Introduction to Fractals and Scaling
Homework for Unit 1:

Introduction to Fractals and the Self-Similarity Dimension

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Beginner

1. Determine the self-similarity dimension of the Sierpiński carpet, shown in Fig. 1.
2. At step $n = 3$ in the construction of the Sierpiński triangle, shown in Fig. 2,
   (a) How many small triangles are there?
   (b) What is the area of each triangle?
   (c) What is the total area of the shape?
   (d) What is the perimeter of each triangle?
   (e) What is the total perimeter of the shape?
3. The steps in the construction of the Cantor set are shown in Fig. 3. Initially, at $n = 0$, the line segment has a length of 1. At each step, the middle third of each line segment is removed. Thus, at $n = 1$, each of the two line segments has a length of 1/3.
   (a) At $n = 2$ what is the length of each line segment?
   (b) At $n = 2$ how many line segments are there?
   (c) At $n = 2$ what is the total length of all the line segments?
   (d) Determine an expression for the length of each line segment at step n.
   (e) Determine an expression for the number of line segments at step n.
   (f) Determine an expression for the total length of all line segments step n.
   (g) As n gets larger and larger, what happens to the number of line segments?
   (h) As n gets larger and larger, what happens to the total length of the line segments?
4. Determine the self-similarity dimension of the Cantor set, shown in Fig. 3.
5. Suppose a circle is scaled up by a factor of 3. What happens to the circle's area?
6. Suppose a tomato is scaled up by a factor of 2.5. What happens to the tomato's volume?
7. Suppose an object with a dimension of 1.81 is scaled up by a factor of 2. What happens to the object's size?

Intermediate

1. Determine the self-similarity dimension of the Sierpiński pyramid, shown in Fig. 4.

2. Determine the self-similarity dimension of the Menger sponge, shown in Fig. 5.

3. The first steps in the generation of a fractal are shown in Fig. 6. This fractal is a variant of the Koch curve we have seen before.

   (a) Make a sketch of the next step in the construction of the fractal.
   (b) What is the self-similarity dimension of this fractal?

Advanced

1. Suppose we were to construct a Cantor set as in Fig. 3, but instead of removing the middle third of each line segment at each step, we remove the middle fifth. The resulting fractal is known as the *middle-fifths Cantor set*.

   (a) Sketch the first several steps in the construction of the middle-fifths Cantor set.
   (b) What is the self-similarity dimension of the middle-fifths Cantor set?

2. The first step in the construction of the Peano curve is shown in Fig. 7.

   (a) Sketch the next step in the construction of the curve.
   (b) What does the curve look like after many steps?
   (c) What is the self-similarity dimension of the Peano curve?

3. Some logarithm puzzles. (These are, in a sense, "trick questions." Give them a try if you want, but don't get stressed out about them. These sorts of questions won't be appearing later in the course.) Solve the equations below for x

$$5^x = -10 \,. \tag{1}$$

$$5^x = \frac{1}{2}x \,. \tag{2}$$

$$5^x = 10x \,. \tag{3}$$

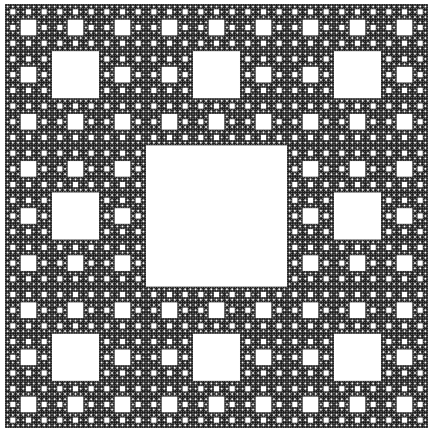

Figure 1: The Sierpiński carpet. Figure source: "Sierpiński carpet" by Josh Greig. Own work by the original uploader. Licensed under Public Domain via Wikimedia Commons—https://commons.wikimedia.org/wiki/File:Sierpinski_carpet.png#/media/File:Sierpinski_carpet.png.

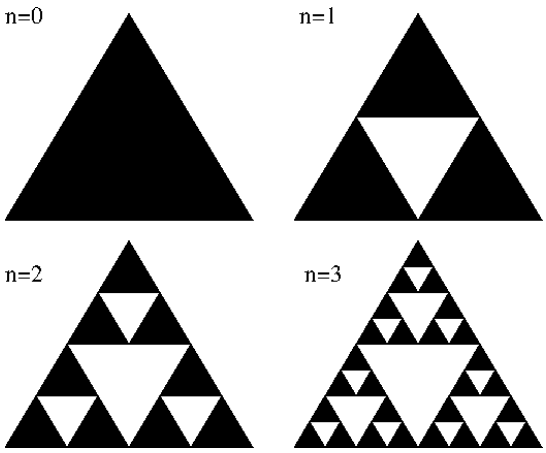

Figure 2: Steps in the construction of the Sierpiński triangle.

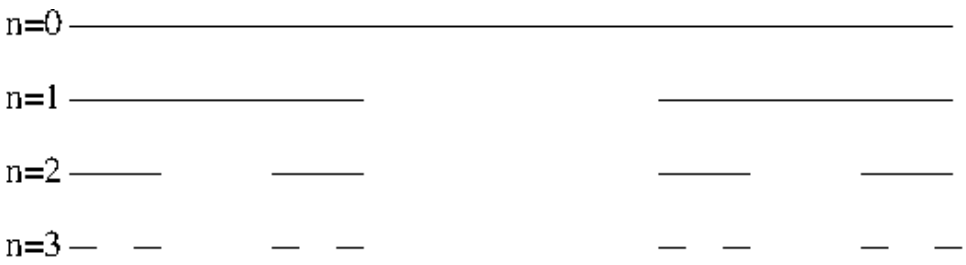

Figure 3: Steps in the construction of the Cantor set.

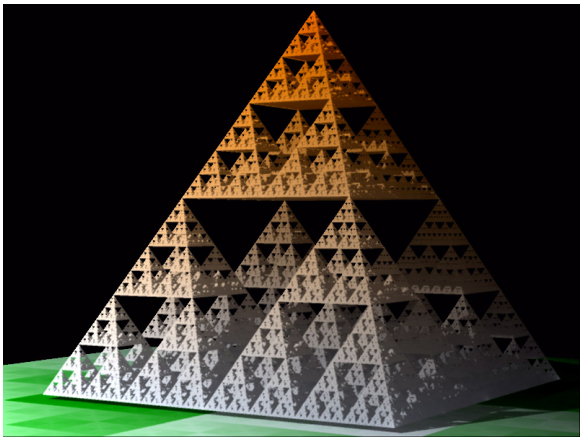

Figure 4: The Sierpiński pyramid. Figure source: "Sierpiński pyramid". Licensed under Public Domain via Wikimedia Commons—https://commons.wikimedia.org/wiki/File:Sierpinski_pyramid.jpg.

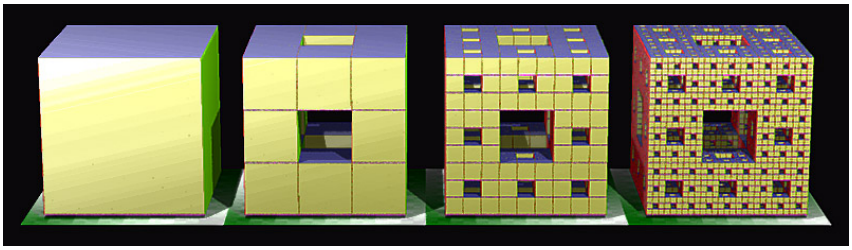

Figure 5: The steps in the construction of the Menger sponge. Figure source: "Menger sponge (Level 0-3)". Licensed under Public Domain via Wikimedia Commons—https://commons.wikimedia.org/wiki/File:Menger_sponge_(Level_0-3).jpg.

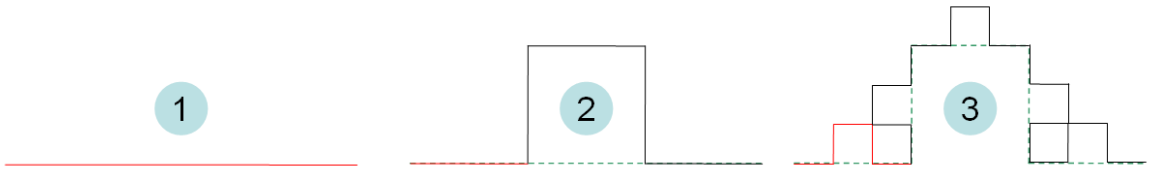

Figure 6: Steps in the construction of a fractal. Figure source: "Quadratic Koch curve type1 iterations" by Prokofiev—Own work. Licensed under GFDL via Commons. https://commons.wikimedia.org/wiki/File:Quadratic_Koch_curve_type1_iterations.png.

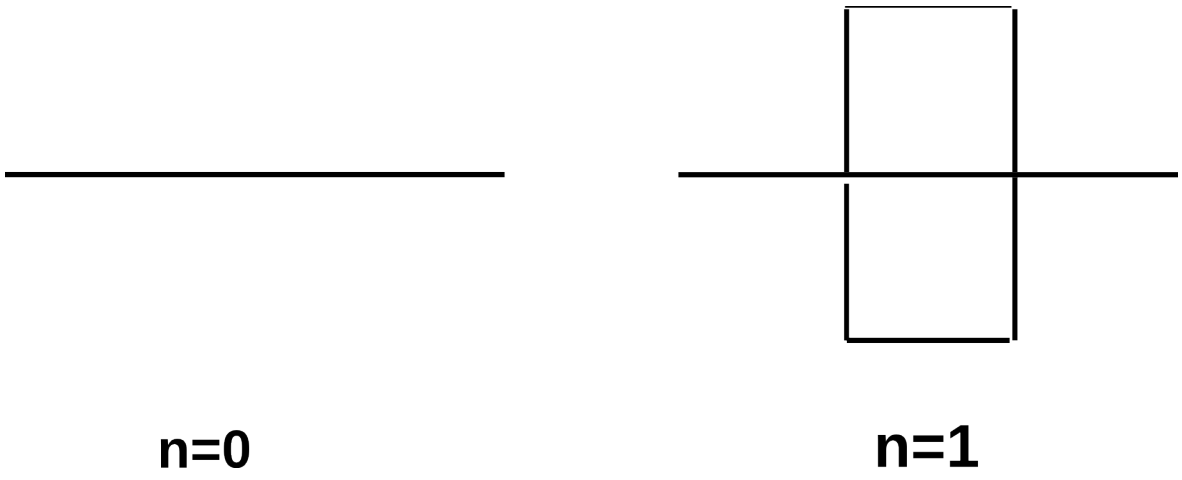

Figure 7: The first step in the construction of a Peano curve.