

Introduction to Fractals and Scaling

Homework Solutions for Unit 8:

Urban Scaling

<http://www.complexityexplorer.org>

Beginner

Let's assume that the urban scaling results hold exactly, so that the GDP of a city scales as $N^{7/6}$ (where N is the population), and that the length of roads in a city scales as $N^{5/6}$.

The population of the New York City metropolitan statistical area (MSA) is 20.1 million. The population of the Los Angeles MSA is 18.2 million.

1. A city has a GDP Of 500 million euro. A city twice this size would have a GDP $2^{7/6} \approx 2.24$ times larger. Thus, this city would have a GDP of approximately $500 \times 2.24 = 1122$ million euro. Similarly, a city half the size would have a GDP $(1/2)^{7/6} \approx 0.45$ times smaller. I.e., the smaller city would have a GDP of approximately $500 \times 0.45 = 223$ million euro.
2. A city has 1000 km of roads. The road length in a city of twice this size would be approximately $2^{5/6} \approx 1.78$ times larger. So the road length would be $1000 \times 1.78 = 1780$ km. A city that was half the size would have a road length approximately $(1/2)^{5/6} \approx 0.56$ times smaller. Thus, the city would have approximately $1000 \times 0.56 = 560$ kilometers of roads.
3. The population of the New York MSA is $20.1/18.2 \approx 1.10$ times larger than the Los Angeles MSA. So we would expect the length of roads in Los Angeles to be $(1/1.10)^{5/6} \approx 0.92$ times smaller. Thus, the expected length of roads in Los Angeles is approximately $6074 \times 0.92 \approx 5592$ miles.
4. As above, the population of the New York MSA is around 1.10 times larger than that of the Los Angeles MSA. We would expect the GDP for New York to be around $1.10^{7/6} \approx 1.12$ times larger than Los Angeles. Thus, we expect a GDP for New York of approximately $8.31 \times 1.12 \approx 9.29$ billion dollars.

Intermediate

There are no intermediate problems for this unit.

Advanced

There are no advanced problems for this unit.