Summary of Unit 4

Bifurcations: Part I

Introduction to
Dynamical Systems and Chaos

http://www.complexityexplorer.org

The Logistic Differential Equation

- A very simple model of a population where there is some limit to growth
- $dP/dt = rP(1-P/K)$
- r is a growth parameter
- K is the carrying capacity
- There is a stable equilibrium at P=K and an unstable equilibrium at P=0

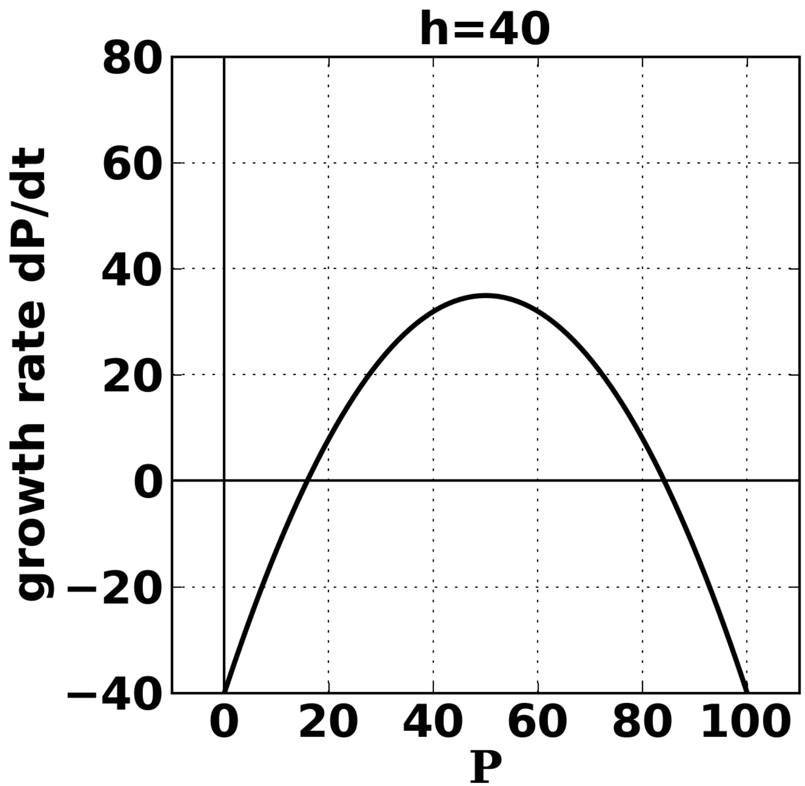
Differential Eqs vs. Iterated Functions

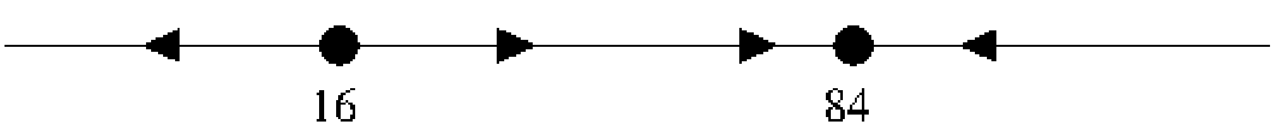
- Time is continuous
- P is continuous
- Cycles and chaos are not possible
- This is due to determinism: for a given P the population can have only one dP/dt

- Time moves in jumps
- P moves in jumps
- Cycles and chaos are possible
- (The logistic iterated function is sometimes called the **logistic map**.)

Logistic Equation with Harvest

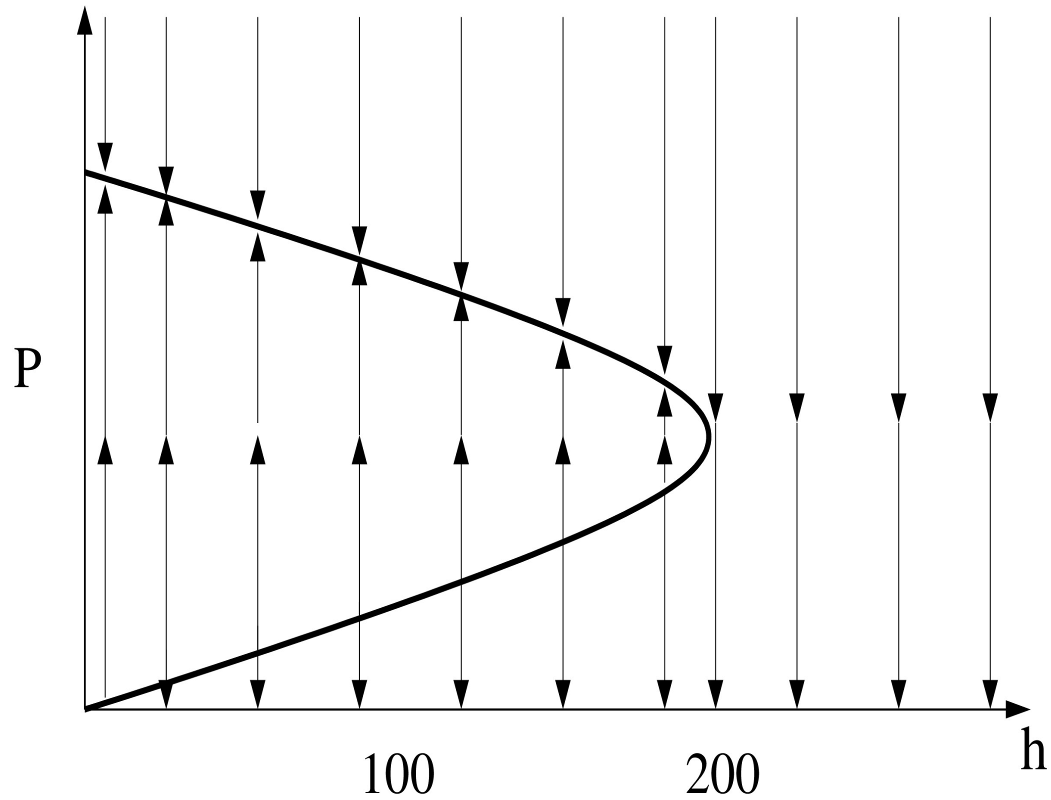
- $dP/dt = rP(1-P/K) - h$
- h is harvest rate
- What happens for different values of h?
- The value of the stable equilibrium decreases as h increases





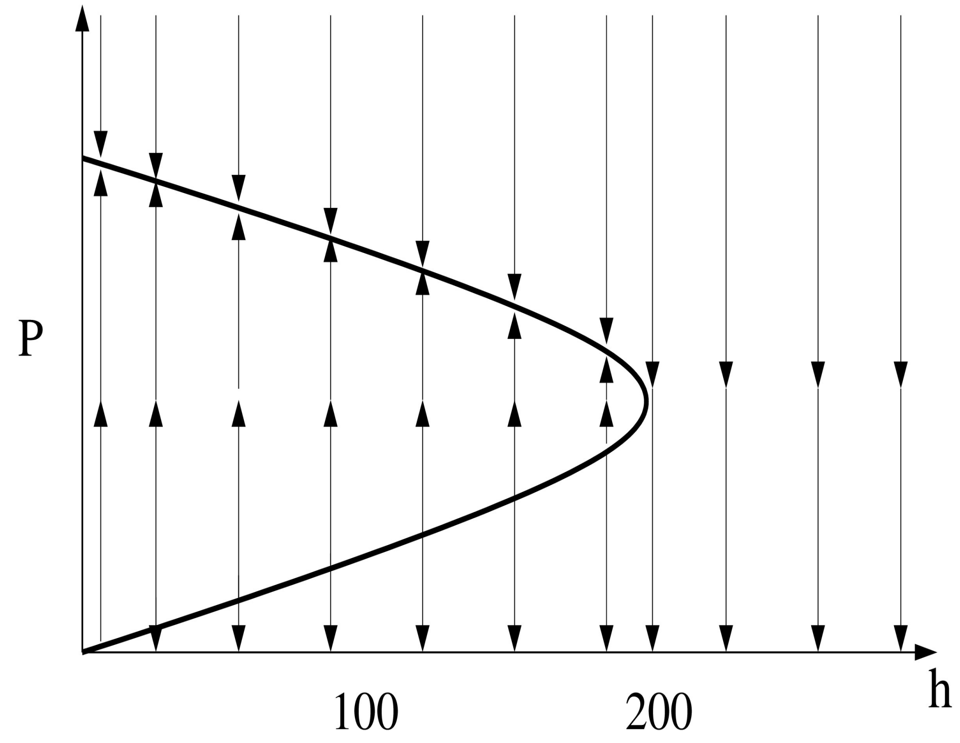
Bifurcation Diagrams

- Take phase lines for different parameter values and “glue” them together
- Shows how the fixed points of a dynamical system change as a parameter is varied



Bifurcations

- A sudden, qualitative change in the behavior of a dynamical system as a parameter is varied
- Sometimes properties of continuous models are discontinuous



Bifurcations

- There is a nice classification of bifurcations into several different types.
- For more, see:
- Strogatz, *Nonlinear Dynamics and Chaos,* Westview Press 2001. Chapter 3.
- http://www.scholarpedia.org/article/Bifurcation
- Most texts on dynamical systems/differential equations