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Geometry and... (units to captivate our captive students)

Create “units” which connect textbook Geometry to mathematical and non-mathematical fields so that we can deepen understanding of shapes, similarity, scale, and proof, as well as aid in the development of “visual strength.” Specific examples include selections from Euclid and Lobachevsky, as well as examples from chemistry, art, and literature.

A My hope for a student who takes a course in Geometry

- 1) See that Geometry is interesting and relates to other areas of study and see that there is no end to the study
- 2) Gain an understanding of properties of basic shapes– particularly the circle, triangle, square, and parallel lines
- 3) Develop a strong understanding of ratio, including similarity (trigonometry) and scale
- 4) Appreciate the nature of proof in mathematics
- 5) Improve visual strength

B Introduction to six “Geometry and...” units

I Geometry and Geometry

- i) Euclid’s Definitions, Common Notions, Postulates
- ii) Euclid’s 1.1, 1.5, 1.47, 6.31
- iii) Lobachevsky
- iv) Variety of Fun Theorems
- v) Fractal Geometry

II Geometry and Art

- i) Linear Perspective
- ii) Similar Figures and the Illusion of Foreground and Background
- iii) Division of Canvas and Frescoes Into Significant Ratios
- iv) The Role of Rectangles in Art
- v) Mechanical Aids for Drawing
- vi) Drawing Based on Simple Geometric Shapes

III Geometry and Chemistry

- i) Congruency, Arrangements, Similarity
- ii) Water, Hydrogen Bonds, and Snow
- iii) Naturally Occurring Forms of Carbon
- iv) Carbon as the Basis of Life
- v) Carbon in a Tetrahedron

IV Geometry and Literature

- i) Plato and Aristotle
- ii) Gulliver’s Travels
- iii) Poetry
- iv) John Cheever’s Geometry of Love

V Geometry and Architecture

- i) Arch
- ii) Dome
- iii) Flying Buttress
- iv) Efficiency of Rectangular Floor Plans

VI Geometry and Proof

- i) Meaning of Proof for Euclid
- ii) Algebraic Proofs
- iii) Proof by Sliding
- iv) Proof Without Words
- v) Meaning of Proof for Science and History

C Visual Strength

Abridged Bibliography

Ames, Lee J	<i>Draw 50 Animals</i>
Asimov, Isaac	<i>Asimov on Chemistry</i>
Asimov, Isaac	<i>The Chemicals of Life</i>
Asimov, Isaac	<i>World of Carbon</i>
Blackwell, William	<i>Geometry in Architecture</i>
Bonola, Roberto	<i>Non-Euclidean Geometry</i>
Bouleau, Charles	<i>The Painter's Secret Geometry</i>
Braver, Seth	<i>Lobachevsky Illuminated</i>
Devaney, Robert	<i>The Mandelbrot and Julia Sets: A Tool Kit of Dynamic Activities</i>
Edgerton, Samuel Y	<i>The Heritage of Giotto's Geometry: Art and Science on the Eve of the Scientific Revolution</i>
Ghyka, Matila	<i>The Geometry of Art and Life</i>
Gonick, Larry and Criddle, Craig	<i>The Cartoon Guide to Chemistry</i>
Hambidge, Jay	<i>The Elements of Dynamic Symmetry</i>
Hockney, David	<i>Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters</i>
King, Ross	<i>Brunelleschi's Dome</i>
King, Ross	<i>Leonardo and the Last Supper</i>
Kline, Morris	<i>Mathematics in Western Culture</i>
Ladis, Andrew	<i>The Brancacci Chapel</i>
Nelsen, Roger	<i>Proof Without Words</i>
Pedoe, Dan	<i>Geometry and the Visual Arts</i>
Steadman, Philip	<i>Vermeer's Camera: Uncovering the Truth Behind the Masterpieces</i>

Recommended Websites

Euclid's Elements (David Joyce, Clark University)	http://aleph0.clarku.edu/~djoyce/java/elements/elements.html
Snowflakes (Kenneth Libbrecht, Caltech)	http://www.its.caltech.edu/~atomic/snowcrystals/
Fractals (Robert Devaney, Boston University)	http://math.bu.edu/DYSYS/index.html
History of Math (John O'Connor and Edmund Roberts, St. Andrews)	http://www-gap.dcs.st-and.ac.uk/~history/

Fourteen suggested “Geometry and…” units with brief descriptions

Geometry and Architecture

Discuss the geometry behind the arch, flying buttress, and dome. Examine the ratios involved in rectangular floor plans--maximization of the ratio of perimeter to area allows for strong natural light and easy air exchange, but adds costs of heat. Look at the geometry of San Patrizio’s Well in Orvieto, Italy (cleverness solves a problem).

Geometry and Art

Discuss important terms of art (such as linear perspective, scale and double-scale) and how they relate to geometry. Examine specific paintings to see that figures are placed carefully according to strict ratios to create a balance which pleases the eye or denotes importance. Introduce the idea of mechanical aids for drawing, such as the Camera Lucida as well as complicated drawings emerging from simple geometric beginnings.

Geometry and Carpentry

Discuss various “tricks” used by carpenters and look at examples such as roof lines and stairs (experienced carpenters use a 3-4-5 triangle to determine maximum steepness for stairs).

Geometry and Chemistry

Discuss specific geometric ideas of chemistry (the varied shapes of the three naturally occurring forms of carbon and why carbon is the basis of life). Look at hydrogen bonding in water and how that affects snowflakes. Explore the vocabulary and ideas of geometric terms which are used in chemistry.

Geometry and Cities

New ideas about the organization of cities have begun to be published such as *A Typology of Street Patterns* by Louf and Barthelemy. Abstract:

“We propose a quantitative method to classify cities according to their street pattern. We use the conditional probability distribution of shape factor of blocks with a given area, and define what could constitute the fingerprint of a city. Using a simple hierarchical clustering method, these fingerprints can then serve as a basis for a typology of cities.”

In addition, students can look at a variety of maps of old European cities or modern American suburbs to look for common traits.

Geometry and Fractals

Fractals introduce students to non-integer dimensions, to objects with infinite perimeter and finite area, and to geometry without local linearity (which therefore defies calculus), as well as the visualization of iteration in the Complex Plane. It also introduces students to iteration in the Complex Plane. In addition, iteration in the Complex Plane is discussed.

Geometry and Future Math Classes in High School

Trigonometry rests on the right triangle, vectors and polar forms rest on trigonometry, “related rates” problems in Calculus often rely on similar triangles, solids created by rotating a given area around a given line require visual strength).

Geometry and Geometry

Where is High School Geometry heading? Compare Euclid’s Geometry with Lobachevsky’s Geometry. Look at Geometry on a Sphere, Fractal Geometry, Projective Geometry, and Topology.

Geometry and the Human Body

Examine the non-obvious differences between males and females (“carrying angle” of a woman’s arm, lower center of gravity for a woman as seen in the “Who can lift the chair?” activity, skeletal differences between men and women), understand why babies can’t support their own heads, and discuss the impact of two ears and two eyes but only one mouth.

Geometry and Literature

Examine famous works from Classics (Plato and Aristotle use ratios to explain their ideas, and Dante has his circles for various sinners in *The Inferno* while Jonathan Swift’s *Gulliver’s Travels* has his protagonist experience life as the smallest or largest creature in the land) as well as more recent writing (John Cheever’s short story “*The Geometry of Love*”). Introduce the idea that some poetry needs to be seen to be appreciated (John Hollander’s “*Swan and Shadow*” or ee cummings’s *r-p-o-p-h-e-s-s-a-g-r*).

Geometry and Nature

Look at Fibonacci numbers in pine cones or pineapples, the patterns in the underside of bird feathers, parallel lines in leaves, the use of hexagons to store honey, the branching of rivers and trees, and lungs as a fractal.

Geometry and Optical Illusions

Examine the properties which make an effective optical illusion and discuss how we make sense of images. Look at visual tricks in movies and art and the geometry which drives them.

Geometry and Proof

Examine the meaning of a proof in Geometry versus law, history, or science. Our legal standard for sending a person to jail is to prove guilt “beyond a reasonable doubt.” Look at what it means to prove a “fact” from history such as who fired the first shot at Lexington, Massachusetts on April 19, 1775. Discuss the meaning of proof in science (in 1840, there was a scientific proof of the existence of planet near Mercury named Vulcan).

Geometry and Visual Strength

Discuss the vocabulary required to look at a painting or photograph (foreground, background, relation of figures to each other), improve the ability to separate images into parts (overlapping triangle diagrams, various pairings of figures in a painting), picture and explain three-dimensional objects created by rotation (a cylinder is created by rotating a segment parallel to the x-axis around the x-axis) or by definition (a cylinder is the set of all points equidistant from a segment).