How do Teachers Make Sense of Student Work for Instruction[1]?

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Recent research on mathematics teaching promotes a view of ambitious instruction that is based more directly on research on student learning and calls for teachers to make sense of student thinking as a regular part of their instruction (Lampert et al, 2010; Stein, et al, 2008). In conceptualizing an integration of research on learning with research on teaching, Sztajn, Confrey, Wilson and Edgington (2012) propose "learning trajectory based instruction" (LTBI) as teaching that draws on developmental progressions of student learning as the basis for instructional decisions. In addition, formative assessment, one of the most promising educational interventions (Black & Wiliam, 1998), involves continually collecting and interpreting evidence of student thinking in order to formulate an instructional response. This interpretive process lies at the core of formative assessment and data use (Coburn & Turner, 2012; Spillane & Miele, 2007). Yet despite the importance of understanding student thinking to both current theories of mathematics instruction and formative assessment, we know relatively little about how teachers understand and interpret evidence of student thinking in school and classroom settings (Coburn & Turner, 2012; Little, 2012) and the relationship between teacher interpretations of data and their planning and execution of instructional responses (Heritage, Kim, Vendlinski, & Hermann, 2009).

In this study, we focus on how grades 3-5 teachers make sense of and interpret artifacts of students' multiplicative thinking, namely the written work that their own students produce on multiplication and division problems. Our research questions focus on how teachers make sense of this evidence of student learning and decide how to respond instructionally:

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1. How do teachers sort and/or organize a classroom set of student work for analysis? What conceptual frames do they draw upon?
2. How do teachers make sense of examples of their own student work? What do they pay attention to and how do they make sense of the information?
3. How do teachers draw on their analysis of student work and other information to formulate instructional responses?

Relevant Literature

How teachers notice student thinking in the context of classroom instruction has been a focus of much recent research on mathematics teaching (Choppin, 2011; Jacobs, Lamb & Philipp, 2010; Mason, 2002; Sherin, Jacobs & Philipp, 2010; Star & Strickland, 2008; van Es & Sherin, 2008). Considered an important component of teaching expertise, noticing student thinking involves focusing or attending to important aspects of complex classroom events (van Es & Sherin, 2008). Jacobs et al. (2010; 2011) build on this work to propose the construct of "professional noticing of children's mathematical understanding" as "a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understanding (p.99). Their research suggests that these skills can improve with professional development experiences focused around student thinking and that learning to attend to student strategies is foundational, but not sufficient, in learning to provide robust teaching responses.

Goldsmith & Seago (2013) extend the concept of noticing to teachers' interaction with student work in professional development settings, noting that it involves "attending to both the mathematical content of the task and students' mathematical thinking." (p. 170). Similarly, Wilson, Lee & Hollebrands (2011) investigated how preservice teachers make sense of students' work on and found that in addition to the actions of describing, comparing and inferring, teachers go through a process of restructuring their own mathematical understandings as they collect evidence of multiple approaches and develop models of student thinking. Kazemi and Franke (2004) studied facilitated conversations with elementary teachers around student work and found that over time teachers began to recognize sophistication of strategies, think about better ways to elicit student thinking in the classroom, and develop "possible instructional trajectories" that built on student thinking. Several studies highlight the fact that teachers can learn to use specific frameworks for making sense of student thinking, and that this can lead to improvement in instructional practice (Carpenter et al, 1988; 1992; Clements et al, 2011; Wilson, 2009).

Taken together this work has established that teachers' analysis of student work can be improved through professional development. Yet there is little research that explores how teachers actually look at and interpret student work in the absence of any intervention. In a large scale study of teachers' capacity for learning trajectory-oriented formative assessment, Supovitz, Ebby and Sirinidies (2013) found that the vast majority of teachers in grades K-12 look at examples of student work from a procedural perspective, focusing on what students do rather than what they understand. There were even fewer teachers (less than 10% overall) who were able to see the student work conceptually *and place it in a developmental context.* Similarly, in a study of sixth grade teachers, Heritage et al, (2009) found that teachers' interpretation of student responses to an algebraic problem were mostly either empty (no explanation) or procedural, rather than conceptual. In both of these studies, teachers were looking at pre-constructed or fictitious examples of student work.

The current study builds on this body of work to further explore the different frameworks, organizing principles, and kinds of knowledge that teachers draw upon in making sense of their own students' work for instruction. Understanding how teachers make sense of the artifacts of student thinking, factors that influence their interpretation, and practices that are more productive for generating instructional interventions is important for both designing professional development experiences to support and improve formative assessment practices.

Methods

This study is part of larger study of how learning trajectory-oriented formative assessment can be used to provide teachers with greater understanding of student thinking and lead to more refined instructional responses, focused specifically on students' multiplicative thinking in grades 3-5 mathematics. In this paper we report on the baseline qualitative inquiry that explores teachers' sense-making around their own student work before any intervention. We used semi-structured cognitive interviews (Fontana & Frey, 2003; Means & Loftus, 1991) to understand how teachers' interpret their own student learning data and draw on that evidence to formulate instructional responses.

We collected data from 20 teachers (in grades 3-5) in three elementary schools in an urban setting. The three schools were purposefully selected to represent different contexts: nine teachers from a high-performing diverse urban public school, five teachers from a low-performing economically disadvantaged urban public school, and six teachers from a diverse bilingual charter school. The teachers vary in terms of professional experience ranging from 2-31 years, with the median of 9.5 years of experience. There is also variation in terms of professional development experiences in mathematics and the primary curriculum being used to teach mathematics.

Each interview consisted of three distinct activities. Teachers were asked to bring a class set of open ended work on a multiplication or division problem to the interview. All but one of the teachers in the sample brought student work on either a word problem or a computation problem where students were expected to show their work. The remaining teacher brought an exit slip that consisted of five closed questions focusing on writing related multiplication and division equations. In the interview teachers were asked to categorize or sort the work in ways that made the most sense them, while talking aloud about their process. Second, researchers asked teachers to highlight at least one example from each category and to speak more in-depth on the work in terms of strengths and weaknesses.

Lastly, teachers were asked to explain what if any instructional responses they may have for the students discussed as well as for the class as a whole. The interviews ranged between 20-45 minutes and were audio-recorded and transcribed. We also collected copies of the student work that was discussed.

The data was analyzed in two primary ways. First, an analytical memo was produced for each participant (Strauss & Corbin, 1990). The memo tracked ways teachers categorized student work, examples and reasoning for the specific student work referenced for each category, and instructional responses respective of the student work. From these memos along with the theoretical framework, we applied inductive and deductive codes to the data. We used a grounded theory approach to the data looking for interactions and relationships among codes and categories and cases (Charmaz, 2006). Following Boyatzis' (1998) thematic analysis, we focused on both deductive and inductive coding around ways teachers made sense of student work and the factors that may influence their sense-making. The final codes were then organized into larger categories.

The largest distinction that emerged was whether the teachers' interpretation of student work drew directly on the evidence or the written work presented on the page or drew on working knowledge of the student, such as the student's work habits, previous performance, or home situation. These categories are not mutually exclusive; teachers most often drew on both direct evidence and working knowledge to interpret student thinking. In our analysis we looked more closely at how they made sense and reconciled these different sources of information to arrive at an interpretation of student performance and develop instructional responses. Teachers' interpretation of the student work ranged along a continuum of depth, from a focus on surface characteristics of students work (correct answer or neatness), to a descriptive focus (how student solved the problem), to a conceptual focus (what the work suggested about what students understood). A final, but less frequently encountered level of interpretation was characterized by drawing together the descriptive and conceptual focus to consider both conceptual, procedural, and developmental aspects of student work. These levels of interpretation are described in more detail in the next section.

Similarly, we developed a set of codes for teachers' planned instructional responses, ranging from general (reteaching content, grouping students), to procedural (focus on procedural mastery, fluency or efficiency) to conceptual (deepening understanding or connecting procedural and conceptual understanding) to developmental (building on student understanding to move towards more sophisticated strategies and understanding). Once the codes were established, all interviews were double coded. The research team met to discuss any discrepancies that emerged and agreed on a final code. The full code list is presented in Table 2.

Results

We first describe the sorting strategies that teachers used to create different categories within the class set of student work and explore what those strategies suggest about the conceptual frames they were drawing on when sorting. We then turn to teachers' analysis of individual examples of student work within the categories they created to illustrate the different levels of analysis of student work and instructional responses. Finally, we explore the connections between sorting frames, analysis of student work and instructional response.

Sorting Student Work

When sorting student work, the strategies teachers used to create different piles fell into three distinct groups (1) *Correctness*, where teachers sorted primarily on whether or not the answer was correct; (2) *Proficiency*, where teachers created categories that described a judgment of students' overall level of proficiency, often using labels to describe the students or their work; and (3) *Strategies*, where teachers sorted the work by the different strategies that students used to solve the problem.

There were also two interviews where teachers did not have a discernible initial sorting frame and instead described pieces of work separately. Table 1 below shows the distribution of these sorting frames across the sample in relation to years of experience.

Table 1. Teachers' Sorting Frames in Relation to Experience.

Sorting Frame	N	Description	Mean years of experience (Range)
Correctness	10	Initial sort by whether or not student got correct answer to the problem or a percentage correct on a set of problems	8.7 (4-17)
Proficiency	4	Combining correctness on task with past performance and/or strategy use to sort by overall judgment of level of proficiency	17.5 (6-31)
Strategies	4	Work sorted by students' use of strategy as evidenced in work	13 (2-22)
No Sorting	2	Teacher did not sort into categories during interview despite being prompted	11.25 (2-20)

While the size of the sample limits our analysis, the sorting frame that teachers used does not seem to be directly related to years of teaching experience. Of the four teachers who sorted student work by strategy, three of them had received extensive professional development; all had experience looking at student work in grade level teams and one was a recent graduate of a university teacher education program that had a substantial focus on the development of student thinking in mathematics in both coursework and fieldwork. However, there were other teachers who had similar experiences and sorted by correctness or overall judgment. This suggests that sorting student work by strategy may be a habit learned only through professional development but that this is necessary but not sufficient condition. Below, we illustrate each of these sorting frames with examples from the interviews.

Correctness. Teachers who sorted student work by correctness either made dichotomous groupings (e.g., correct, incorrect) or if there was more than one problem on the page, based the groupings on the number correct (e.g., “the kids who didn’t have any of the answers correct; then one correct, two correct and all correct”). Often they also considered another aspect of the student work along with correctness, such as whether or not the student showed work to support the answer. Underneath this framework is a conceptualization of learning as meeting the standard that has been set, and students achieve mastery by solving problems and getting the correct answer. This conceptualization has behaviorist roots, and student thinking is less important than student performance; in fact student thinking is only important when students are not reaching mastery as a means for determining the dosage of remediation.

Proficiency. Several teachers made groups by considering multiple factors, such as correctness and effective use of strategy, along with their knowledge of how the student typically performed in mathematics or performed on other assignments. They often used terms such as “proficient,” “advanced,” or “below basic” to describe the students’ performance, vocabulary that is used to describe students’ performance on standardized tests and often students themselves. These judgments of students’ overall proficiency were often based on both the work under discussion and patterns in the student’s performance over time. Teachers often drew on their knowledge of students beyond the work itself, such as specific learning profiles or habits they had observed over time. Several teachers separated out students who had been formally classified with learning differences.

Underneath the proficiency framework is a conceptualization of learning as multifaceted--one that includes factors in addition to correctness, such as conceptual understanding or use of multiple strategies. However, others have noted that in the NCLB era, the terms used to describe different levels of proficiency can sometimes suggest a frame of "instructional triage where attention is focused on students that are just below the proficiency level " (Horn, Kane & Wilson, 2013). We did not observe this directly in our discussions with teachers of their own student work, though often the language they used to describe student proficiency seemed to come from NCLB testing and data use practices.

Strategies. A smaller group of teachers sorted their student work primarily based on the strategy that the students used to solve the problem, for example, skip counting, repeated addition, or use of a known or derived fact. Sorting by strategy indicated a view of learning that went beyond mastery to consider not just whether students solve the problem correctly, but *how* they solved it. This view has its roots in a constructivist notion of learning, where the process of coming to know mathematics is as important as the result. As noted above, three of the teachers who demonstrated this sorting strategy were at the school where there was a history of looking at student work in grade level meetings, professional development focused on research on student learning, and a curriculum that reflected a student-centered, inquiry-based and conceptually-oriented approach to learning mathematics. The fourth teacher had done her student teaching at that school and was a recent graduate of a teacher education program with an emphasis on student thinking. Notably, the other teachers at that school used either a proficiency framework or did not sort but none of them used a correctness framework. At the other two schools where there had not been a practice of looking at student work by strategy, most teachers used a correctness framework.

In the next section we explore how teachers analyzed specific examples of student work from the different piles that they made to represent high, medium, and low level work and the degree to which there was a relationship between their initial sort or the underlying conceptual framework of learning mathematics and the depth of their analysis of student work.

Interpreting Evidence of Student Thinking

In analyzing the individual examples of student work in each pile, teachers drew on a combination of the evidence on the page and the previous knowledge they had constructed about the student. The codes that were generated from the inductive analysis of the interviews in relation to teachers' analysis of the evidence in the student work confirmed and refined a four-level framework we had previously developed for teachers analysis of student work : (1) *General*, for analyses that focused only on surface aspects of the work; (2) *Descriptive*, for analyses that focused on how students solved the problem; (3*) Conceptual*, for analyses that focused on conceptual understanding connected to use of strategy; and (4) *Developmental*, for analyses that situated the student strategy and understanding within a developmental progression of multiplicative thinking. (See Ebby in press; Ebby, Sirinides & Supovitz, 2013).

However, because this project had teachers looking at their own student work rather than a set of pre- constructed responses, teachers often drew upon knowledge they had of the student along with the actual work shown on the page to make sense of the student work, resulting in an additional category we call *working knowledge of students*. Kennedy (1982) defines working knowledge as a "special domain relevant to one's job" that is "tentative, subject to change as the worker encounters new situations or evidence." (p. 194). We draw on this definition to conceptualize *working knowledge of students* as a subset of working knowledge focused specifically on what teachers understanding of individual students across time and contexts (both in school and out of school).

Once teachers analyzed the example of student work, we asked them to describe what they thought that student needed to progress in their multiplicative thinking. The levels of the instructional response codes were similar to the analysis codes: (1) *General*, for responses that did not focus on mathematical content; (2) *Procedural*, for responses that focused on teaching a specific procedure or skill; (3*) Conceptual*, for responses that focused on developing conceptual understanding; and (4) *Developmental*, for responses that aimed to move students from their current understanding to more sophisticated strategies and/or understanding. See Table 2 for the complete list of coding categories along with descriptions of each category and overall frequencies over the set of 20 interviews.

Table 2. Coding Categories for Teachers' Analysis of Student Work

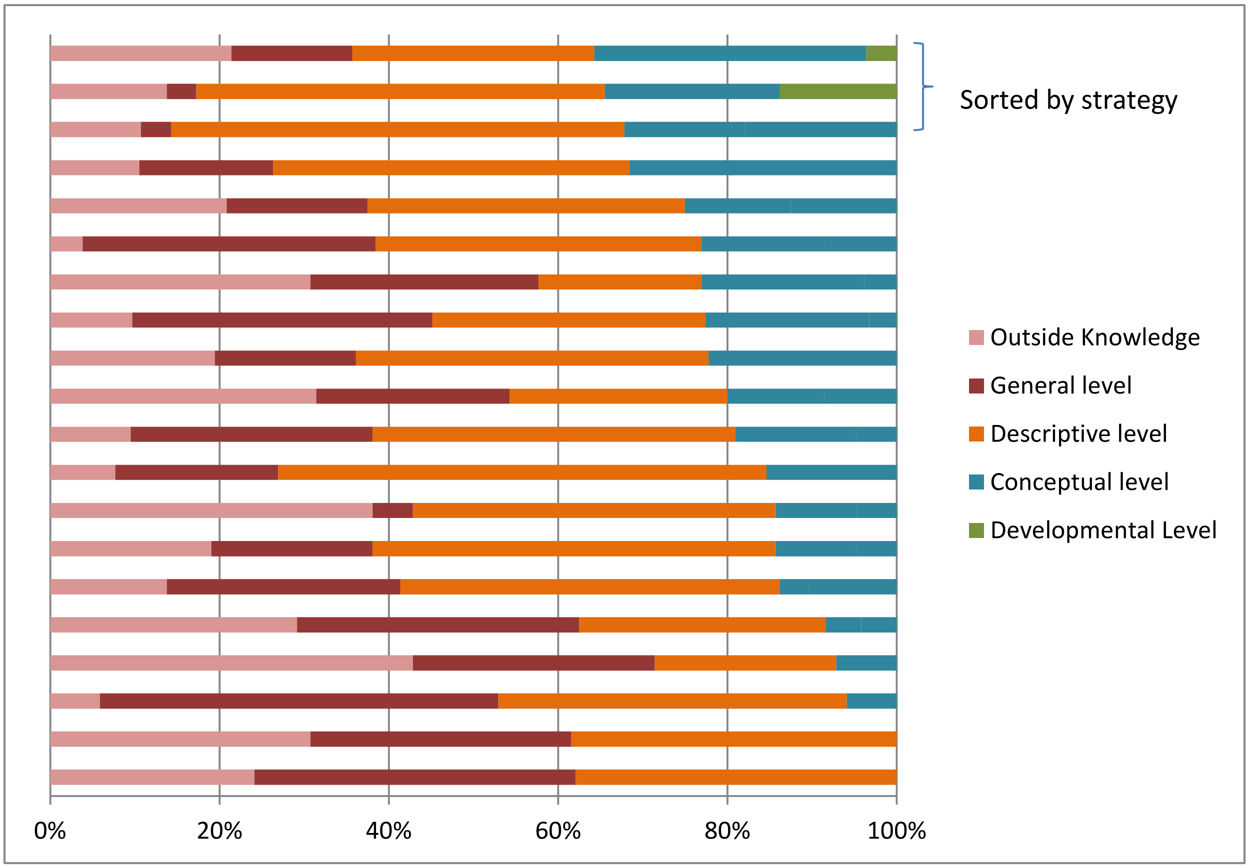
Code	Description	Freq.	# of Teachers
Working Knowledge of Students (N=174)			
Work Habits	Descriptions of how student attends to mathematical tasks, not directly related to work shown on the page	46	19
Previous Work	Descriptions of prior lessons taught or previous performance on assignments or assessments	43	16
Performance Level	Descriptions of students' performance l level relative to others, aptitude, or overall ability	38	12
Learning Profile	Description of student status within the school (e.g., Special Education, ELL)	16	10
Home Life	Descriptions of students' life outside of school as it relates to student performance	12	7
Dispositions	Descriptions of students' emotional response to mathematical work or school	10	8
Classroom Behavior	Descriptions of students' behavior during class, as it relates to student performance	8	6
Levels of Analysis of Student Work (N=411)			
General	Focus on format of answer, correctness or nonspecific judgment	116	20
Descriptive	Focus on how student solved problem, description of strategy or use of procedure	198	20
Conceptual	Focus on evidence of understanding or misunderstanding of concepts underlying or connected to use of strategy.	91	17
Developmental	Situating student work within developmental progression from additive to multiplicative reasoning or concrete to abstract thinking, drawing on both evidence of conceptual and procedural understanding	6	3
Levels of Instructional Response (N=243)			
General	Not focused specifically on development of content or student strategy or student understanding	90	19
Procedural	Focus is on helping students master procedure or strategy, or procedural fluency	92	20
Conceptual	Some focus on developing deeper understanding of concepts, operation, or understanding behind procedure	56	17
Developmental	Developing instructional responses that are based on analysis of student thinking and aim to move students towards more sophisticated strategies	5	3

The participants in ours study worked closely with their students over the course of the school year and so it is not surprising that in making sense of the student work, they often brought in previous experience to inform their understanding of the evidence of student performance. Depending on the particular piece of student work they were looking at, teachers sometimes referenced working knowledge of the student from multiple categories and often not at all. See Ebby & Sam (2015) for a more detailed description and analysis of the way teachers drew on working knowledge of students to arrive at an analysis.

The four categories of analysis of student work represent increasing levels of depth, with *general* representing a superficial level, *descriptive* focusing on the work itself, and *conceptual* and *developmental* requiring the use of frameworks to make sense of the student work in relation to student learning. As illustrated by the examples in the next section, these levels of analysis are not mutually exclusive but rather most often they are cumulative. A teacher's analysis of student work might remain at the general level or reflect both general and descriptive elements, or contain general, descriptive and conceptual elements.

In Figure 1, we show the relative number of codes that were applied in each category for each teacher on the interview. Each row in the chart represents the codes applied to a single teacher interview. Because teachers may have discussed different numbers of samples of student work in the interview, we present these as percentages of the total number of codes that were applied in the interview. As Figure 1 shows, all 20 teachers in the sample drew on their working knowledge of the students to some degree and analyzed their student work at both general and descriptive levels. There was variation, however, in the degree to which conceptual and developmental frameworks were utilized to make sense of the student work. In the figure, the profiles are ordered in relation to the relative proportion of codes that went beyond descriptive (either conceptual or developmental). As the data shows, there is no direct relationship between the relative amount of working knowledge codes and the depth of the analysis. Rather, we found that it was the different ways in which this working knowledge interacted with the evidence in the student work which was important for both the level of analysis and instructional responses (Ebby & Sam ,2015).

Figure 1. Analysis of Student Work, Profiles by Teacher



In the following sections, we illustrate each of these categories of levels of analysis of student work with examples from the interviews. Outside knowledge is not considered separately as it always occurred in conjunction with one of the other categories of analysis. It is also important to remember that these levels are cumulative rather than mutually exclusive: the level of analysis indicates the greatest depth that was reached during the discussion of a particular piece of student work.

*General**.*** The most superficial level of analysis was characterized by reference to non-mathematical characteristics of the student work, format or correctness. Correctness was included in this category because it is the most immediate or obvious characteristic of the work that could be observed. Responses that were coded at this level did not contain any substantive analysis of work the student had done or strategy used to get to the answer. Often, as in the following example, teachers drew upon outside knowledge of the student along with observations about the format or the correctness of their answer. In this example, the teacher had first sorted her work into correct and incorrect and then within those created subcategories of students who showed their work and students who did not show their work. Here she describes a piece of student work that was judged to be incorrect.

> With this student who knows his multiplication but doesn't always focus, I could see that started to draw the pictures, got sidetracked, drew some more pictures but never really, maybe looked at a friend's paper. This doesn't give me a clear picture as to what was going on in his head when he was at that point. . . . So yeah, this would be incorrect. This individual has some challenging behavioral issues. So I'm not surprised. It looked like he tried. He started to do something and then just got sidetracked.

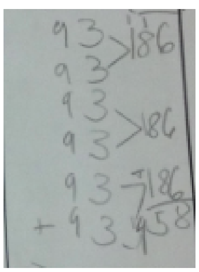
In her analysis of the strengths and weaknesses in the work, the teacher noted that the student had drawn pictures, but did not look to those pictures for any more information. When asked what she would do next with the student, she focused on his need for help:

> Maybe go back and give him some simple multiplication problems, some simple division problems because he, out of all these students you can see he clearly… there were a few that got it. You can see the mindset was there but he doesn't have it and so giving him some extra work, working with him one on one, sitting down with him.

Like the analysis, this instructional response also remained at a general level, giving the student easier division problems and one-on-one help. There was no diagnosis of what the student needed help with.

It is important to note that teacher's analysis of student work rarely remained only at the general level and no teacher produced only general analyses across a set of student work. There were also times when a more general analysis was warranted, for example when the student did not show enough work to draw any conclusions or the teacher could not make sense of the student work.

Descriptive. Teachers' comments about individual pieces of student work were considered descriptive when they included details that went beyond surface characteristics to focus on *how* the students solved the problem. After sorting her work based on the number of correct answers on a page of 6 word problems, the following teacher focused on the pile of papers where the student had gotten 3 problems out of 6 incorrect, noting that she was surprised to find one student in that pile because “she doesn’t really struggle with math.” She focused on a problem involving 6 groups of 93 where the student had used a building up additive strategy:



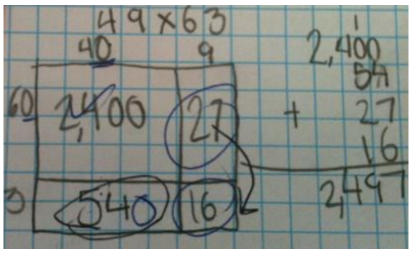
> Yeah, [she] doesn’t usually struggle with math but she can be careless so…I think she was probably… Yeah, she was doing something. She gets easily frustrated too, so… So it looks like she did 93, she did it 6 times.

After noting that the student was adding 93 six times, the teacher did not go further to consider what this meant about the students’ understanding of multiplication. Rather, she focused her instructional response on the computational error and planned to introduce her to the standard algorithm

> So probably with her I’m going to focus on not being careless, and when you do do these strategies where you’re adding something up like six times it’s much easier to make a mistake because there’s so many things at play. So I’ll probably try to sort of push her in the direction of the vertical strategy. And then also if you don’t have your basic multiplication facts memorized then it makes everything so much harder.

In this case the descriptive analysis led to an instructional response focused on teaching the student a specific procedure and focusing on fact fluency, rather than recognizing and building on the fact that the student was understanding the multiplicative situation additively and beginning to group by larger amounts.

In the next example, the teacher analyzed the array model that the student had used to multiply 49 and 73 and recognized that the student was not using the model correctly to determine the four partial areas or products, but did not go beyond description to reflect on the implications for the student's understanding of multiplication, place value or the distributive property:



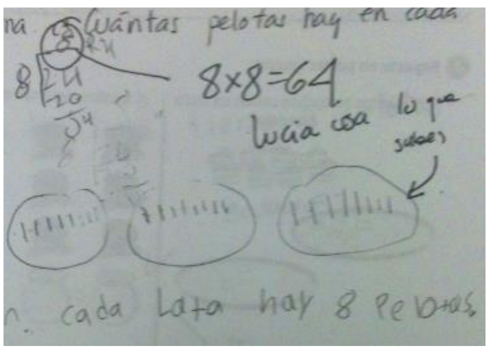
This is right, this is written correctly but they don't seem to understand how to use kind of like the columns and the rows in order to figure out which factors to multiply in which part. And then here she seems to have gotten totally lost. Like she knows she has to multiply different numbers at the 1s and the 10s from the two different numbers, but she seems to not understand where each part goes.

Again this analysis of the student work remained at the procedural level—"which factors to multiply" and "where each part goes" rather than focusing on the conceptual underpinnings of the open area model for multiplication. She concluded, "This student seems very lost" and went on to say, "So this is a student I'd definitely pull out for re-teaching with my assistant or with me" (a general instructional response) but did not give any details about what she would be focusing on in this remediation. In this case, the descriptive and procedural focus of the analysis highlighted that the student was not using the procedure correctly, but did not illuminate any direction to go in to help the student develop a more powerful understanding of the array and how it represents multiplication.

Teachers' analyses of student work were most frequently at the descriptive level and the instructional responses developed from descriptive analyses were most often either at the general or procedural level. There were a few cases where the instructional response was conceptual, but were not connected to evidence in the work itself. (For example, one teacher concluded that a student needed to work on place value when the work did not show any place value errors).

Conceptual. When teachers went beyond description to analyze not only what the student did, but what they understood about multiplication, we categorized the analysis as conceptual. Most conceptual analyses also contained descriptive and general elements but they were categorized as conceptual if there was some focus on student understanding. However, the depth of this analysis ranged from a general conceptual focus to more articulated analysis of the conceptual understanding evidenced in the student work.

For example, consider the following interpretation of a third grade student's solution to a partitive division problem in a Spanish immersion classroom (24 tennis balls in 8 containers). The student partitioned the total amount into three equal groups by ones using hash marks but then wrote "each can has 8 balls" and divided 24 by 8 to get 8.

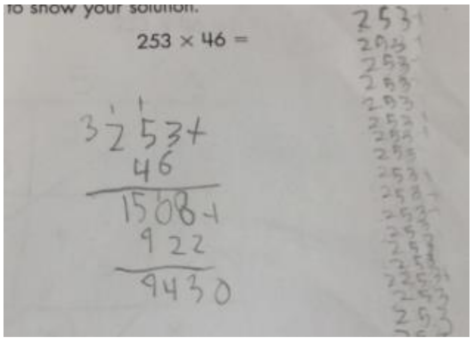


> . . . the strategy that she used is not showing me that she understands. So she has, I mean, she knows division but she doesn't have the skill and I know that student so for sure I know. She has a concept of division but she's not developed in the concept. She just knows the basics that you have to separate them in equal groups. She's not using the rest of the strategies that we have been using in the classroom. So, and she makes those careless mistakes to let me know that she is inattentive as well. She's not connecting the concepts.

In this example, the teacher considers the disconnect between the strategies shown on the page along with what she knows already about the student to conclude that her understanding of "the concept of division" is not "developed." However, she does not identify more precisely what it is that the student understands or fails to understand in relation to division. Not surprisingly, the instructional response, while conceptual in nature, was also less specific: "I guess drawing is fine but she's below. [I would tell her] Use what you know. That is, [use] this part to respond to the question."

In contrast, sometimes teachers were able to look at the student work and identify specific concepts that the student either understood or didn't understand, such as place value or properties of operations. In the following example, part of which is shown below, the student had multiplied 253 and

46 incorrectly, recognized it was incorrect, tried to add 253 up 46 times, and then gave up. The teacher's interpretation of this work is shown next to the image below:



> I don't think he really understands what he's doing. It's that procedural thing and if you lose one step in the process or the procedure you can't find your way back and I don't think he knows what exactly he's doing in this. I don't think there's any real evidence in place value awareness.

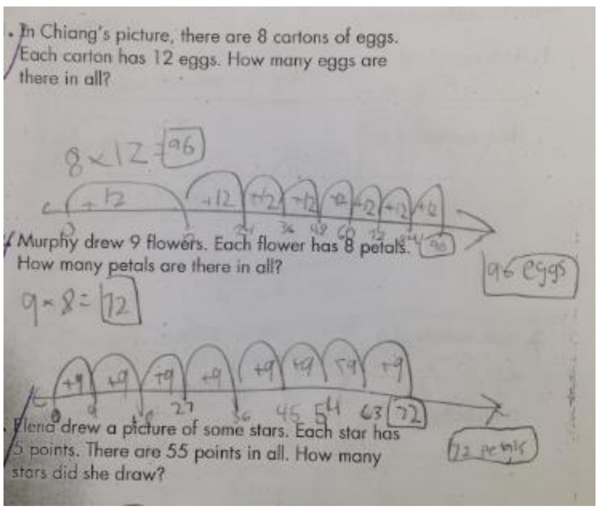
The instructional response described by the teacher builds on this idea that the student has not developed an understanding of "place value awareness" in his use of the algorithm:

> I think what I might do with him is to kind of look, before we set this up look and see what we're doing. We're multiplying 200 by six and we're multiplying 50, just to break it down so that he sees that what every one of these numbers has a value and it's being multiplied by this and it's going to be a big long addition problem and then work with this. Or maybe even just work with the, I think maybe the arrays with him. He likes things very organized, very boxed. So possibly that might be a way to pull him into this and then that's a way to show things broken up for him.

In this case, the teacher made a conceptual analysis of the students' work and then used that analysis to formulate a response that focused on building conceptual understanding of an alternate procedure, namely partial products or "breaking up" the numbers. In this way, she was integrating conceptual and procedural aspects of multiplication.

Developmental. Only two teachers in the sample drew on their understanding of the developmental sequence or learning trajectory to make sense of their student work. Although these teachers stood out in our sample in that they had more significant professional development experiences in mathematics teaching and learning over the years, their approach to analyzing student work can offer important insights into more productive practices. We illustrate the confluence of sorting strategy, analysis of student work and instructional response through two examples from one of these teachers.

This teacher sorted her third grade student work based on strategies: whether the students used repeated addition, skip counting or knowledge of facts to determine the answer to two equal groups multiplication word problems. When she got to the following example, she was not sure whether to put it with her pile of repeated addition or the pile of skip counting:



This one is interesting… Well, they do repeated but then they're doing a number line of the skip counting – 9, 18, 27, 36 because I think they don't know how to skip count automatically in their head so they're using that number line to jump by 9s, jump by 12s. We've done number lines like in other units to add and subtract and but I haven't showed them this way, like just to do repeated addition to skip count. So I think he just feels comfortable using that number line to organize his thinking.

> So I don't know what pile I would put this in. I'm thinking he's still using… You know what, no, it is skip counting because he's not adding 12 + 12 together, or 9 +9 + 9… You know, I still don't know. I'm trying to think, he's adding but he's also putting the skip counting on the bottom. I feel like he's doing both. I feel like he's adding the number and as he gets to it he puts it down, but I don't think he knows it automatically that it's like 9, 18, 27, 36.

As this teacher looked at what the student had put on the paper and reasoned it out, she thought carefully about whether that strategy represented additive thinking or the beginnings of multiplicative thinking or skip counting. Ultimately she determined that it was in between repeated addition and skip counting, reflecting an overall sense of the development from additive to multiplicative thinking. This level of analysis integrates both procedural and conceptual elements to view learning as a process of developing more sophisticated strategies that are built on conceptual understanding.

When she got to an example of the work she had sorted into the multiplicative pile, the teacher interpreted the student's strategy of breaking 8 x 12 up into (4 x 12) and (4 x 12) as evidence that she was beginning to understand the distributive property:

> This one looks like, she's breaking up the 8. She does 8 x 12 is 96 but then she's thinking 4 and 4, and I feel like this is the distributive property here – the 4 x 12 is 48 and then she's doing 48 and 48 which leads to this too. And this is what, in Investigation 3 we'll talk more about this with the distributive property and breaking that up to lead to that. Even though she got [a different problem] wrong here because it should be 72… That's where she's made mistakes but I see her thinking here, but she got it right on this one. Even though the answer's wrong, her thinking is getting there, it's getting more efficient there.

In addition, the teacher noticed that this understanding of the distributive property was reflected in her solution to another problem even though the answer was incorrect. Importantly, this teacher went beyond determining whether the student had mastered the skill of multiplying 8 x 12 to get the correct answer to think about what her strategy suggested about her underlying understanding of the properties of multiplication. She also recognized that the strategy of breaking up one factor is more efficient than the skip counting strategy that most of the students were using. This suggests a more sophisticated view of learning mathematics that has procedural, conceptual, and developmental elements, one that requires both understanding and valuing the concepts that are involved in multiplicative reasoning.

When it came to describing instructional responses, the teacher used her understanding of the developmental trajectory from counting to additive to multiplicative strategies to think about how she could use opportunities in the curriculum to help the students advance. In particular, knowing that the curriculum would shift from a focus on skip counting to the array model, she understood how this model could be used to help them progress in understanding and procedural fluency:

> With the array cards I feel like those kids will break up the array. So they might see an array of an 8 x 12 and if they know what a 4 x 12 is and then doubling that they'll be able to see that array. And I'll have them share out too like when we talk about like arrays, when we do that game.

In sum, teachers' analyses of student work represented different levels of depth that reflected an overall focus on procedural, conceptual and/or developmental frameworks. Teachers who made general or descriptive interpretations were focusing only on the work on the page. Teachers who made conceptual or developmental interpretations were drawing on frameworks of student understanding as well as the work on the page to make sense of their student work. These different levels of analysis have important implications for the kind of instructional responses that teachers generate after looking at their student work.

Discussion

Our analysis of over 150 instances of teachers making sense of student work suggests that the process of looking at student work to inform instruction involves three interpretive activities: sorting, extracting relevant cues from the work on the page, and analyzing that evidence, and that these activities are influenced by the conceptual frames that teachers bring to the task. Three conceptual frames that emerged as being influential were (1) teachers beliefs about learning mathematics (e.g., whether learning was a process of mastery of skills versus a process of development, and whether that development includes conceptual as well as procedural aspects) (2) sorting mechanisms (e.g., correctness, proficiency or strategies) that are learned through experience and (3) working knowledge of students constructed from experiences with students across contexts and time. These conceptual frames are often interrelated, for example if one views learning mathematics as a process of mastery one is more likely to sort student work based on correctness and may construct a profile of the student that revolves around the degree to which the student has mastered previous skills and concepts. If on the other hand, a teacher views learning mathematics as process of development, she may be more likely to sort the student work based on strategy, look for evidence of both conceptual and procedural understanding, and view a student's learning profile as malleable.

Figure 2 illustrates our emerging conceptualization of the way in which these conceptual frames shape the interpretive processes involved in looking at student work for instruction—in the way teachers extract cues from the evidence on the page, construct an analysis of the student work, and bring it all together to formulate and instructional response.

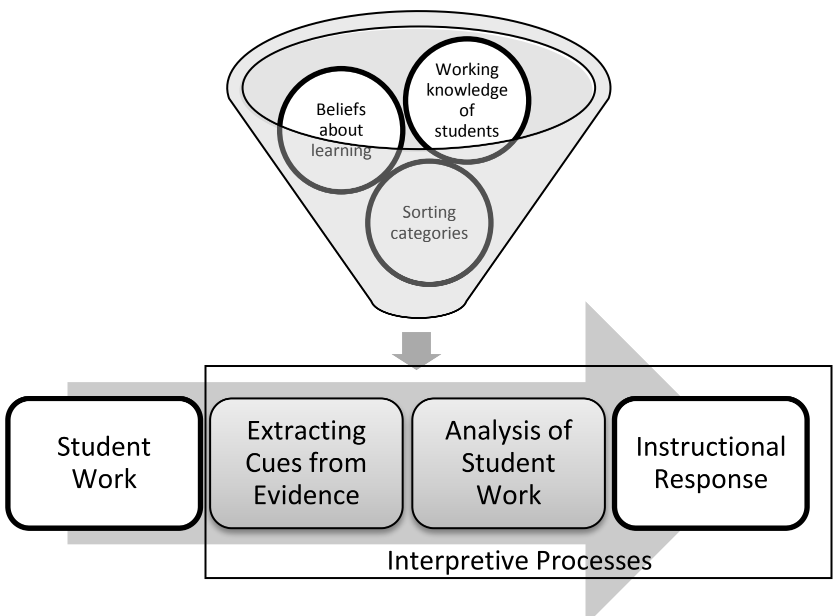


Figure 2. The Interpretive Process of Looking at Student Work for Instruction

Our analysis also demonstrates that the interpretation of student work can reflect different levels of depth of analysis, ranging from general or superficial, to descriptive, to conceptual to developmental. The few teachers that demonstrated a developmental approach to both analysis of student learning and instruction exhibited what Sztajn et al, (2012) define as learning trajectory based instruction (LBTI) or ”teaching that uses learning trajectories as the basis for instructional decisions.” These teachers were able to identify both conceptual and procedural aspects in their student work to develop instructional responses that both built on student thinking and were directed towards deeper understanding and competency. An LTBI response emerges from the integration of procedural and conceptual analyses of student work into a developmental framework. Yet the vast majority of the teachers in our study described what students were doing (a procedural analysis) without considering either conceptual or developmental aspects of their strategies.

Implications

The Common Core State Standards in mathematics (CCSSM) have substantially increased expectations for both students and teachers. The CCSSM reflect a balance of conceptual and procedural skills, as well as a developmental perspective. As stated in the introduction, “the development of the standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time” (Common Core State Standards Initiative, 2010, p. 4). This focus on balancing conceptual and procedural understanding and learning trajectories places new demands on teaching, as teachers must not only understand the mathematical ideas and skills embodied in the Standards, but also assess where students are in the trajectory of learning those concepts and skills, and then use that information to design and enact instructional responses that support students’ movement along that trajectory. In other words, teachers must be able to implement formative assessment processes based on, and supportive of, the development of student thinking.

Looking at student work and using data to inform instruction are common practices in schools and professional development experiences in mathematics, yet we know little about how to make these processes most effective. Our findings highlight both more and less productive ways that teachers think about evidence of student thinking and how instructional responses are related to differences in depth of the interpretation of student work. In particular, our work illustrates the limits of general and descriptive analyses of student work in generating substantive instructional responses for students that are based on their mathematical thinking.

Our work also highlights the importance of conceptual frames to the process of making sense of evidence and data use. Understanding the analysis of student work as an interpretive process raises some important issues for professional development and the improvement of practice. First, how can we support teachers to move from descriptive analyses of student work to incorporate conceptual analyses and a learning trajectory framework? Having teachers learn to sort student work by strategy rather than by correctness seems to be a promising direction, but teachers will also need new conceptual frames to help interpret that work. We hypothesize that if new frames were introduced, such as those exemplified by learning trajectories, teachers could incorporate those frames and focus on the strategies that students are using to better inform and enhance their working knowledge of the students rather than drawing primarily on their existing working knowledge to interpret the work on the page. But will introducing a learning trajectory framework for analysis of student work be sufficient? What kinds of knowledge and skills do teachers need to be able to construct learning trajectory responses, not only in their planned responses but in their actual classroom practices?

If teachers have no alternative frames through which to look at student work other than a view of learning math as a process of mastery along their working knowledge of the student and a procedural understanding of mathematics, then they are unlikely to be able to effectively draw on the evidence of student thinking in the student work to construct analyses and instructional responses that could move students forward in both depth of understanding and sophistication of strategy use. The deeper levels of analysis, which led to more substantive instructional implications, involve the use of conceptual frameworks that reflect learning as a constructive process and therefore are in conflict with a correctness sorting strategy or a view of learning as mastery. Yet many ubiquitous practices in schools that teachers are expected to participate in—grading or "correcting" student work, using results of standardized testing to make decisions, writing behavioral objectives in lesson plans, "covering" the curriculum—reflect a view of learning as mastery. When teachers shift to a developmental view of learning to look at student work, how do they make sense of and negotiate these different frameworks in their daily practice?

References

Black, P. B., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education, 5*(1), 7-74.

Boyatzis, R.E. (1998), *Transforming Qualitative Information: Thematic Analysis and Code Development*. Thousand Oaks, CA: Sage Publications.

Carpenter, T. P., & Fennema, E. (1992). Cognitively Guided Instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research*, *17*, 457-470.

Carpenter, T.P., Fennema, E., Peterson, P. L., & Carey, D.. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19*, 385-401

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Thousand Oaks, CA: Sage Publications.

Choppin, J. (2015). The impact of professional noticing on teachers' adaptations of challenging tasks. *Mathematical Thinking and Learning, 13*, 175-197.

Clements, D. H. , Sarama, J., Spitler, M. E., Lange, A. A., & Wolfe, C. B. (2011). Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education*, *42*, 127-166.

Cobb, P. & Steffe, L.P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education*, 14, 83-94.

Coburn, C. & Turner, E. (2012). The practice of data use: An introduction. *American Journal of Education*, 118 (2), 99-111.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers.

Desimone, L. M., & Le Floch, K. C. (2004). Are we asking the right questions? Using cognitive interviews to improve surveys in education research. *Educational evaluation and policy analysis*, *26*(1), 1-22.

Ebby, C.B., & Sam, C. (2015). Understanding How Math Teachers Make Sense of Student Work for Instruction. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago.

Ebby, C.B., & Sirinides, P.M. (in press). Conceptualizing Teachers' Capacity for Learning Trajectory-Oriented Formative Assessment in Mathematics. In J. A. Middleton, J. Cai, and H. Hwang (Eds.) *Large Scale Studies in Mathematics Education*. Springer.

Ebby, C.B., Sirinides, P.M., Supovitz, J., & Oettinger, A. (2013). Teacher analysis of student knowledge (TASK): A measure of learning trajectory-oriented formative assessment. Technical Report, Consortium for Policy Research in Education

Fontana, A., & Frey J. (2003). The interview: From structured questions to negotiated texts. In N. Denzin & Y. Lincoln (Eds.). *Collecting and interpreting qualitative materials* (2^{nd} ed., pp. 645-672). Thousand Oaks, CA: Sage.

Goldsmith, L. T., & Seago, N. (2010). Using Classroom Artifacts to Focus Teachers' Noticing: Affordances and Opportunities. In M. Sherin, R. Philipp, & V. Jacobs (Eds.) *Mathematics Teacher Noticing: Seeing through Teachers' Eyes*. New York: Routledge.

Heritage, M., Kim, J., Vendlinkski, T. & Herman, J. (2009). From evidence to action: A seamless process in formative assessment? *Educational Measurement Issues and Practice*, 28(3), 24-31.

Horn, I.S., Kane, B.D., & Wilson, J. (2013). *Making Sense of Student Performance Data: Mathematics Teachers' Learning through Conversations with Assessments.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Jacobs, V. R., Lamb, L. L. C., Philipp, R. A., & Schappelle, B. P. (2011). Deciding How to Respond on the Basis of Children's Understandings. In M. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics Teacher Noticing: Seeing Through Teachers' Eyes* (pp. 97–116). New York: Routledge.

Kazemi, E. & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*, 203–35.

Kennedy, M. (1983). Working knowledge. *Knowledge: Creation, Diffusion, Utilization*, 5, 193-211.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, K., & Franke, M. L. (2010). Instructional explanations in the disciplines. In M. K. S. L. Kucan (Ed.), *Using designed Instructional activities to enable novices to manage ambitious mathematics teaching* (pp. 129-141). New York: Springer

Little, J. W. (2012). Understanding data use practice among teachers: The contribution of micro-process studies. *American Journal of Education, 118*(2), 143-166.

Mason, J. (2002). *Researching your own practice. The discipline of noticing*. Londres: Routhledge Falmer

Means, B., & Loftus, E. F. (1991). When personal history repeats itself: Decomposing memories for recurring events. *Applied cognitive psychology*, *5*(4), 297-318.

Sherin, M. Jacob, V. & Philipp, R. (2010). *Mathematics Teacher Noticing: Seeing Through Teachers' Eyes. New York*: Routledge.

Spillane, J. P., & Miele, D. B. (2007). Evidence in practice: A framing of the terrain. In P. A. Moss (Ed.), *Evidence and decision making* (pp. 46–73). Malden, MA: National Society for the Study of Education.

Star, J.R., & Strickland, S.K. (2008). Learning to observe: using video to improve preservice mathematics teacher's ability to notice. *Journal of Mathematics Teacher Education, 11*, 107-125.

Strauss, A., & Corbin, J. M. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Thousand Oaks, CA: Sage Publications.

Stein, M.K., Engle, R. A., Smith, M. & Hughes, E.K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*, 313–340.

Supovitz, J., Ebby, C.B., Sirinides, P. (2014). Teacher Analysis of Student Knowledge: A measure of learning trajectory-oriented formative assessment. Retrieved from http://www.cpre.org/sites/default/files/researchreport/1446_taskreport.pdf.

Sztajn, P., Confrey, J., Wilson, P.H., & Edgington, C. (2012). Learning trajectory based instruction: Towards a theory of teaching. *Educational Researcher, 41*, 147-156.

van Es, E. A. & Sherin, M. G. (2008). Mathematics teachers "learning to notice" in the context of a video club.*Teaching and Teacher Education*, 244-276.

Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. New York: Falmer Press.

Wilson, P.H. (2009). *Teacher's uses of a learning trajectory for equipartitioning* (Unpublished doctoral dissertation). North Carolina State University, Raleigh, N.C.

Wilson, P.H., Lee, H.S., & Hollebrands, K. (2011). Understanding prospective mathematics teachers' processes for making sense of students' work with technology. *Journal for Research in Mathematics Education*, *42*, 39-64.