Introduction to Fractals and Scaling
Solutions to Unit 5 Homework

Empirical Power Laws

http://www.complexityexplorer.org

Beginner

1. If x is distributed according to $p(x) = Ax^{-\alpha}$, then the cumulative distribution function $P(x)$ is a power law with exponent $\alpha - 1$. I.e., $P(x) = Cx^{-(\alpha-1)}$. Thus, if $\alpha = 2$, the exponent for the cumutlative distribution function is 1, because $\alpha - 1 = 2 - 1 = 1$.

2. Consider the rank/frequency plot shown in Fig. 1.

   (a) There are 5 data points are equal to or larger than 1.
   (b) There are 4 data points are equal to or larger than 5.
   (c) There is 1 data point are equal to or larger than 15.

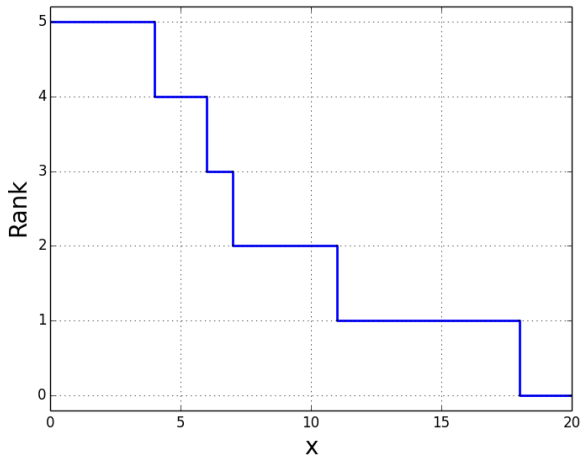

Figure 1: A rank-frequency plot.

3. We are given the following data: $3, 5, 9, 12, 13, 25, 31, 41, 43, 58$. Let $P(x)$ be the cumulative distribution function for these data. Then,

   (a) $P(1) = 1$.
   (b) $P(10) = 0.7$.
   (c) $P(25) = 0.5$.
   (d) $P(72) = 0.0$.

Intermediate

1. Suppose you found the best power-law fit for a set of data. You then estimated the p-value for this fit and obtained a value of 0.85. This provides evidence for the proposition that the data is well-described by a power law. This means that it is unlikely that we would have obtained such a good fit to the power law by chance. If the p-value had been 0.04, this would be taken as evidence against a power law, since it indicates that the fit is not good.

Advanced

I will post solutions to the advanced problems by the end of the week.