Introduction to Fractals and Scaling
Homework for Unit 3:

Introduction to Fractals and the Self-Similarity Dimension

http://www.complexityexplorer.org

Beginner

1. It takes 300 boxes with a side of $s = 1/4$ to cover a square. Approximately how many boxes of side $s = 1/8$ would be needed to cover this square?

2. The Sierpiński triangle has a dimension of approximately 1.585. Suppose that, for a certain Sierpiński triangle, 200 boxes of side $s = 1/4$ are required to cover the triangle. Approximately how many boxes of side $s = 1/8$ would be needed to cover the triangle?

3. (a) Fig. 1 shows a Sierpiński triangle covered with a grid of boxes of side $s = 1/2$. How many boxes are needed to cover the Sierpiński triangle?

   (b) Fig. 2 shows a Sierpiński triangle covered with a grid of boxes of side $s = 1/4$. How many boxes are needed to cover the Sierpiński triangle?

   (c) Based on your answers to the above two questions, estimate the box-counting dimension of the Sierpiński triangle.

Intermediate

1. We have been working with the equation $N(s) = c(1/s)^D$. Equivalently, one can write this as $N(s) = cs^-D$. Suppose we have box-counted for boxes of two different sides: s_1 and s_2, obtaining N_1 and N_2 respectively. Then we know that

$$N_1 = cs_1^{-D} , \tag{1}$$

$$N_2 = cs_2^{-D} . \tag{2}$$

   Use these two equations to derive the following relationship:

$$N_1 = N_2 \left(\frac{s_2}{s_1}\right)^D . \tag{3}$$

2. Suppose a Sierpiński carpet is covered by 344 boxes of side 0.15. Approximately how many boxes of side 0.14 would be needed to cover it? (The Sierpiński carpet has a dimension of approximately 1.829.)

3. Suppose a triangle is covered by 214 boxes of side 0.2. Approximately how many boxes of side 0.19 would be needed to cover the triangle?

s	$N(s)$
0.5	44
0.25	128
0.125	349
0.0625	985
0.03125	2830

Table 1: Data from box-counting

Advanced

1. You are using box-counting to estimate the dimension of an object and obtain the data in Table 1.

   (a) Make a table of $\log(s)$ and $\log(N(s))$.

   (b) Plot $\log(N(s))$ and $\log(s)$, either by hand or using a spreadsheet or some other program.

   (c) Use this plot to estimate the dimension of the object.

Looking Ahead

Here are some questions that are will get you thinking about some of what we'll be covering later. (The answer to some of these will have to wait until we get to some of the later units in the course.)

1. Suppose you have an animal and then scaled it up by a factor of two. What would happen to its

   (a) Mass?

   (b) Volume?

   (c) Surface Area?

   (d) Metabolic Rate?

   (e) Average Lifetime?

2. Suppose you have a city and then doubled its population. On average, what would happen to its

   (a) Total road surface?

   (b) Total length of electrical cables?

   (c) GDP?

   (d) Number of gasoline stations?

   (e) Number of patents filed by city residents?

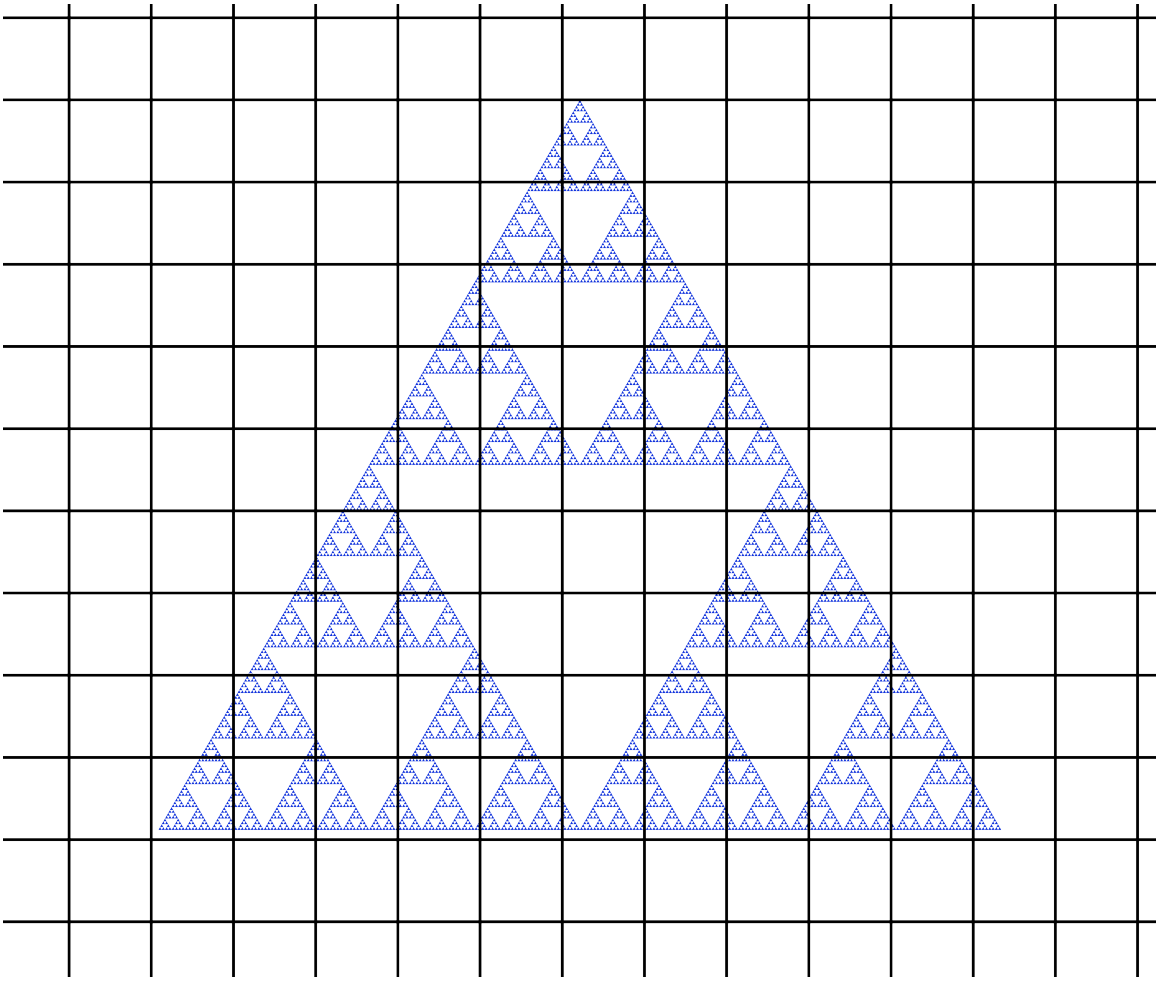

Figure 1: The Sierpiński triangle covered with boxes of side $s = 1/2$.

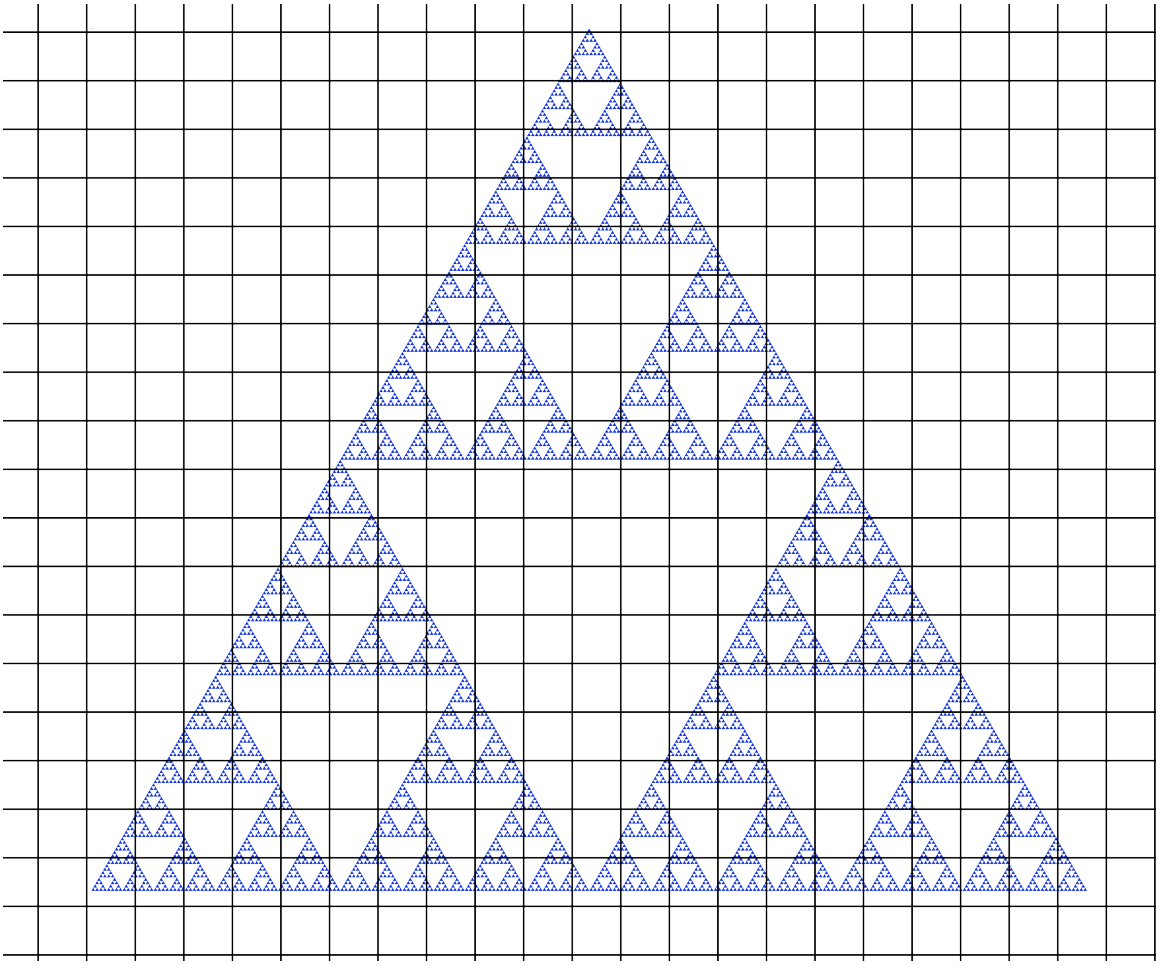

Figure 2: The Sierpiński triangle covered with boxes of side $s = 1/4$.