

**2015 NCTM ANNUAL MEETING
& EXPOSITION**

April 15–18 • Boston

Removing the Fear of Fractions from Your Students

Thursday, April 16, 2015: 9:30 AM-10:30 AM

157 A (BCEC)

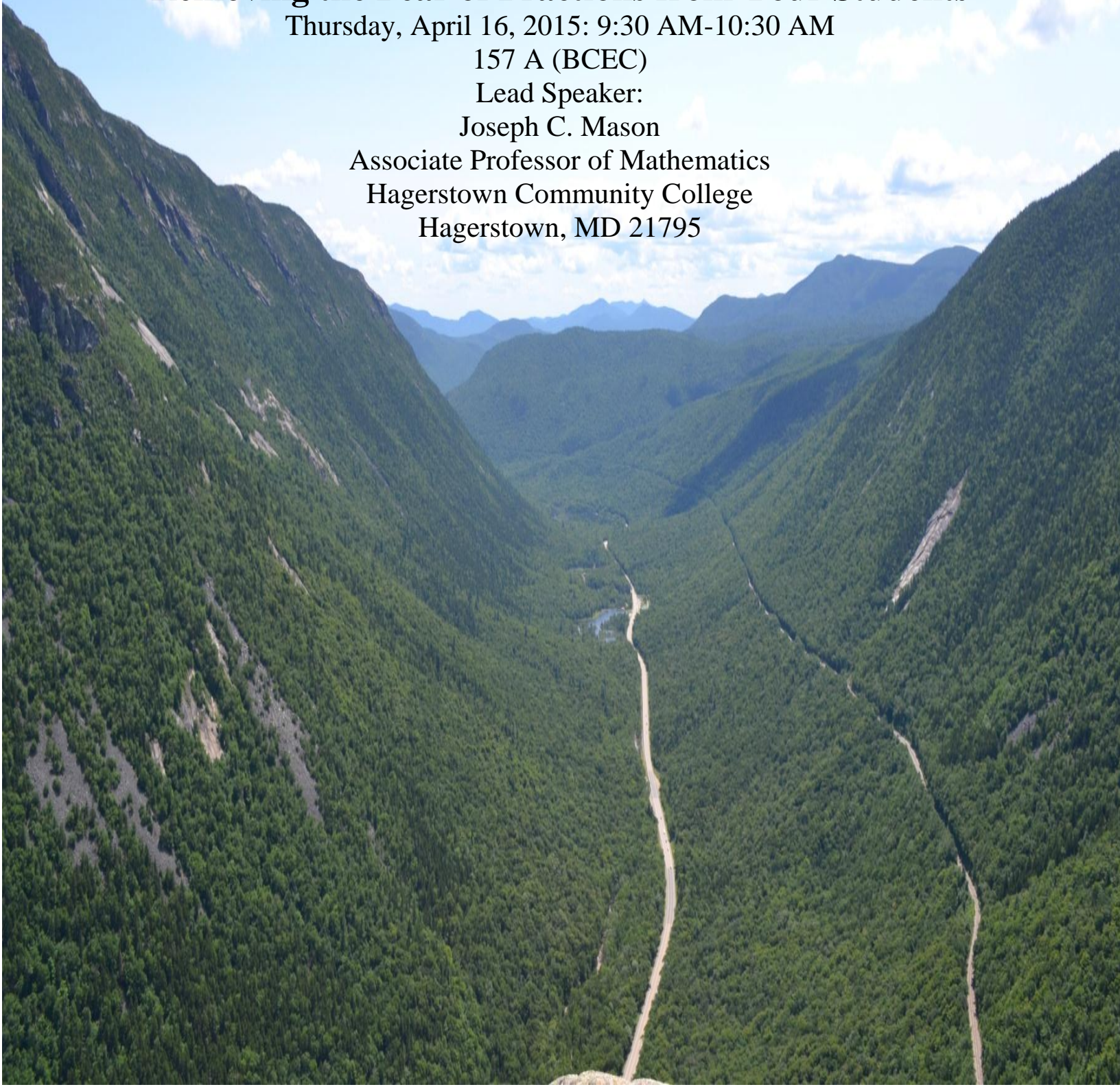
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How would I describe Math Education today?

Uses a one-dimensional approach with students directed to memorize an algorithm, always use the same algorithm, think and perform in the same manner, plug in numbers, chug to an answer, and show all work. This tends to make math non-interesting, non-challenging, and produces a robotic mind incapable of excelling in problem solving.

How would I like to see Math Education described?

Uses a multi-dimensional approach with students analyzing the problem first, using a conceptual understanding to decide on a strategy, using different strategies on different problems, and performing most work mentally. This will make math intriguing, challenging, and abstract. It will also create a society of critical thinkers, decision makers, problem solvers, and individuals who enjoy math because they are not always told what to do or to perform as a robot.

Start-up Activity

Divide the following rectangle into fourths



Strategies for Introducing Fractions

In order to be proficient in working with fractions it is important to understand the basics and terminology.

Parts of a fraction

$\frac{3}{4}$ ← numerator – tells how many parts you have

4 ← denominator – tells how many parts are in a whole

Types of fractions

proper fraction – a fraction in which the numerator is smaller than the denominator.

Used to represent an amount smaller than a whole. **Example:** $\frac{3}{8}$

improper fraction – a fraction in which the numerator is larger than the denominator.

Used to represent an amount larger than a whole. **Example:** $\frac{3}{2}$

mixed fraction – a whole number along with a fraction. Also used to represent an

amount larger than a whole. **Example:** $5\frac{3}{4}$

Representing a fraction with a shaded region.

First – decide how many parts make up a whole (this will be your denominator)

Second – determine how many parts are to be shaded (this is your numerator)

Third – if you have an improper fraction or mixed fraction, you will have more than one whole.

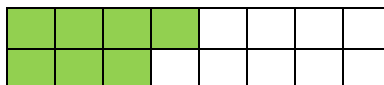
Examples: 1. $\frac{3}{4}$ The denominator is 4, therefore you need 4 parts in a whole.

The numerator is 3, therefore you need to shade 3 parts.



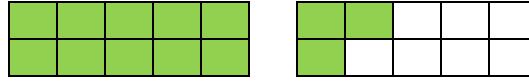
2. $\frac{7}{16}$ The denominator is 16, therefore you need 16 parts in a whole.

The numerator is 7, therefore you need to shade 7 parts.



3. $\frac{13}{10}$

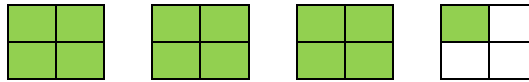
The denominator is 10, therefore you need 10 parts in a whole.
The numerator is 13, this is larger than 10 indicating you have more than a whole, draw enough wholes to contain 13 parts.



Do you see how this is also $1\frac{3}{10}$?

4. $3\frac{1}{4}$

The denominator is 4, therefore you need 4 parts in a whole.
You have a whole number of 3 and a numerator of 1, therefore you need 3 whole and 1 part of a fourth whole



Do you see how this is also $\frac{13}{4}$?

Strategies for Simplifying Fractions

First: Emphasis should be placed on simplifying fractions that one may run into often and the GCD is readily seen. Repetition of simpler problems will lead to speed and accuracy in the future.

Simplify

1. $\frac{6}{8} =$

2. $\frac{6}{10} =$

3. $\frac{2}{4} =$

4. $\frac{10}{16} =$

Second: Simplifying fractions in which a common divisor is obvious but the GCD may not be. We will use what is known as the vertical approach, divide numerator and denominator by the common divisor, cross the numerator/denominator out and place the result of the division above the numerator and below the denominator. Remember, it is OK to simplify more than once to obtain simplified answer.

Examples:

1. $\frac{\cancel{12}}{\cancel{20}} = \frac{3}{5}$

- numerator/denominator both even, divide by 2, cross out and place answers above/below problem.
- new numerator/denominator both even, divide by 2, cross out and place answers above/below problem.
- GCD of 3 and 5 is 1 so answer is 3/5

Some individuals prefer to cross out the numbers as they simplify as illustrated above and others do not as illustrated below

$$2. \frac{45}{120} = \frac{3}{8}$$

• numerator/denominator both divisible by 5, divide, and place answers above/below problem.
 • new numerator/denominator both divisible by 3, divide, and place answers above/below problem.
 • GCD of 3 and 8 is 1 so answer is 3/8

Simplify

1. $\frac{25}{30} =$

2. $\frac{24}{32} =$

3. $\frac{18}{24} =$

Third: What about the harder fractions like $\frac{39}{52}$ in which a common divisor is not readily seen?

You can use the Euclidean algorithm on page 21

Examples:

1. $\frac{39}{52} = \frac{3}{4}$ Euclidean algorithm

$$\begin{array}{r}
 1 \\
 39 \overline{) 52} \\
 \underline{- 39} \quad 3 \\
 13 \overline{) 39} \\
 \underline{- 13} \quad 3 \\
 \underline{- 39}
 \end{array}$$

13 is GCD $13 \overline{) 39} \quad 3$ $13 \overline{) 52} \quad 4$

2. $\frac{54}{81} = \frac{2}{3}$ Euclidean algorithm

$$\begin{array}{r}
 1 \\
 54 \overline{) 81} \\
 \underline{- 54} \quad 2 \\
 27 \overline{) 54} \\
 \underline{- 27} \quad 2 \\
 \underline{- 54}
 \end{array}$$

27 is GCD $27 \overline{) 54} \quad 2$ $27 \overline{) 81} \quad 3$

Simplify

1. $\frac{34}{51} =$

2. $\frac{33}{132} =$

Simplifying an improper fraction and changing an improper fraction to a mixed fraction should not be taught as meaning the same thing. Many times in higher mathematics, answers are expected to be left as improper fractions but simplified. Changing an improper fraction to a mixed fraction should be taught as a totally different concept.

We will use the same methods previously discussed for simplifying proper fractions to simplify improper fractions. Remember to leave your answer as an improper fraction.

Simplifying (leave answers as improper fractions)

1. $\frac{8}{6} =$

2. $\frac{16}{10} =$

3. $\frac{10}{4} =$

Changing an Improper Fraction to a Mixed Fraction

We will use what is traditionally taught as simplifying improper fractions as the method for changing an improper fraction to a mixed fraction.

To change an improper fraction to a mixed fraction, divide denominator into numerator to get whole number part, place remainder over original denominator to get fractional part and then simplify fraction part.

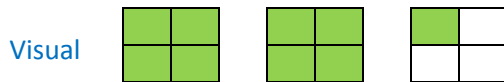
Examples:

1. $\frac{9}{4} = 2\frac{1}{4}$

$$\begin{array}{r} 2 \\ 4 \overline{) 9} \\ - 8 \\ \hline 1 \end{array}$$

Hence $\frac{9}{4} = 2\frac{1}{4}$

Why does this work? Recall: The denominator tells how many parts in a whole
 The numerator tells how many parts you have
 Hence, you end up with 2 whole and 1 part of a whole

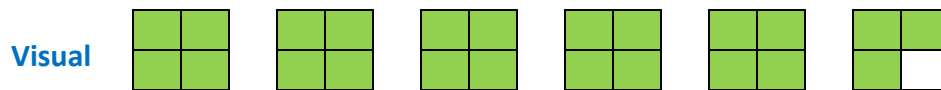


Changing a Mixed Fraction to an Improper Fraction

To change a mixed fraction to an improper fraction, multiply the whole number times the denominator of the fraction, add the numerator to get the new numerator, and leave the denominator the same.

Example: 1. $5\frac{3}{4} = \frac{5 \cdot 4 + 3}{4} = \frac{23}{4}$

Why does this work? Recall that $5\frac{3}{4}$ is the same as $5 + \frac{3}{4}$ or 5 wholes plus $\frac{3}{4}$ of another



Recall: The denominator tells how many parts in a whole (4).

5 whole implies $5 \cdot 4 = 20$ parts

$\frac{3}{4}$ of a whole implies 3 parts

Therefore we have **23 parts in all**

Addition and Subtraction of Proper Fractions

Strategy 1: Fractions with small denominators in which finding a common denominator seems rather easy. This strategy should always be taught first. **(Traditional)**

1. $\frac{1}{5} + \frac{2}{3}$

$$\frac{3}{15} + \frac{10}{15}$$

$$\frac{13}{15}$$

$$\frac{1}{4} + \frac{3}{10}$$

$$\frac{5}{20} + \frac{6}{20}$$

$$\frac{11}{20}$$

2. $\frac{4}{5} - \frac{2}{3}$

$$\frac{12}{15} - \frac{10}{15}$$

$$\frac{2}{15}$$

$$\frac{3}{4} - \frac{2}{3}$$

$$\frac{9}{12} - \frac{8}{12}$$

$$\frac{1}{12}$$

Strategy 2: Fractions with larger denominators in which finding a common denominator is not always easy. (Traditional with a non-traditional twist, “Mason Box” described on page 20)

$$1. \quad \frac{5}{12} + \frac{7}{15}$$

$$\frac{25}{60} + \frac{28}{60}$$

$$\frac{53}{60}$$

$$3 \left| \begin{array}{cc} 4 & 5 \\ 12 & 15 \end{array} \right.$$

$$\text{lcd} = 5 \cdot 12 \text{ or } 4 \cdot 15 = 60$$

$$2. \quad \frac{11}{18} - \frac{7}{24}$$

$$\frac{44}{72} - \frac{21}{72}$$

$$\frac{23}{72}$$

$$3 \left| \begin{array}{cc} 3 & 4 \\ 9 & 12 \end{array} \right. \\ 2 \left| \begin{array}{cc} 18 & 24 \end{array} \right.$$

$$\text{lcd} = 3 \cdot 24 \text{ or } 4 \cdot 18 = 72$$

Try this strategy for yourself on the following.

$$\frac{5}{12} + \frac{5}{16}$$

$$\frac{13}{21} - \frac{5}{24}$$

Strategy 3: Fractions in which the numerator and denominator are single digit numbers and the denominators do not have a common factor. **(Non-Traditional)**

	transitioned to	
Traditional <i>Show all work</i>	→	Non-Traditional <i>mental math</i>
1. $\frac{1}{4} + \frac{2}{3}$	$\begin{array}{r} 3 + 8 \\ \frac{1}{4} + \frac{2}{3} \\ 12 \\ \hline \frac{11}{12} \end{array}$	$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$
$\frac{3}{12} + \frac{8}{12}$	$\frac{11}{12}$	$\frac{11}{12}$
$\frac{11}{12}$		
2. $\frac{4}{5} - \frac{2}{3}$	$\begin{array}{r} 12 - 10 \\ \frac{4}{5} - \frac{2}{3} \\ 15 \\ \hline \frac{2}{15} \end{array}$	$\frac{4}{5} - \frac{2}{3} = \frac{2}{15}$
$\frac{12}{15} - \frac{10}{15}$	$\frac{2}{15}$	$\frac{2}{15}$
$\frac{2}{15}$		

Try this strategy for yourself on the following.

1. $\frac{3}{4} + \frac{1}{3}$

2. $\frac{2}{7} + \frac{1}{2}$

3. $\frac{3}{5} - \frac{1}{4}$

4. $\frac{4}{5} - \frac{3}{4}$

Strategy 4: Fractions in which the numerator and denominator are single digit numbers and the denominators have a common factor. **(Non-Traditional)**

	transitioned to			
Traditional <i>Show all work</i>	→			Non-Traditional <i>mental math</i>
1. $\frac{1}{4} + \frac{5}{6}$	$\frac{1}{4} + \frac{5}{6}$	$\frac{1}{4} + \frac{5}{6}$	$\frac{1}{4} + \frac{5}{6}$ <small>6 + 20 24</small>	$\frac{1}{4} + \frac{5}{6}$
$\frac{3}{12} + \frac{10}{12}$	$\frac{6}{24} + \frac{20}{24}$	$\frac{26}{24}$	$\frac{26}{24}$	$\frac{26}{24}$
$\frac{13}{12}$	$\frac{26}{24}$	$\frac{13}{12}$	$\frac{13}{12}$	$\frac{13}{12}$
$1\frac{1}{12}$	$\frac{13}{12}$	$1\frac{1}{12}$	$1\frac{1}{12}$	$1\frac{1}{12}$
	$1\frac{1}{12}$			
2. $\frac{7}{8} - \frac{5}{6}$	$\frac{7}{8} - \frac{5}{6}$	$\frac{7}{8} - \frac{5}{6}$	$\frac{7}{8} - \frac{5}{6}$ <small>42 - 40 48</small>	$\frac{7}{8} - \frac{5}{6}$
$\frac{21}{24} - \frac{20}{24}$	$\frac{42}{48} - \frac{40}{48}$	$\frac{2}{48}$	$\frac{2}{48}$	$\frac{2}{48}$
$\frac{1}{24}$	$\frac{2}{48}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$
	$\frac{1}{24}$			

Try this strategy for yourself on the following.

1. $\frac{1}{4} + \frac{1}{6}$

2. $\frac{2}{9} + \frac{1}{6}$

3. $\frac{5}{8} - \frac{3}{10}$

4. $\frac{5}{6} - \frac{5}{8}$

Strategy 5: Fractions in which one denominator is a multiple of the other. **(Non-Traditional)**

transitioned to

Traditional <i>Show all work</i>		Non-Traditional <i>mental math</i>
<p>1. $\frac{5}{6} + \frac{7}{12}$</p> <p>$\frac{10}{12} + \frac{7}{12}$</p> <p>$\frac{17}{12}$</p> <p>$1\frac{5}{12}$</p>	<p>$\overset{10}{\frac{5}{6}} + \frac{7}{12}$</p> <p>$\frac{17}{12}$</p> <p>$1\frac{5}{12}$</p>	<p>$\frac{5}{6} + \frac{7}{12}$</p> <p>$\frac{17}{12}$</p> <p>$1\frac{5}{12}$</p>

$$2. \quad \frac{5}{6} - \frac{7}{12}$$

$$\frac{10}{12} - \frac{7}{12}$$

$$\frac{3}{12}$$

$$\frac{10}{6} - \frac{7}{12}$$

$$\frac{3}{12}$$

$$\frac{5}{6} - \frac{7}{12}$$

$$\frac{3}{12}$$

Try this strategy for yourself on the following.

$$1. \quad \frac{1}{4} + \frac{1}{16}$$

$$2. \quad \frac{7}{9} + \frac{1}{3}$$

$$3. \quad \frac{5}{8} - \frac{1}{4}$$

$$4. \quad \frac{5}{6} - \frac{3}{18}$$

Addition of Mixed fractions

Traditional #1

$$3\frac{2}{3} + 8\frac{4}{5}$$

$$3\frac{10}{15} + 8\frac{12}{15}$$

$$11\frac{22}{15}$$

$$12\frac{7}{15}$$

Traditional #2

$$3\frac{2}{3} + 8\frac{4}{5}$$

$$\frac{11}{3} + \frac{44}{5}$$

$$\frac{55}{15} + \frac{132}{15}$$

$$\frac{187}{15}$$

$$12\frac{7}{15}$$

Non-Traditional Strategy: Add the whole numbers and use one of the five strategies discussed earlier on addition of the fractions.

Try this strategy for yourself on the following.

1. $3\frac{5}{8} + 6\frac{1}{4}$

2. $8\frac{2}{3} + 7\frac{3}{4}$

Subtraction of Mixed fractions without the need to regroup/borrow

Traditional #1

$$13\frac{2}{3} - 8\frac{1}{5}$$

$$13\frac{10}{15} - 8\frac{3}{15}$$

$$5\frac{7}{15}$$

Traditional #2

$$13\frac{2}{3} - 8\frac{1}{5}$$

$$\frac{41}{3} - \frac{41}{5}$$

$$\frac{205}{15} - \frac{123}{15}$$

$$\frac{82}{15}$$

$$5\frac{7}{15}$$

Non-Traditional Strategy: Subtract the whole numbers and use one of the five strategies discussed earlier on subtraction of the fractions.

Try this strategy for yourself on the following.

1. $13\frac{5}{16} - 6\frac{1}{4}$

2. $12\frac{4}{5} - 7\frac{2}{3}$

Subtraction of Mixed fractions with the need to regroup/borrow

Traditional #1

$$11\frac{1}{3} - 8\frac{3}{5}$$

$$11\frac{5}{15} - 8\frac{9}{15}$$

$$10\frac{20}{15} - 8\frac{9}{15}$$

$$2\frac{11}{15}$$

Traditional #2

$$11\frac{1}{3} - 8\frac{3}{5}$$

$$\frac{34}{3} - \frac{43}{5}$$

$$\frac{170}{15} - \frac{129}{15}$$

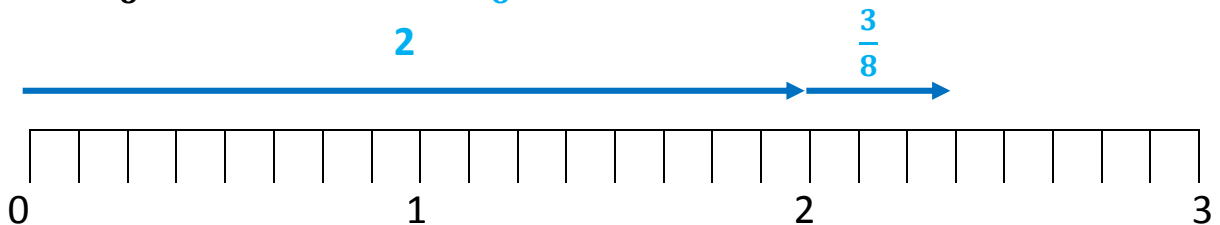
$$\frac{41}{15}$$

$$2\frac{11}{15}$$

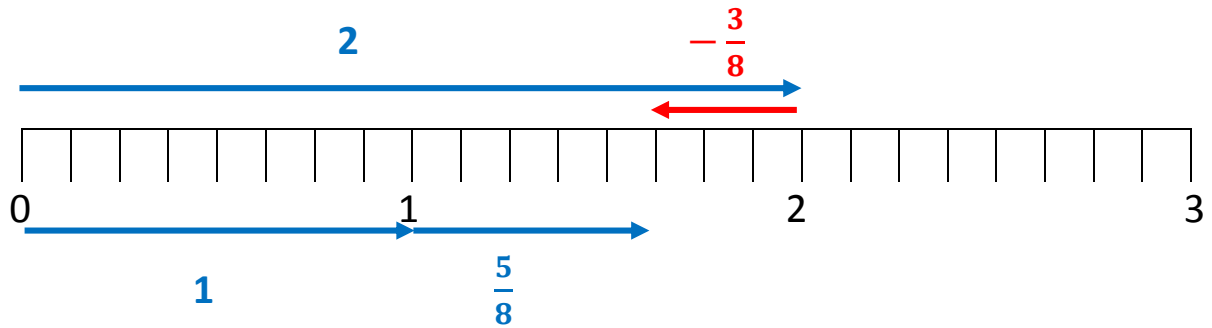
Non-Traditional Strategy: Subtract the whole numbers and use one of the five strategies discussed earlier on subtraction of the fractions.

Need to understand:

$2\frac{3}{8}$ is the same as $2 + \frac{3}{8}$



$2\frac{-3}{8}$ means $2 - \frac{3}{8}$ which is the same as $1 + \frac{5}{8}$ which is $1\frac{5}{8}$



Try this strategy for yourself on the following.

1. $11\frac{3}{8} - 6\frac{3}{4}$

2. $10\frac{1}{4} - 7\frac{2}{3}$

3. $12\frac{1}{3} - 5\frac{5}{8}$

4. $16\frac{2}{5} - 7\frac{4}{9}$

Divisibility Rules

Traditional Divisibility Rules that are usually presented

Two: A number is divisible by **two**, if the number is even. (ends in 0, 2, 4, 6, 8)

Examples: 24 ends in 4, therefore even and divisible by two
2108 ends in 8, therefore even and divisible by two

Five: A number is divisible by **five**, if the number ends in a 5 or 0.

Examples: 25 ends in 5, therefore even and divisible by five
2105 ends in 5, therefore even and divisible by five

Ten: A number is divisible by **ten**, if the number ends in a 0.

Examples: 20 ends in 0, therefore even and divisible by ten
2100 ends in 0, therefore even and divisible by ten

Non-Traditional Divisibility Rules that are not usually presented but are good to know

Three: A number is divisible by **three**, if the sum of its digits is divisible by 3.

Examples: 24 sum of digits is $2 + 4 = 6$ and **6 is divisible by 3**, therefore 24 is divisible by three
2118 sum of digits is $2 + 1 + 1 + 8 = 12$ and **12 is divisible by 3**, therefore 2118 is divisible by three

Four: A number is divisible by **four**, if the number formed by the last two digits is divisible by 4.

Examples: 212 number formed by last two digits is 12 and **12 is divisible by 4**, therefore 212 is divisible by four
2104 number formed by last two digits is 04 and **4 is divisible by 4**, therefore 2104 is divisible by four

Six: A number is divisible by **six**, if the number is divisible by 2 and 3 both.

Examples: 312 ends in 2, therefore even and divisible by 2
sum of digits is 6 and **6 is divisible by 3**, therefore 312 is divisible by six
2154 ends in 4, therefore even and divisible by 2
sum of digits is 12 and **12 is divisible by 3**, therefore 2154 is divisible by six

Seven: Remove the last digit of the number, double it, and then subtract it from the rest of the number (not including that last digit which was removed). If you get a number divisible by 7, then your original number is divisible by 7.

- Examples:**
- 273 Take the last digit 3, double it to get 6, and subtract it from 27 ($27 - 6 = 21$ and 21 is divisible by 7), therefore 273 is divisible by seven
 - 3983 Take the last digit 3, double it to get 6, and subtract it from 398 ($398 - 6 = 392$ and is 392 divisible by 7? Not sure repeat process on 392) Take the last digit 2, double it to get 4, and subtract it from 39 ($39 - 4 = 35$ and 35 divisible by 7), therefore 3983 is divisible by seven

Eight: A number is divisible by **eight**, if the number formed by the last three digits is divisible by 8.

- Examples:**
- 5240 number formed by last three digits is 240 and 240 is divisible by 8, therefore 5240 is divisible by eight
 - 32,104 number formed by last three digits is 104 and 104 is divisible by 8, therefore 32,104 is divisible by eight

Nine: A number is divisible by **nine**, if the sum of its digits is divisible by 9.

- Examples:**
- 2547 sum of digits is $2 + 5 + 4 + 7 = 18$ and 18 is divisible by 9, therefore 2547 is divisible by nine
 - 2115 sum of digits is $2 + 1 + 1 + 5 = 9$ and 9 is divisible by 9, therefore 2115 is divisible by nine

Eleven: A number is divisible by **eleven**, if the difference between the sum of the digits in the odd positions of the number and the sum of the digits in the even positions of the number is divisible by 11. A lot of the time, the two sums will be equal, so their difference equals 0, which is divisible by everything, so certainly is divisible by 11.

- Examples:**
- 583 Sum of odd positions ($5 + 3 = 8$), sum of even positions (8), difference is 0 and 0 is divisible by 11 therefore 583 is divisible by eleven
 - 6160 Sum of odd positions ($6 + 6 = 12$), sum of even positions ($1 + 0 = 1$) difference is 11 and 11 is divisible by 11 therefore 6160 is divisible by eleven

Mason Box Strategy for Finding LCM (Non-Traditional Method)

Place the two numbers you are trying to find the least common multiple of inside of an open box

Find a number that divides into both numbers

Place this number outside of the box in front of the numbers

Divide each number and place answer above number in open box

Repeat above process until the only number that divides both is one

Multiple numbers in opposite corners of open box to find LCM

It does not matter which diagonal you use, they both result in same LCM

Examples:

1. LCM (8, 12) = 24

$$4 \left| \begin{array}{cc} 2 & 3 \\ 8 & 12 \end{array} \right. \quad \text{or} \quad 2 \left| \begin{array}{cc} 2 & 3 \\ 4 & 6 \\ 8 & 12 \end{array} \right.$$

2. LCM(20,15) = 60

$$5 \left| \begin{array}{cc} 4 & 3 \\ 20 & 15 \end{array} \right.$$

In either case, the outside corners are the same 2 & 12 or 3 & 8.

LCM (20,15) = 4 · 15 or 3 · 20 = 60

LCM (8, 12) = 2 · 12 or 3 · 8 = 24

What if you are finding the LCM of three numbers?

Find the LCM of two numbers

Then find the LCM of this number (LCM of first two numbers) and the third number

Example:

LCM (20, 15, 24) = 120

$$5 \left| \begin{array}{cc} 4 & 3 \\ 20 & 15 \end{array} \right. \quad \begin{array}{c} 6 \\ 2 \end{array} \left| \begin{array}{cc} 5 & 2 \\ 30 & 12 \\ 60 & 24 \end{array} \right.$$

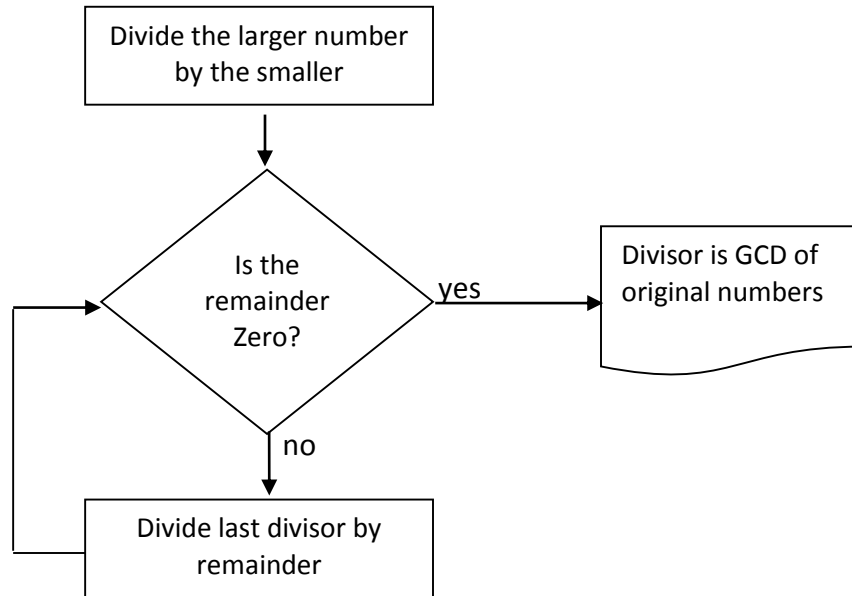
LCM (20, 15) = 3 · 20 = 60

LCM (60, 24) = 2 · 60 = 120

Hence LCM (20, 15, 24) = 120

Euclidean Algorithm for Finding Greatest Common Divisor

Euclidean algorithm is found in Book IV of Euclid's Elements (300 BC) but is seldom taught.



Using this method on the following examples

$$\text{GCD}(20, 30) = 10$$

$$\begin{array}{r} 1 \\ 20 \overline{)30} \\ \underline{20} \quad 2 \\ 10 \overline{)20} \\ \underline{20} \\ 0 \end{array}$$

$$\text{GCD}(24, 84) = 12$$

$$\begin{array}{r} 3 \\ 24 \overline{)84} \\ \underline{72} \quad 2 \\ 12 \overline{)24} \\ \underline{24} \\ 0 \end{array}$$

What if you are finding the GCD of three numbers?

This method only works on two numbers at a time.

Find the GCD of two of the numbers and then find the GCD of this answer and the third number.

Example:

$$\text{GCD}(120, 250, 200) = 10$$

$$\begin{array}{r} 2 \\ 120 \overline{)250} \\ \underline{240} \quad 12 \\ 10 \overline{)120} \\ \underline{120} \\ 0 \end{array}$$

$$\begin{array}{r} 20 \\ 10 \overline{)200} \\ \underline{200} \\ 0 \end{array}$$

$$\text{GCD}(120, 250) = 10 \quad \text{GCD}(10, 200) = 10$$

$$\text{Hence GCD}(120, 250, 200) = 10$$

Closing Activity

This activity will involve how a fundamental understanding of multiplication could change the future for multiplying whole numbers.

Multiply each of the following using strategies you teach.

1. 10002×4326

2.
$$\begin{array}{r} 321 \\ \times 242 \\ \hline \end{array}$$

Material Available on Using Non-Traditional Strategies

**Fundamental Concepts of Mathematics I and Teaching Strategies for Teachers
Part 1: Numeration Systems, 2014, Lulu Publishing.**

Paperback, 71 pages in full color, 211 practice problems with answers, \$10.99

**Fundamental Concepts of Mathematics I and Teaching Strategies for Teachers
Part 2: Whole Numbers, 2014, Lulu Publishing.**

Paperback, 117 pages in full color, 368 practice problems with answers, \$14.99

**Fundamental Concepts of Mathematics I and Teaching Strategies for Teachers
Part 3: Integers, 2014, Lulu Publishing.**

Paperback, 53 pages in full color, 219 practice problems with answers, \$10.99

**Fundamental Concepts of Mathematics I and Teaching Strategies for Teachers
Part 4: Fractions, 2014, Lulu Publishing.**

Paperback, 121 pages in full color, 374 practice problems with answers, \$14.99

**Fundamental Concepts of Mathematics I and Teaching Strategies for Teachers
Part 5: Decimals, 2014, Lulu Publishing.**

Paperback, 96 pages in full color, 498 practice problems with answers, \$12.99

**Fundamental Concepts of Mathematics I and Teaching Strategies for Teachers,
2014, Lulu Publishing.**

Paperback, 408 pages in full color, Contains Parts 1 - 5 listed above, 1670 practice problems with answers, \$49.99

The material outlined above is exclusively available through Lulu Publishing and is presently on sale for 20% off the prices listed above.

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