Introduction to Fractals and Scaling
Homework Solutions for Unit 6:

Generating Power Laws

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Beginner

1. It is equally likely to connect to any node. There are 8 nodes; thus, the probability of connecting to each node is 1/8:
   (a) The probability that the new node links to node 1 is 1/8.
   (b) The probability that the new node links to node 2 is 1/8.
   (c) The probability that the new node links to node 4 is 1/8.

2. The probability of connection is proportional to the number of in-links a node has. There are 7 links, 4 of which point to node 2, and 3 of which point to node 1. Thus,
   (a) The probability that the new node links to node 1 is 3/7.
   (b) The probability that the new node links to node 2 is 4/7.
   (c) The probability that the new node links to node 4 is 0.

Intermediate

1. With probability p the link connects to a node at random (as was the case in Beginner problem 1 and with probability $1-p$ the link connects to a node with a probability proportional to the number of in-links the node has (as was the case in Beginner problem 2. Thus
   (a) The probability that the new node links to node 1 is given by:
   $$p \times 1/8 + (1-p) \times 3/7 \;=\; (0.8)(1/8) + (1-0.8)(3/7) \;\approx 0.1857\;. \tag{1}$$
   (b) The probability that the new node links to node 2 is given by:
   $$p \times 1/8 + (1-p) \times 4/7 \;=\; (0.8)(1/8) + (1-0.8)(4/7) \;\approx 0.2143\;\;. \tag{2}$$
   (c) The probability that the new node links to node 4 is given by:
   $$p \times 1/8 + (1-p) \times 0 \;=\; (0.8)(1/8) + 0 \;=\; 4/35 \;=\; 0.1\;\;. \tag{3}$$

2. The exponent α is given by
   $$\alpha \;=\; 1 + \frac{1}{1-p}\;. \tag{4}$$
   (a) Plugging in $\alpha = 0.8$, one obtains $\alpha = 6$.
   (b) As p gets closer and closer to one, α gets larger and larger. A larger α means that $p(x)$ decays more rapidly. In terms of the model, as p approaches 1, the attachment in the model is more and more random—the preferential attachment occurs with a very small probability. Thus, nodes with a large number of in-links are very unlikely. This is consistent with a larger α.