Sensitive Dependence on Initial Conditions

A dynamical system has sensitive dependence on initial conditions (SDIC) if arbitrarily small differences in initial conditions eventually lead to arbitrarily large differences in the orbits.

More formally

- Let f be a function and let x_0 and y_0 be two possible seeds.
- The f has *sensitive dependence on initial conditions* if there is some number δ such that for any x_0 there is a y_0 that is not more than ϵ away from x_0, where the initial condition y_0 has the property that there is some integer n such that after n iterates, the orbit of y_0 is more than δ away from the orbit of x_0. That is, $|x_n - y_n| > \delta$.

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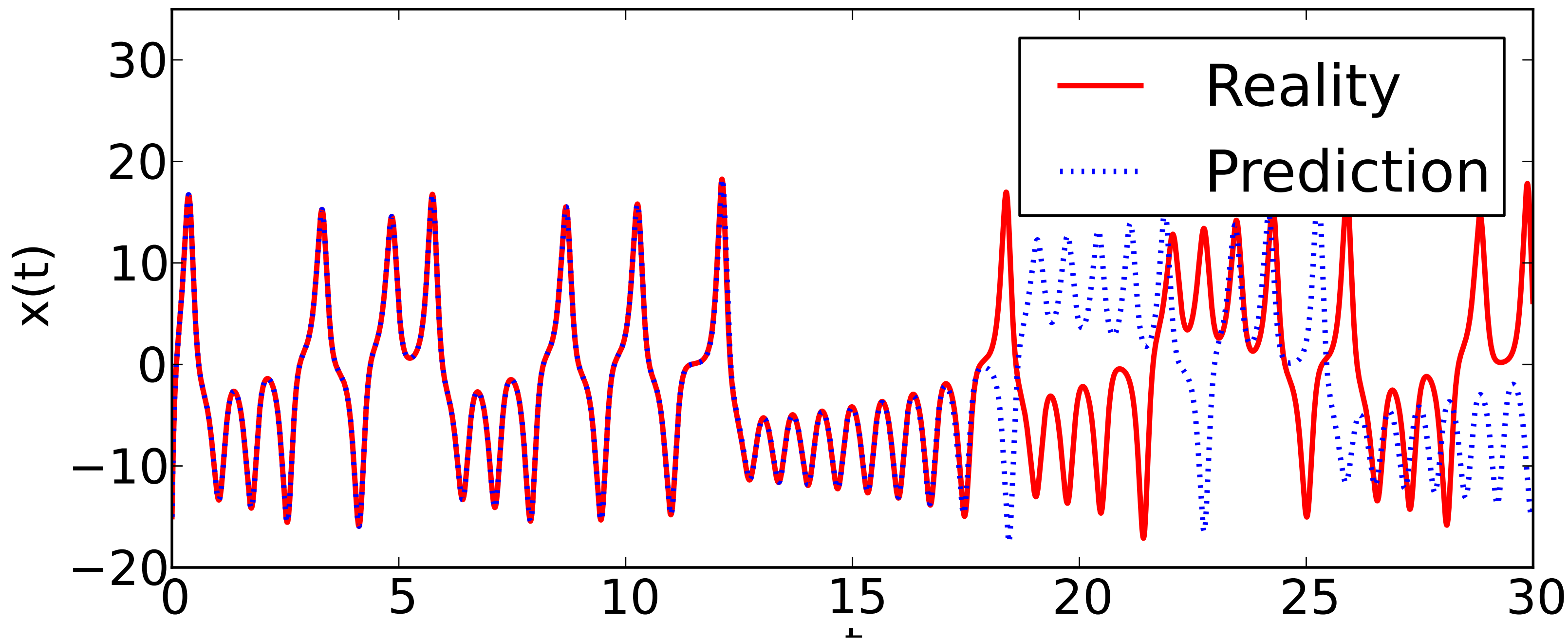
Example: You choose $x_0 = 0.7$, $\delta = 0.2$, and $\epsilon = 0.1$.

Can I find a y_0 within 0.1 of 0.7 such that $|x_n - y_n| > 0.2$ for some n?

Yes. $y_0 = 0.72$. Then $|x_5 - y_5| = 0.626 > 0.2$.

For a function to have SDIC, I must be able to answer yes to a question of this sort for **any** x_0, δ, and ϵ, (provided they are all between 0 and 1).

The Butterfly Effect



- Two different initial conditions: 15 and 15.00000001.
- A tiny change, like the effects of a butterfly flapping its wings, can make a large difference later on.
- Hilborn, R. "Sea gulls, butterflies, and grasshoppers: A brief history of the butterfly effect in nonlinear dynamics." Am. Journal of Physics 72 (2004): 425.

Definition of Chaos

A dynamical system is **chaotic** if:

1. The dynamical system is **deterministic**.
2. The system's orbits are **bounded**.
3. The system's orbits are **aperiodic**; i.e., they never repeat.
4. The system has **sensitive dependence on initial conditions**.