Examining Student-Centered Instruction in High School Mathematics

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Abstract

As districts implement rigorous standards for student learning in mathematics that emphasize deep understanding, procedural fluency, problem solving, and reasoning, attention is being placed on student-centered approaches to instruction. This study uses qualitative and quantitative methods to (a) explore student-centered approaches to mathematics instruction in high school, (b) examine the relationship between implementation of these approaches and instructional context, and (c) investigate the relationship between student-centered instruction and student outcomes. Through this mixed methods approach, this study provides a nuanced look at the ways in which teachers implement student-centered approaches to mathematics instruction and found that teachers who use these approaches more regularly believed in the importance of these types of instructional approaches, worked in schools that focused on preparing students for a variety of life pathways, and had flexibility in lesson design. On the quantitative side, the study found a statistically significant positive relationship between student-centered instruction and students' (a) problem solving performance and (b) reported engagement. The study's finding may be useful to teachers, teacher leaders, and teacher educators who are interested in more student-centered approaches to mathematics instruction.

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Far too many students see mathematics as a subject that must be endured, not as a subject that provides tools for making sense of the world around them. Too often, they view mathematics as a set of procedures that should be memorized, not understood. This may be the result of the way in which students engage with mathematics in the classroom. Traditionally, instruction in the United States has consisted of teacher lecture followed by student practice, with a focus on application of procedures (Hiebert et al., 2003; Stigler & Hiebert, 2004). Although this approach to instruction promotes the development of mathematical skills, it does not necessarily promote deep understanding of mathematics (Hiebert & Grouws, 2007). As states and districts implement more rigorous College and Career Readiness Standards that emphasize procedural fluency, reasoning, understanding and problem solving, including the Common Core State Standards in Mathematics, teachers will need to find ways to modify instruction to teach for sense-making and the development of mathematical problem solving, communication, and critical thinking skills.

Advocates of student-centered approaches to teaching and learning argue that, in order to reach these more rigorous standards, instructional environments should (a) focus on the skills and understandings important to the discipline as well as 21st century skills, (b) engage students in sense-making, (c) have well-established norms of trust and respect, and (d) provide opportunities for differentiation to support student learning (Bransford, Brown, & Cocking, 2000; Fischer, 2009; Hinton, Fischer, & Glennon, 2013; Piaget, 1952; Trilling and Fadel, 2009; Vygotsky, 1978). For decades, mathematics educators have also advocated for these features of instruction and, over time, the National Council of Teachers of Mathematics has endorsed these approaches through the release of various standards documents (1989, 1991, 2000), including the recent *Principles to Actions: Ensuring Mathematical Success for All* (2014).

Taken together, these recommendations present a vision for student-centered mathematics instruction. Although the challenges associated with realizing that vision are well documented, more work is needed to understand the ways in which teachers are able implement instruction that realizes some, if not all, of that vision and how these approaches relate to student outcomes, particularly in high schools. By investigating the following research questions the study presented here addresses this need:

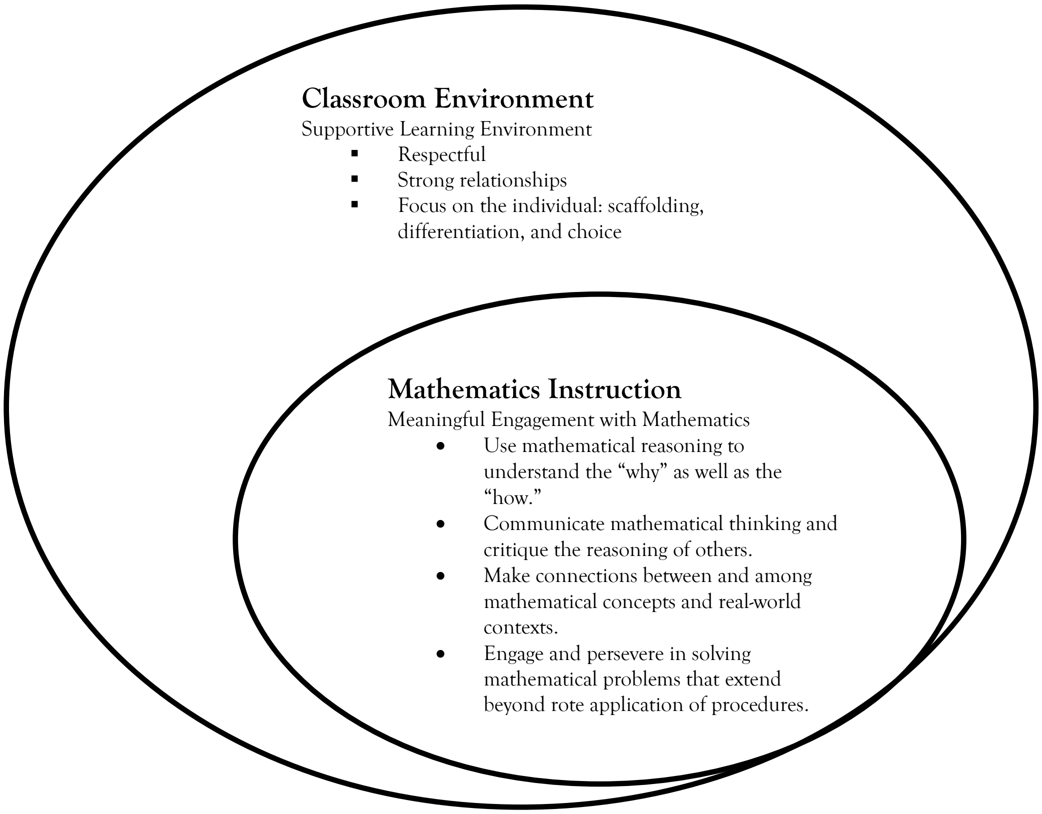
1. What are different ways in which highly regarded high school mathematics teachers implement student-centered instructional practices?
2. How do high school mathematics teachers' teaching philosophy and instructional environments relate to the degree to which they implement student-centered instruction?
3. What types of instructional approaches do high school students say help them succeed in mathematics?
4. What is the relationship between the degree to which student-centered instructional practices are implemented and high school students' (a) engagement and (b) problem-solving skills?

Lens for Studying Student-Centered Mathematics Instruction

To analyze student-centered approaches to mathematics instruction, this study distinguished student-centered features of the *classroom environment* from student-centered features of *mathematics instruction*. As illustrated in Figure 1, student-centered features of the classroom environment are those that are supportive of students, but not related to the way in which students interact with mathematics. These characteristics include: a caring, respectful environment; responsiveness to individual student needs and interests; and the establishment of strong relationships between and among teachers and students. Student-centered approaches to mathematics instruction are those that provide opportunities for all students to *meaningfully* engage with mathematics. That is, these approaches provide all students with opportunities to

- Use mathematical reasoning to understand the "why" as well as the "how."
- Communicate mathematical thinking and critique the reasoning of others.
- Make connections between and among mathematical concepts and real-world contexts.
- Engage and persevere in solving mathematical problems that extend beyond the rote application of procedures.

Figure 1. Characteristics of Student-Centered Mathematics Classrooms



The distinction between the classroom environment and instruction is important. Consider a supportive, student-centered classroom environment where norms of respect and trust have been established and students receive individual support to scaffold their learning inside and outside of the classroom. Within this supportive environment, the way in which students interact with the mathematics can take many forms. In one classroom, instruction might focus on rote application of mathematical procedures through teacher lecture followed by student practice. In another classroom, instructional activities might provide students with opportunities to explore, problem solve, reason, and communicate about mathematics. Both classrooms offer supportive learning environments, but the second classroom provides more opportunities for students to meaningfully engage with mathematics and, thus, is more student-centered.

With this lens for examining the features of student-centered classrooms, it is possible to hold general features of student-centered classroom environments constant so that differences in approaches to mathematics instruction and associated student outcomes could be analyzed and compared. This study investigates the instructional approaches, instructional context, and student outcomes of highly-regarded teachers who had well-established student-centered classroom environments but who varied in the degree to which they implemented instruction that provides opportunities for all students to meaningfully engage with mathematics.

Method

The nature of the study's research questions required a mixed methods approach. Case study methodology was used to investigate the first three research questions, which are intended to provide insight into the different ways in which student-centered approaches are implemented in high school mathematics classrooms, the contextual factors that support or hinder their use, and how students react to those approaches. Quantitative methods were used to address the fourth research question, which examines the relationship between student-centered instructional practices in mathematics and student outcomes.

Sample and Selection

The sample of participating teachers was drawn from New England and New York. Recruitment of teachers for both components of the study involved reaching out to district and school leaders, as well as representatives from student-centered school networks and organizations dedicated to promoting student-centered teaching approaches to solicit nominees. To ensure that the final sample of participants included teachers who had established supportive, student-centered learning environments for students but who represented a range of instructional approaches, the study team collected data from the teachers as well as the instructional leaders with whom they worked. Table 1 illustrates the data sources used to select participants for the study.

Table 1

Data Sources Used to Select Teacher Participants

Data Sources	Information Obtained
Teacher Application/Interview	Approach to and design of mathematics instruction; ways of supporting struggling students
Instructional Leader Survey	Degree to which they consider teacher to be “one of the best;” teacher’s approach to supporting struggling students; teachers’ instructional style and learning environment; teacher’s ability to support student success
Classroom Observation (case study candidates, only)	Implementation of mathematics instruction

Ultimately, 22 teachers who were highly regarded, created supportive learning environments for students, but represented a range of instructional approaches were identified for the quantitative component of the study, with 7 of those teachers participating in the case study component. Table 2 shows the characteristics of the final sample of teachers

Table 2

Demographics and Professional Context of Participating Teachers

Characteristics of Teachers	Overall (*n* = 22)
Gender (%)	
Male	27.3
Female	72.7
High School Teaching Experience (%)	
1-3 years	13.6
4-10 years	18.2
11 years or more	68.2
Degree in Mathematics or Mathematics Education (%)	
Bachelor’s	63.6
Master’s	27.3
School Location (%)	
Large or small city	22.7
Suburb	22.7
Rural or town	54.5
School Enrollment (%)	
Fewer than 300 students	13.6
300-999 students	68.2
1,000 or more students	18.2
School Demographics (mean %)	
Minority	22.8
Free or reduced price lunch	30.0

Source: Common Core of Data public and private school data 2010-2011, 2011-2012 school years, state department of education websites, and study records.
Note: Gender, high school teaching experience, and degree in mathematics or mathematics education were collected as part of the study’s teacher survey.

About three-quarters of the 22 teachers were female and one-quarter were male. They had an average of 16 years of experience, although five of the teachers had been teaching for fewer than five years. Eighteen of the teachers had an undergraduate degree in mathematics, and eight held a master’s degree in mathematics or mathematics education.

In terms of the school context, all but a few of the teachers taught in regular public schools. A few schools were public charter or magnet schools, and a few schools were private schools. The schools were located across the six New England states (Vermont, Maine, Rhode Island, Massachusetts, Connecticut, and New Hampshire) and New York and were situated in a wide range of geographic settings. About one half of the schools were located in rural or small town jurisdictions; the other half were located in suburban, small city, or large city settings.[1] The average number of students per school was approximately 650, with the smallest school enrolling 115 students and the largest enrolling more than 1,500 students. The percentage of students from minority families and students eligible for free or reduced-price lunch across all study schools was 22 percent and 30 percent, respectively. On average, schools participating in the study were somewhat smaller and less racially or ethnically diverse than national averages, but they were representative of their host cities or states.[2]

Data Sources

To answer the study's research questions, several different types of qualitative and quantitative data were collected from the participating teachers, students, and schools. Table 3 shows the data sources for the study. Student-level data collection focused on one, pre-determined "target" class from each teacher. Identification of the target class for each teacher was informed by the study's problem-solving assessment. The assessment was created from publically released items from the Programme for International Student Assessment (PISA), which is administered internationally to students aged 15-16 years. Classes that were composed of mostly 15-16 year old students were, therefore, chosen as the target classes for this study.

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[1] These numbers were taken from the most recently available local education agency universe and public elementary/secondary school universe data files released as part of the Common Core of Data, a program of the U.S. Department of Education's National Center for Education Statistics that annually collects fiscal and non-fiscal data about all public schools, public school districts, and state education agencies in the United States.

[2] Nationally, the average high school enrollment is 684 students, the overall minority percentage is 48 percent, and the percentage of high school students eligible for free or reduced-price lunch is 38 percent. For the six New England states, the average high school enrollment, the overall mean percent minority, and the mean high school percent eligible for free or reduced-price lunch are 704, 22 percent and 30 percent, respectively.

Table 3

Study Data Sources

Data Source	Nature of Data	Target Classrooms	Research Questions
Videos of mathematics instruction	Instructional practices implemented in lessons where a new mathematics concept is introduced (three lessons per teacher)	Case study teachers	1
Instructional logs	Description and examples of instructional activities used throughout a week of instruction (one week per month per teacher for eight months)	Case study teachers	1
Teacher interview	Teachers' perceptions of their school and mathematics department, philosophy of mathematics instruction, planning process, instructional practices, and the challenges faced in implementing instruction aligned with their philosophy	Case study teachers	2
Student focus groups	Students' perceptions of their experiences in mathematics class and the factors that contribute to student success in mathematics (three to five students per teacher)	Case study teachers	3
Administrative records	Demographic data (e.g., the percentage of students from minority families, English language learners, students in special education, and students who free or reduced-price lunch as well as grade 8 achievement on state mathematics tests) at the student and school levels	All teachers	2, 4
Teacher survey	Frequency of instructional practices implemented with the target class	All teachers	4
Challenging assignments	Examples of the most challenging assignment (to be completed by individual students) offered to the target class over a specified period of time	All teachers	4
Student survey	Students' perceptions of their school and their experiences in the target mathematics class	All teachers	4
Mathematical problem-solving assessment	Student responses to publicly released items from PISA, an international assessment given to 15- and16-year-old students.	All teachers	4

Findings

Student-Centered Approaches to Mathematics Instruction

The first research question focused on the different ways in which highly regarded high school mathematics teachers implement student-centered instructional practices. Video and instructional log data were used to identify the range of student-centered approaches implemented by the seven case study teachers. Analysis of this data focused on the tasks and activities offered to students as well as the communication around those tasks and activities. These aspects of instruction were analyzed in two common phases of mathematics lessons: the development of new mathematical ideas and the reinforcement of previously introduced mathematics.

During the development phase, students are presented with and time is spent fully developing a new mathematics concept or rule. This can happen at any point in the lesson and often happens at the beginning of a lesson, after a review of homework, or after a warm-up problem. This type of instruction might happen more than once during a lesson and can take any amount of time to complete. In some cases, development activities can take the entire class period.

The reinforcement phase is when students have the opportunity to strengthen their understanding and practice applying mathematics content. Like the development of new mathematics, reinforcement opportunities can occur at any point in the lesson—during the warm up, homework review, classwork, homework—and may appear at several points in any lesson.

As expected, the analyses identified a range of instructional approaches that provide the learning opportunities outlined in Figure 1. Teacher-led, whole-class discussion was the most commonly implemented structure for the development of new mathematics. Within this structure, teachers used different types of activities and discussion techniques to engage students in thinking about the material. Some teachers implemented instructional approaches that focused the discussion on mathematical procedures, only. Other teachers implemented instructional approaches that emphasized the conceptual underpinnings of the mathematical concept being introduced. Some approaches allowed for a minimal amount of student contribution, which was often focused on stating mathematical facts. Others allowed for stronger student contribution, often with opportunities for students to explain their thinking. The strongest student-centered approach to the development of new mathematics observed was one in which students explored the new material prior to formal presentation from the teacher.

Analyses of the reinforcement of previously introduced mathematics indicated that teachers engaged students in different types of problems and tasks. Some teachers used problems that required the rote application of procedures, while others used problems that required students to reason, think critically and communicate their thinking. In talking with the students about those tasks, some of the teachers focused only on the procedures and occasionally asked students for the next step in solving the problem. Others would ask students to share their thinking. These teachers would frequently ask “Why?” and “Can you explain?” as students provided the answers, illustrating a stronger student-centered approach to mathematics instruction.[3]

Linking Instruction, Philosophy and Environment

The second research question focused on how a teacher’s expressed philosophy of mathematics teaching and learning and instructional environment relate to the extent to which student-centered approaches to mathematics instruction were implemented. Data collected from

[3] To read detailed classroom vignettes that elaborate on these approaches, see the full report at http://www.nmefoundation.org/resources/student-centered-learning/an-up-close-look-at-student-centered-math-teaching

the videos, instructional log and interviews with the seven case study teachers was used to address this question. Analyses indicated that, although each of the case study teachers had established supportive, student-centered learning environments for their students (e.g. environments characterized by support and respect for all students, strong relationships, and willingness provide help when needed), they differed in in the degree to which they implemented student-centered mathematics instruction. Some implemented these approaches more regularly. Others did not. They all, however, implemented student-centered approaches to instruction to some extent. Analysis of the interview data indicated that those who implemented student-centered approaches to mathematics instruction more regularly believed in the importance of providing students with opportunities to explore, communicate, and reason in mathematics class. These teachers also believed their schools to be more focused on supporting a variety of pathways for students than on standardized test scores and had access to textbooks that were full of exploratory activities and complex mathematics problems.

Student Perspectives

The third research question addressed the types of instructional approaches that students believe help them to succeed in mathematics. Data collected via the student focus groups was used to address this research question. Analyses focused on what students felt had helped them to be successful and whether or not their opinion of mathematics had changed as a result of being in the class. Because the study targeted highly regarded teachers, who had established supportive learning environments, it is not surprising that students felt very positively about their experiences in the teachers' classrooms, regardless of instructional approach. Students cited organization, opportunity for additional help, and the teachers' ability to explain things well as features that helped them to be successful. However, the data also indicated that students assigned to teachers who implemented student-centered approaches more regularly appreciated the opportunity to explore mathematics, see the connections between the mathematics and the real world, and be engaged in the lesson. These students reported finding the content interesting and meaningful, and some indicated that they had grown to love mathematics over the course of the year.

Relationship to Student Outcomes

The final research question examined differences in student engagement and problem-solving skills among students assigned to teachers who varied in the degree to which they implemented student-centered instructional practices. In order to conduct the analysis, it was necessary to create a measure of student-centered practice (SCP). This measure was created using information about instructional practice collected via the teacher survey and challenging assignments collected.[4] Information regarding student engagement was collected via the student survey. Information regarding students' problem-solving skills was collected with the problem-solving assessment that contained publically-released items from the PISA.

The following mixed model was used to examine the relationship between instruction and student engagement.

$$\boldsymbol{y}_{SS} = \alpha + \boldsymbol{SCP} \cdot \delta + \boldsymbol{\epsilon}$$

In this equation, y_{SS} is a vector of the students' score on that construct (engagement). SCP is the composite measure of student-centered instruction described previously, α is a regression coefficient for the intercept, δ is a regression coefficient for the effect of student-centered teaching on the students' engagement or interest, and ϵ is an error term that includes a

[4] For a more detailed description of the composite measure of student centered practice (SCP), see the full report at http://www.nmefoundation.org/resources/student-centered-learning/an-up-close-look-at-student-centered-math-teaching

component for the teacher using a linear mixed model. This model measures survey outcomes as a function of the SCP measure and does not adjust for any baseline covariates. The analysis indicated a statistically significant positive relationship between the SCP measure and both student engagement.

The following model was used to examine the relationship between instruction and problem-solving skills:

$$\boldsymbol{y}_{PISA} = \alpha + \boldsymbol{y}_{NECAP-8} \cdot \beta + \boldsymbol{SCP} \cdot \delta + \boldsymbol{\epsilon}$$

In this equation, y_{PISA} is a vector of the students' end of year PISA scale scores, $y_{NECAP-8}$ is the vector of students' grade eight math scale scores, SCP is the composite student-centered instruction measure defined previously, α is a regression coefficient for the intercept, β is a regression cefficient for the slope of the grade eight NECAP, δ is a regression coefficient for the effect of student-centered teaching on the students, and $\boldsymbol{\epsilon}$ is an error term that includes a component for the teacher using a linear mixed model. This model compares the value-added results of students in classrooms with varying degrees of student-centered instruction, using the study's SCP measure. The analysis indicated a statistically significant positive relationship between the SCP measure and student problem-solving skills.

Taken together, these results indicate that the benefits to students of having a highly regarded mathematics teachers, who create supportive learning environments are even greater when the teacher implements student-centered approaches to instruction as defined in this study.

Conclusion & Implications

Overall, several conclusions emerge from this study of student-centered mathematics instruction. The qualitative analyses identified a range of instructional approaches that provided opportunities for students to engage meaningfully with mathematics. The data indicated that there are many variations in the types and frequencies with which teachers implement student-centered approaches in high school mathematics classrooms. The study showed that even teachers who report implementing more traditional approaches to instruction will often implement some aspects of student-centered instruction at times. The study also found that the instructional context—the philosophy of the teachers and the school, the curricular materials available—may be related to the degree to which teachers implement student-centered approaches to mathematics instruction. The study also found that students appreciate being taught by highly regarded teachers, who create supportive learning environments and are able to identify specific aspects of instruction that help them succeed in mathematics classrooms.

The quantitative analyses showed positive, significant relationships between the study's measure of student-centered practices and students' engagement and problem-solving skills, suggesting that the benefits of having a highly regarded mathematics teacher, who creates a positive learning environment may be even greater if the teacher is more student-centered in his or her approach.

Drawing on these conclusions, this study has at least three concrete implications for policymakers and practitioners who are interested in promoting student-centered instruction in mathematics.

- The fine-grained definition of student-centered mathematics instruction used in this study may help promote this type of instruction, providing multiple entry points for teachers who are trying to move in this direction.
- Even highly abstract concepts (such as those taught in high school) can be presented in student-centered ways.

- Teaching philosophy and instructional context may interact and affect how strongly and consistently teachers enact student-centered approaches to mathematics instruction.

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