions about rotations.

A Slightly Longer Sketch of the Unit

Students might pass through something like these sequence of skills on their way through this unit (brief examples in parentheses.)

- Doing arithmetic with negative numbers (“2 + -4 = ___”; “4 x - 2 = ___”)
- Performing transformations on the number line (“2 translated by -3 goes to ___”)
- Following transformation rules on the number line (“x--->2x would send -3 to ___”)
- Performing transformations in the plane (“Dilate this figure by a factor of 2.”)
- Following transformation rules in the plane (“Draw the result of (x,y) --> (-x, y) on this figure”)
- Performing 90 degree rotations fluently (“Rotate (10,3) by 90 degrees 5 times. Where does it end up?”)
- Understand and graph complex numbers (“Which of these are complex numbers?”; “Graph 2+3i on the plane”)
- Multiplying by i (“Multiply 2+3i by i and graph the result”)
- Adding and subtracting complex numbers (“Represent (2+3i) + (2-3i) on the graph”)
- Solve simple quadratic equations with complex solutions (“Solve for x: x^2 + 3 = 0”)
- Dividing by i (“Divide 2+3i by i and graph the result”)
- Perform 45 degree rotations fluently (“(10,3) is rotated by 45 degrees. It lands at ___.”)
- Perform 30 degree rotations fluently (“(10,3) is rotated by 30 degrees. It lands at ___.”)
- Perform n degree rotations fluently (“(10,3) is rotated by 70 degrees. It lands at ___.”)
- Multiply by a+bi (“What multiplication would rotate 2+3i by 50 degrees?”)
- Divide by a+bi (“3+4i divided by 5+2i is ____.”)
- Solve any quadratic equations with complex solutions (“Solve for x: x^2 + 3x + 3 = 0”)

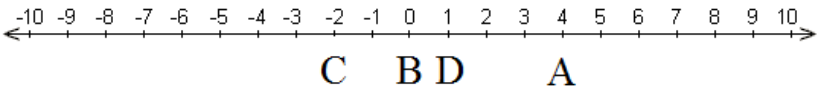
For a fuller sketch of the materials and unit plans and lesson plans, go to http://bitly.com/complexnumbers2015 or get in touch with Max and Michael.

Thanks for checking out our work!

Special NCTM Edition
Fewer problems! Squished to fit 2 pages!!

Simon Says - Problems

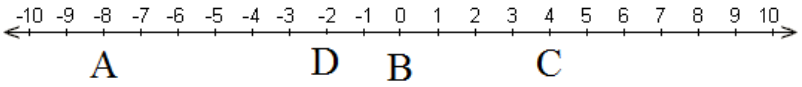
1. Before the caller calls, Amanda, Billy, Carlos and Denice are all standing on the number line as shown in the image below.



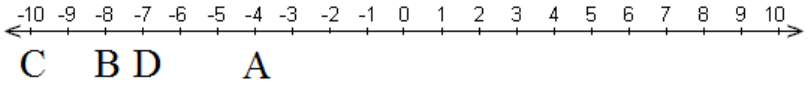
After the caller calls out instructions, everyone follows the caller's instructions and moves if they need to.

For each number line below write the arithmetic (like "add 2") that the caller called out and then write the transformation (like "dilation by -1/2") that describes their motion.

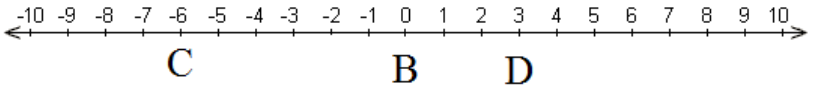
a.



b.

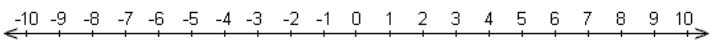


c.



Questions 2-4 relate to following the same rule twice.

2. The caller calls out $x \to 2 \cdot x$ twice.
   a. Use the number line to show how Amanda, Billy, Carlos and Denise would move after each calling.



   b. What transformation have they performed after following this rule twice?

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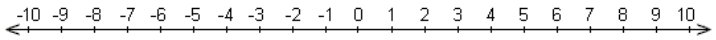
3. The caller calls out $x \to \sqrt{2} \cdot x$ twice.

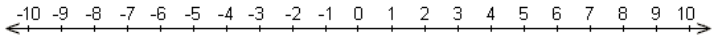
    a. Use the number line to show how Amanda, Billy, Carlos and Denise would move after each calling.



    b. What transformation have they performed after following this rule twice?

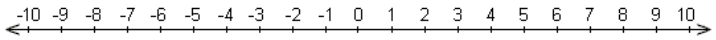
4. The caller calls out $x \to \sqrt{-1} \cdot x$ twice, but nobody on the number line is quite sure what the caller means. After talking it over, everyone on the number line agrees to guess what the caller wants them to do.

    a. Use the number line to show how Amanda, Billy, Carlos and Denise would move after each calling.

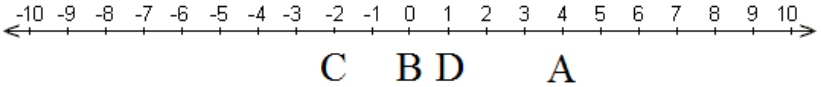


    b. What transformation have they performed after following this rule twice?

5. You're standing at 10 on the number line when the caller calls out, "Rotate by 90 degrees." Where do you end up?

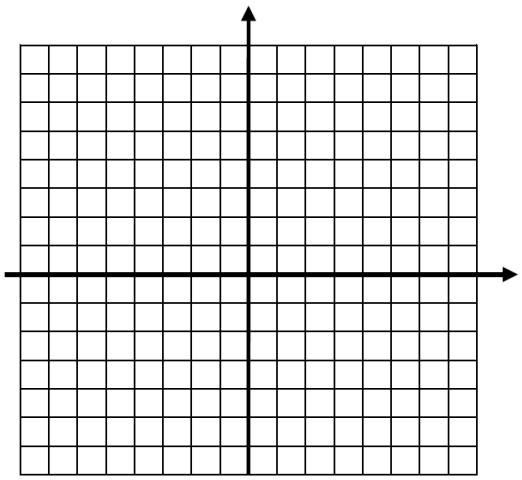


6. **Challenge**: What if the caller calls out, "Rotate clockwise by 45 degrees"? Where would A, B, C and D end up standing? Use the number line to show where they'd stand, and then try to be precise about their exact positions.



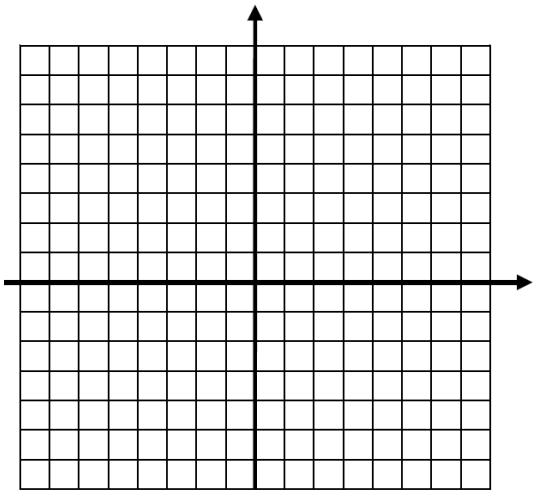
In Class Experiment: Multiplying by *i*

1. Plot these complex numbers on the plane. Check with your group members to make sure you all agree.



a. 2i
b. -4i
c. i + 1
d. -2 + 2i
e. 8
f. 4 + 0i
g. 0 + -5i
h. 0 + 0i

2. Each member of your group should pick a different complex number and plot it on the plane.
    a. Multiply your number by i and plot the result on the plane. (Don't forget: $i \times i = -1$ when it comes to complex numbers.)
    b. Multiply your result by i again and plot the result.
    c. Multiply your result by i *again* and plot the result.
    d. Multiply your result by i again (again!) and plot the result.
    e. The group should confer. Share what you notice about multiplying by i.



3. Repeat your group's experiment with multiplication by a different complex number. [Some suggestions: multiplication by 2i, multiplication by -i, multiplication by 1+i.]

Challenges

1. We know that there's a rule for R_{90} but is there also a rule for R_{270°?

   Use complex numbers to find a rule for how R_{270° effects any point (x,y).

2. What does multiplying by $1+1i$ do to a complex number?

3. Solve the equation $(a+bi)(a+bi)=i$ for a and b. What transformation can multiplication by $a+bi$ do to a complex number?

4. What does multiplying by $\sqrt{3}+1i$ do to a complex number?