Introduction to Fractals and Scaling
Homework for Unit 6:

Generating Power Laws
http://www.complexityexplorer.org

Beginner

A network is growing according to the preferential attachment model as described in the lectures. Recall that in this model when a new node is created it makes a link to an existing node one of two ways:

1. With probability p it connects to an existing node at random.

2. With probability $1 - p$ it connection to an existing node with a probability proportional to the number of in-links the node has.

Shown in Fig. 1 is a network with eight nodes. The questions below involve connecting the new node to the network using one of the two ways.
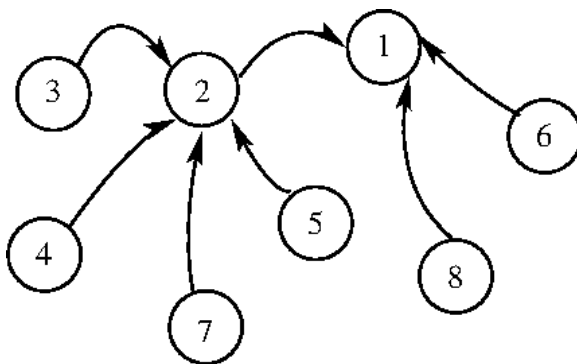

Figure 1: A network.

1. Suppose that the new node connects to an existing node at random—i.e. following option 1, above.

   (a) What is the probability that the new node links to node 1?
   (b) What is the probability that the new node links to node 2?
   (c) What is the probability that the new node links to node 4?

2. Suppose that the new node connects to an existing node in proportion to the number of in-links the existing node has. That is, the new node makes a connection by following option 2, above.

   (a) What is the probability that the new node links to node 1?
   (b) What is the probability that the new node links to node 2?
   (c) What is the probability that the new node links to node 4?

Intermediate

1. Continuing with the preferential growth model described above and the network shown in Fig. 1, suppose that the value of p is 0.8.

   (a) What is the probability that the new node links to node 1?
   (b) What is the probability that the new node links to node 2?
   (c) What is the probability that the new node links to node 4?

2. Let $p(k)$ denote the probability that a node has k in-links. As many nodes are added to the network, the distribution approaches a power law, $p(k) = Ax^{-\alpha}$.

   (a) If $p = 0.8$, what is α?

   (b) As p gets closer and closer to one, what happens to α? What happens to the tail of the distribution? Explain.

Advanced

There are no advanced problems for this unit.