

The Exact Algorithm for Multi-sensor Asynchronous Track-to-Track Fusion

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Abstract – Track-to-track fusion is an important topic for distributed tracking. Compared with the centralized measurement fusion, the track-to-track fusion requires less communication resources and is suitable for practical implementation. Although having been widely investigated in the literature, the majority of track-to-track fusion algorithms assume synchronous communication. However, in practice, local sensors might communicate to the fusion center with arbitrary communication intervals, which raises the problem of multi-sensor asynchronous track-to-track fusion. In this paper we develop an exact fusion algorithm to solve this problem, under the condition that at each time step only parts of the sensors send their local estimation tracks to the fusion center. Formulas are derived to obtain the exact cross-covariances between the local tracks by taking into consideration the impact of the potential feedback from the fusion center. Based on the derived formulas, the scalable fusion algorithm is developed and validated with extensive Monte Carlo simulations.

Keywords: Tracking, track-to-track fusion, Kalman filtering, estimation.

1 Introduction

In a multisensory [1, 2] environment where each sensor is able to obtain measurements and maintain local tracks of the target, it is assumed that sensors are able to communicate the local data with a fusion center (FC) [3, 4]. One of the main tasks of the FC is to decide how to fuse the local information together. One straightforward method is to gather all measurements from sensors, then the FC could fuse them directly via a measurement model and use centralized Kalman filter [5] to estimate the target track. However, while the centralized measurement fusion (CMF) [6-8] mentioned above is optimal, it is not practical for most tracking systems due to the high communication requirements of CMF. An alternative way is to adopt the track-to-track fusion

(T2TF) [9], where the FC fuses local estimation tracks generated from sensors to improve the global estimation accuracy. Compared with the CMF, the major advantage of T2TF is that it could effectively reduce the communication requirements on the system and it can be performed at a lower rate [10, 11]. The problem of track-to-track correlation due to the common process noise was first observed in [12]. Based on the formula derived for the exact calculation of the cross-covariances in [12], the T2TF accounting for the fact that local estimation tracks have dependent state estimation errors was developed in [13] by making use of a static linear estimation model. In [14] the approximation made in [13] was examined and it was shown that the result from [13] is optimal only in maximum likelihood (ML) sense. The information matrix filter (IMF) [15, 16] is another type of T2TF algorithms, unlike the one-scan algorithm proposed in [13], IMF uses tracks from the previous fusion steps. IMF is optimal only at full rate and it doesn't require the cross-covariances among local tracks, which is well known to be difficult to calculate [17]. However, it is to be noted that IMF is not optimal when the communication rate is less than full rate [16].

There has been a great deal of work [18-20] in developing T2TF algorithms with different information configurations including no, partial, and full information feedbacks. However, the existed algorithms are generally not scalable for the multi-sensor case. Although it was mentioned in [19] that there is no theoretical limit on the number of local tracks to fuse, the derived formulas only apply to the two-sensor case. In this paper, we develop scalable algorithms for the multi-sensor T2TF with exact calculation of the cross-covariances, where the fusion operates in an asynchronous manner under the condition that local sensors communicate with the FC at different and arbitrary rates, so that at each time step only parts of the sensors send their local estimation tracks to the FC. Furthermore, we derive exact formulas to update the cross-covariances by taking into account the impact of the potential feedback from the FC. Finally we conduct extensive Monte Carlo simulations to validate the performance of the proposed algorithm.

2 Problem Formulation

Consider a scenario with N sensors at different locations. Each sensor obtains the measurement and updates its local tracks with a sampling interval T . While the local sampling system may or may not be synchronous, the communication system operates in an asynchronous fashion, namely, each sensor is allowed to communicate with the FC with a different communication interval T_i , where $i \in \{1, \dots, N\}$. We define $J(k) \subseteq \{1, \dots, N\}$ as the subset of N sensors that transmit their local tracks to the FC at the time step k . Note that in this configuration, it is assumed that the sensors will receive feedback from the FC when the fusion is completed. In this paper, the out-of-sequence problem [21-23] is omitted for the sake of brevity, that is, the communication links between sensors and the FC is assumed without delay and no data loss in this paper.

If there is a communication link available between the FC and the sensor i at step k , it means that the sensor i is able to send the following data to the FC, including the current local estimation track $\hat{x}_i(k|k)$, $P_i(k|k)$, the local Kalman gains history $\{K_i(t)\}_{t=l_i+1}^k$, in which l_i is defined as the previous communication time at which the sensor i communicated with the FC, and an indicator of local feedback pedigree $\{FDBK_i(t)\}_{t=l_i}^{k-1}$, where $FDBK_i(t) = 1$ if the sensor i receives the feedback from the FC at step t , otherwise $FDBK_i(t) = 0$.

At the FC, let \hat{x}_c, P_c represent the fused track, and P_{ij} represent the cross-covariance between local tracks from the sensor i and j , then the fusion of the tracks from $J(k)$ at step k is formulated as follows

$$\begin{aligned} & [\hat{x}_c(k|k), P_c(k|k)] \\ & = f(\{\hat{x}_i(k|k)\}, \{P_i(k|k)\}, \{P_{ij}(k|k)\}) \end{aligned} \quad (1)$$

where $i, j \in J(k)$ and $i \neq j$.

Note that once a sensor receives a feedback from the FC, its local track should be updated to $\hat{x}_i^*(k|k)$, $\hat{P}_i^*(k|k)$. After that, the corresponding cross-covariances stored in the FC should be updated to $P_{ij}^*(k|k)$ as soon as the FC accesses the up-to-date local communication pedigree from sensor i and j , namely $\{FDBK_i(t)\}_{t=1}^k$ and $\{FDBK_j(t)\}_{t=1}^k$.

3 Multi-sensor Asynchronous track-to-track fusion Algorithm

This section develops the exact algorithms for the multi-sensor asynchronous track-to-track fusion problem. In Sec. 3.1, the maximum likelihood (ML) fusion rules for both of the two-sensor and multi-sensor cases are introduced. Sec 3.2 presents an exact formulas to update the cross-covariances between the local tracks by accounting for the impact of the potential feedback from the fusion center. Based on the derived formulas, we

summarize the scalable fusion algorithm for arbitrary number of sensors at the end of the section.

3.1 Fusion Rules

Given local tracks $\hat{x}_i(k|k)$, $P_i(k|k)$ and $P_{ij}(k|k)$ at FC, where $i, j \in J(k)$, the optimal fusion in the ML sense [14] can be done according to the following formulas.

For the 2-sensor case, namely $|J(k)| = 2$, it was derived in [13] that

$$\begin{aligned} & \hat{x}_c(k|k) \\ & = \hat{x}_1(k|k) + K_{fus}(k)(\hat{x}_2(k|k) - \hat{x}_1(k|k)) \end{aligned} \quad (2)$$

$$\begin{aligned} & P_c(k|k) \\ & = P_1(k|k) - K_{fus}(k)(P_1(k|k) - P_{21}(k|k)) \end{aligned} \quad (3)$$

where

$$\begin{aligned} & K_{fus}(k) \\ & = (P_1(k|k) - P_{12}(k|k)) \\ & \cdot (P_1(k|k) + P_2(k|k) - P_{12}(k|k) - P_{21}(k|k))^{-1} \end{aligned} \quad (4)$$

For the multi-sensor case, namely $|J(k)| > 2$, the ML estimator [24] is given by

$$\hat{x}_c(k|k) = K_{fus}(k) \hat{X}_{J(k)}(k|k) \quad (5)$$

$$P_c(k|k) = (I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)})^{-1} \quad (6)$$

where

$$K_{fus}(k) = (I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)})^{-1} (I_{J(k)}^T P_{J(k)}^{-1}) \quad (7)$$

and I is an $n \times n$ identity matrix, $I_{J(k)} = \overbrace{(I, I, \dots, I)}^{|J(k)|}$ is an $(|J(k)| \cdot n)$ matrix.

Let $\hat{X}_{J(k)} = (\hat{x}_{J(k)_1}, \hat{x}_{J(k)_2}, \dots, \hat{x}_{J(k)_{|J(k)|}})^T$, where $J(k)_i$ is defined as the i th element of $J(k)$. Denote $P_{J(k)}$ as an $(|J(k)| \cdot n) \times (|J(k)| \cdot n)$ matrix with blocks as follows

$$P_{J(k)} = \begin{pmatrix} P_{J(k)_1} & P_{J(k)_1, J(k)_2} & \dots & P_{J(k)_1, J(k)_{|J(k)|}} \\ P_{J(k)_2, J(k)_1} & P_{J(k)_2} & \dots & P_{J(k)_2, J(k)_{|J(k)|}} \\ \dots & \dots & \ddots & \dots \\ P_{J(k)_{|J(k)|}, J(k)_1} & P_{J(k)_{|J(k)|}, J(k)_2} & \dots & P_{J(k)_{|J(k)|}} \end{pmatrix} \quad (8)$$

3.2 Exact Calculation of Cross-covariances

The key issue of the ML track-to-track fusion is how to compute the up to date cross-covariances between the local estimation errors exactly. In general the calculation of cross-covariances could be summarized in two distinct phases. In the first phase, assuming that at step k , the local Kalman gains $K_i(k)$, $K_j(k)$ and cross-covariance $P_{ij}^*(k-1|k-1)$ are all available at the FC, then we could update P_{ij} to step k , based on the fact that the actual cross-covariance between local estimation tracks i

and j evolves similarly to the local Kalman filter step by step in sensor i and j respectively. It was originally derived in [12] that

$$P_{ij}(1|1) = (I - K_i(1)H)Q(I - K_j(1)H)^T \quad (9)$$

$$\begin{aligned} P_{ij}(k|k) &= (I - K_i(k)H)AP_{ij}^*(k-1|k-1)A^T(I - K_j(k)H) \\ &\quad + (I - K_i(k)H)Q(I - K_j(k)H)^T \end{aligned} \quad (10)$$

where A is the transition matrix of the target model and Q is the covariance of the process noise.

In the second phase, it requires to take into account the impact of the potential feedback from the FC, for the reason that the actual cross-covariance might change as the local sensor updates its estimated mean and covariance with the received feedback. Supposing that the fusion center has already accessed the communication history of both sensor i and j until step k , namely $\{FDBK_i(t)\}_{t=1}^k$ and $\{FDBK_j(t)\}_{t=1}^k$, then it could update P_{ij}^* to step k according to the following formulas:

Case 1, both sensor i and j receive the feedback from the FC at step k , then

$$\hat{x}_i^*(k|k) = \hat{x}_j^*(k|k) = x_c(k|k) \quad (11)$$

$$P_i^*(k|k) = P_j^*(k|k) = P_c(k|k) \quad (12)$$

$$P_{ij}^*(k|k) = P_c(k|k) \quad (13)$$

Case 2, neither sensor i nor sensor j receives the feedback from the FC at step k , then

$$\hat{x}_i^*(k|k) = \hat{x}_j^*(k|k) \quad (14)$$

$$\hat{x}_j^*(k|k) = \hat{x}_j(k|k) \quad (15)$$

$$P_i^*(k|k) = P_i(k|k) \quad (16)$$

$$P_j^*(k|k) = P_j(k|k) \quad (17)$$

$$P_{ij}^*(k|k) = P_{ij}(k|k) \quad (18)$$

Case 3, sensor i receives the feedback from the FC at step k , while sensor j does not

$$\hat{x}_i^*(k|k) = x_c(k|k) \quad (19)$$

$$\hat{x}_j^*(k|k) = \hat{x}_j(k|k) \quad (20)$$

$$P_i^*(k|k) = P_c(k|k) \quad (21)$$

$$P_j^*(k|k) = P_j(k|k) \quad (22)$$

For $|J(k)| = 2$, it was shown in [19] that

$$\begin{aligned} P_{ij}^*(k|k) &= (I - K_{fus}(k))P_{J(k),j} \\ &\quad + K_{fus}(k)P_{J(k)_2,j} \end{aligned} \quad (23)$$

For $|J(k)| > 2$, it can be shown that

$$P_{ij}^*(k|k) = K_{fus}(k) \begin{pmatrix} P_{J(k)_1,j} \\ P_{J(k)_2,j} \\ \vdots \\ P_{J(k)_{|J(k)|},j} \end{pmatrix} \quad (24)$$

Proof.

The error of the fused track is defined as

$$\tilde{x}_c(k|k) = \hat{x}_c(k|k) - x(k) \quad (25)$$

Substituting (5) into (25) it gives that

$$\begin{aligned} \tilde{x}_c(k|k) &= \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right)^{-1} \left(I_{J(i)}^T P_{J(i)}^{-1} \right) \hat{X}_{J(k)} \\ &\quad - x(k) \\ &= \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right)^{-1} \left(I_{J(i)}^T P_{J(i)}^{-1} \right) \hat{X}_{J(k)} \\ &\quad - \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right)^{-1} \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right) x(k) \\ &= \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right)^{-1} \\ &\quad \cdot \left(\left(I_{J(i)}^T P_{J(i)}^{-1} \right) \hat{X}_{J(k)} - \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right) x(k) \right) \\ &= \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right)^{-1} \\ &\quad \cdot \left(I_{J(i)}^T P_{J(i)}^{-1} \right) \left(\hat{X}_{J(k)} - I_{J(k)} x(k) \right) \\ &= \left(I_{J(k)}^T P_{J(k)}^{-1} I_{J(k)} \right)^{-1} \\ &\quad \cdot \left(I_{J(i)}^T P_{J(i)}^{-1} \right) \tilde{X}_{J(k)}(k|k) \\ &= K_{fus}(k) \tilde{X}_{J(k)}(k|k) \end{aligned} \quad (26)$$

where

$$\tilde{X}_{J(k)}(k|k) = \begin{pmatrix} \tilde{x}_{J(k)_1}(k|k) \\ \tilde{x}_{J(k)_2}(k|k) \\ \vdots \\ \tilde{x}_{J(k)_{|J(k)|}}(k|k) \end{pmatrix} \quad (27)$$

Let

$$P_{ij}^*(k|k) = Cov(\hat{x}_i^*(k|k), \hat{x}_j^*(k|k)) \quad (28)$$

Substituting (19) and (20) into (28) it gives that

$$\begin{aligned}
P_{ij}^*(k|k) &= E\left(K_{fus}(k)\tilde{X}_{J(k)}(k|k)(\tilde{x}_j(k|k))^T\right) \\
&= K_{fus}(k)E\left(\tilde{X}_{J(k)}(k|k)(\tilde{x}_j(k|k))^T\right) \quad (29) \\
&= K_{fus}(k)\begin{pmatrix} E\left(\tilde{x}_{J(k)_1}(\tilde{x}_j(k|k))^T\right) \\ E\left(\tilde{x}_{J(k)_2}(\tilde{x}_j(k|k))^T\right) \\ \vdots \\ E\left(\tilde{x}_{J(k)_{|J(k)|}}(\tilde{x}_j(k|k))^T\right) \end{pmatrix}
\end{aligned}$$

Namely,

$$P_{ij}^*(k|k) = K_{fus}(k)\begin{pmatrix} P_{J(k)_1,j} \\ P_{J(k)_2,j} \\ \vdots \\ P_{J(k)_{|J(k)|},j} \end{pmatrix} \quad (30)$$

The proof is completed.

Case 4, sensor j receives the feedback from the FC at step k , while sensor i does not

$$\hat{x}_i^*(k|k) = x_i(k|k) \quad (31)$$

$$\hat{x}_j^*(k|k) = \hat{x}_c(k|k) \quad (32)$$

$$P_i^*(k|k) = P_i(k|k) \quad (33)$$

$$P_j^*(k|k) = P_c(k|k) \quad (34)$$

For $|J(k)| = 2$, it can be easily shown that

$$\begin{aligned}
P_{ij}^*(k|k) &= P_{iJ(k)_1}(I - K_{fus}(k))^T \\
&\quad + P_{iJ(k)_2}(K_{fus}(k))^T \quad (35)
\end{aligned}$$

For $|J(k)| > 2$, we have

$$P_{ij}^*(k|k) = \begin{pmatrix} P_{J(k)_1,i} \\ P_{J(k)_2,i} \\ \vdots \\ P_{J(k)_{|J(k)|},i} \end{pmatrix}^T (K_{fus}(k))^T \quad (36)$$

Proof.

Substituting (31) and (32) into (28) it gives that

$$\begin{aligned}
P_{ij}^*(k|k) &= E\left(\tilde{x}_i(k|k)(K_{fus}(k)\tilde{X}_{J(k)})^T\right) \\
&= E\left(\tilde{x}_i(k|k)(\tilde{X}_{J(k)})^T\right)(K_{fus}(k))^T \quad (37) \\
&= \left(E\left(\tilde{X}_{J(k)}(\tilde{x}_i(k|k))^T\right)\right)^T (K_{fus}(k))^T
\end{aligned}$$

Namely,

$$P_{ij}^*(k|k) = \begin{pmatrix} P_{J(k)_1,i} \\ P_{J(k)_2,i} \\ \vdots \\ P_{J(k)_{|J(k)|},i} \end{pmatrix}^T (K_{fus}(k))^T \quad (38)$$

The proof is completed.

It is noteworthy to point out that the algorithm is scalable in the sense that there is no theoretical limit on the number of local sensors. Finally, the exact algorithm for multi-sensor asynchronous track-to-track fusion can be summarized as follows.

For the FC at each fusion step k ,

- (1) update $J(k)$;
- (2) update $\{FDBK_i(t)\}_{t=l_i}^{k-1}$, where $i \in J(k)$;
- (3) update $\{K_i(t)\}_{t=l_i+1}^k$, where $i \in J(k)$;
- (4) update all available P_{ij} and P_{ij}^* with **Algorithm 1** (described below);
- (5) fuse local tracks with eqns. (2) to (7);
- (6) send the fused track to the local sensors set $J(k)$;
- (7) if a local sensor receives a feedback from the FC, update its estimated mean and covariance with the fused track.

Algorithm 1 calculate P_{ij} and P_{ij}^* at each step k

```

1: for all  $t \in \{1, \dots, k\}$ 
2:   for all  $i \in \{1, \dots, N\}$ 
3:     for all  $j \in \{1, \dots, N\}$ 
4:       if  $i \neq j$  &&  $P_{ij}(t|t)$  is empty
5:         calculate  $P_{ij}(t|t)$ 
with Algorithm 2
6:       end if
7:     end for
8:   end for
9:   for all  $i \in \{1, \dots, N\}$ 
10:    for all  $j \in \{1, \dots, N\}$ 
11:      if  $i \neq j$  &&  $P_{ij}^*(t|t)$  is empty
12:        calculate  $P_{ij}^*(t|t)$ 
with Algorithm 3
13:      end if
14:    end for
15:  end for
16: end for

```

Algorithm 2 calculate $P_{ij}(t|t)$

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1: if both  $K_i(t)$  and  $K_j(t)$  are available
2:   if  $i == 1$ 
3:     initial  $P_{ij}$  with eqn. (9)
4:   else
5:     if  $P_{ij}^*(t-1|t-1)$  is available
6:       calculate  $P_{ij}(t|t)$  with eqn. (10)
7:     end if
8:   end if
9: end if

```

Algorithm 3 calculate $P_{ij}^*(t|t)$

- 1: **if** both $FDBK_i(t)$ and $FDBK_j(t)$ are available
 - 2: calculate $P_{ij}^*(t|t)$ with eqns. (11) to (24) and (31) to (36)
 - 3: **end if**
-

4 Simulations

To evaluate the performance of the proposed multi-sensor asynchronous track-to-track fusion algorithm, considering a tracking scenario with one target, four sensors and a fusion center. A 1D constant velocity model [25] for the target is given as

$$\dot{x}(t) = Fx(t) + Lq(t) \quad (39)$$

where $x = (x, \dot{x})$ and $q(t)$ is a white process noise with a power spectral density [26] q_c and

$$F = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (40)$$

$$L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (41)$$

To implement the Kalman filter [27, 28] for local sensors, the model is discretized as in [25],

$$x(k) = Ax(k-1) + w(k-1) \quad (42)$$

where

$$A = \exp(F\Delta t) \quad (43)$$

$$Q = \int_0^{\Delta t} \exp(F(\Delta t - \tau)) L q_c L^T \exp(F(\Delta t - \tau))^T d\tau \quad (44)$$

The measurement model is given as

$$z_i(k) = Hx(k) + v_i(k) \quad (45)$$

where $v(k) \sim N(0, R_i)$.

The following set of parameters is used for the simulation. For the target, let $\Delta t = 1s$ and $q_c = 1m^2 / s^4$. For the sensors, it is assumed that these sensors are available to observe the positions of the target with a sampling interval of $1s$ and the variance of the measurement noise is $R_1 = R_2 = R_3 = R_4 = R = 1m^2$. Furthermore, the communication configuration between the sensors and the FC is set as the following: sensor 1 communicates with the FC every 2 steps, starting from step 2; sensor 2 communicates with the FC every 4 steps, starting from step 2; sensor 3 communicates with the FC every 4 steps, starting from step 4; and sensor 4 communicates with the FC every 6 steps, starting from step 6. For the FC, it would send the fused track back to the sensors immediately after fusing their local tracks at the FC.

A 100 runs Monte Carlo simulation is performed to verify the effectiveness of the proposed fusion algorithm. We use the mean squared error (MSE) [27, 29, 30] as the

performance metric. From Figure 1 it can be seen that the estimation error for the fused estimation is significantly lower than the local estimation errors, it illustrates that the fusion of local tracks improves the estimation accuracy effectively at each fusion step.

We further compare the performance of the proposed multi-sensor track-to-track fusion algorithm against the Naive fusion and the centralized Kalman filter. The Naive fusion [31] is the simplest fusion algorithm in which it is assumed that the correlation between the local estimations is negligible. The centralized Kalman filter is a full-rate fuser, and it offers the optimal estimation result since it requires all the local measurements to be available. Table 1 shows that the proposed multi-sensor track-to-track fusion algorithm is better than the Naive fusion, while it still cannot achieve the performance of the centralized Kalman filter.

With respect to the filter consistency, Figure 2 shows that the proposed track-to-track fusion algorithm is consistent since most of the values are found inside the 95% confidence interval (3 of 50 are found outside) with the NEES [34] test, which illustrates that the calculation of the cross-covariances is appropriate. In addition, although the Naive fusion performs very closely to the proposed track-to-track fusion algorithm in the sense of MSE, it is not a consistent estimator, where the fused covariances are much smaller than the true ones which could lead to overconfidence.

Table 1: Comparison between Naive fusion, centralized Kalman filter and the multi-sensor track-to-track fusion algorithm with the averaged mean squared error (MSE)

Fusion Algorithm	Averaged MSE	Consistency
Naive fusion	1.2486	No
Centralized Kalman filter	0.8941	Yes
Multi-sensor Track-to-Track fusion	1.2209	Yes

5 Conclusion

In this paper, we develop a scalable algorithm for the multi-sensor asynchronous track-to-track fusion with exact calculation of the cross-covariances, where the fusion operates in an asynchronous manner under the condition that local sensors communicate with the fusion center at arbitrary rates. As a result, only parts of the sensors send their local tracks to the fusion center at each time step. Furthermore, exact formulas were developed to update the cross-covariances between the local tracks by taking into account the impact of the potential feedback from the fusion center. Based on the derived formulas, we present a scalable fusion algorithm and an

extensive Monte Carlo simulation validates the proposed algorithm.

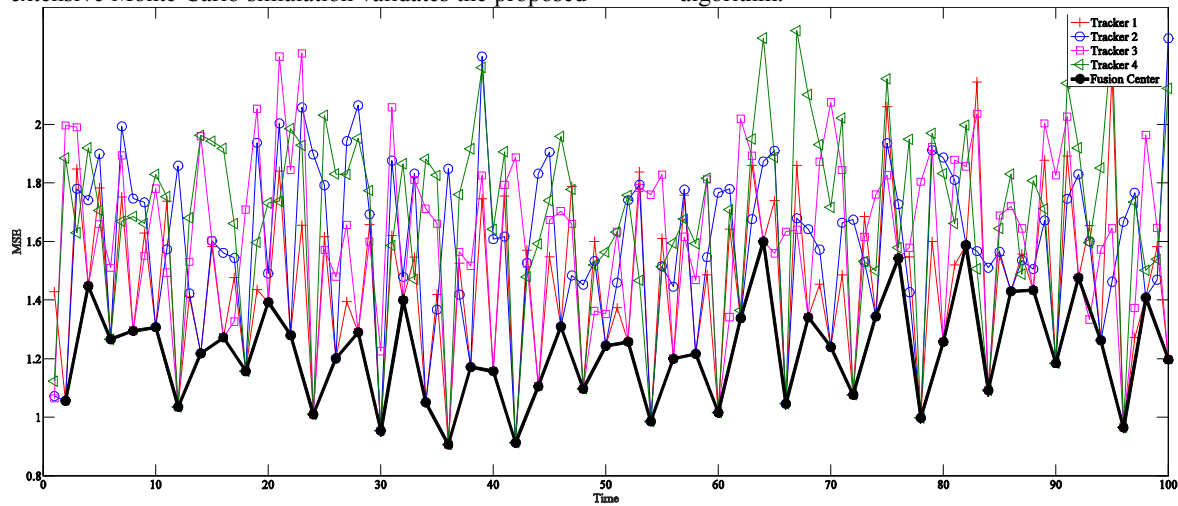


Figure 1. MSE for the multi-sensor asynchronous track-to-track fusion algorithm

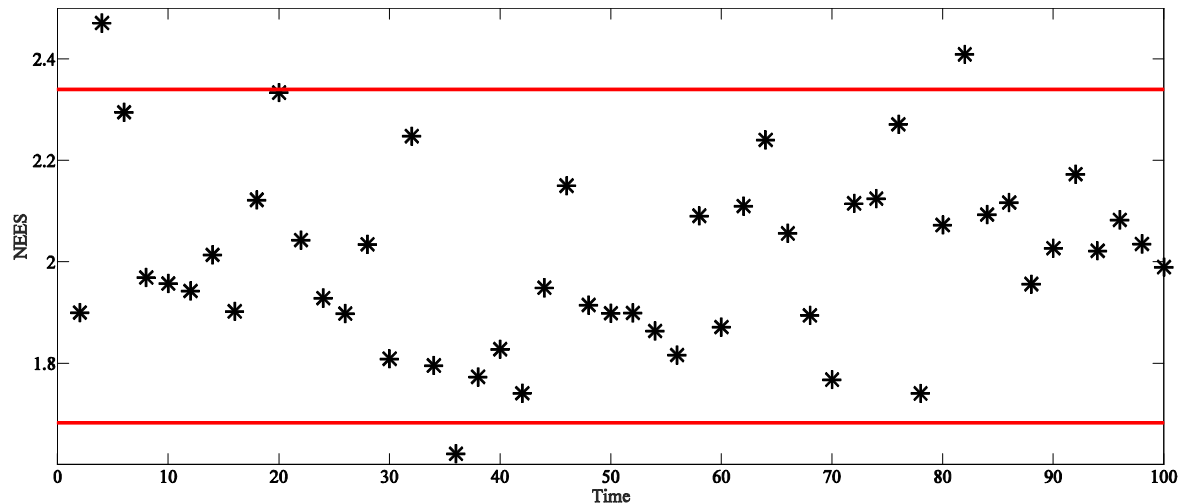


Figure 2. NEES test for the multi-sensor asynchronous track-to-track fusion algorithm

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