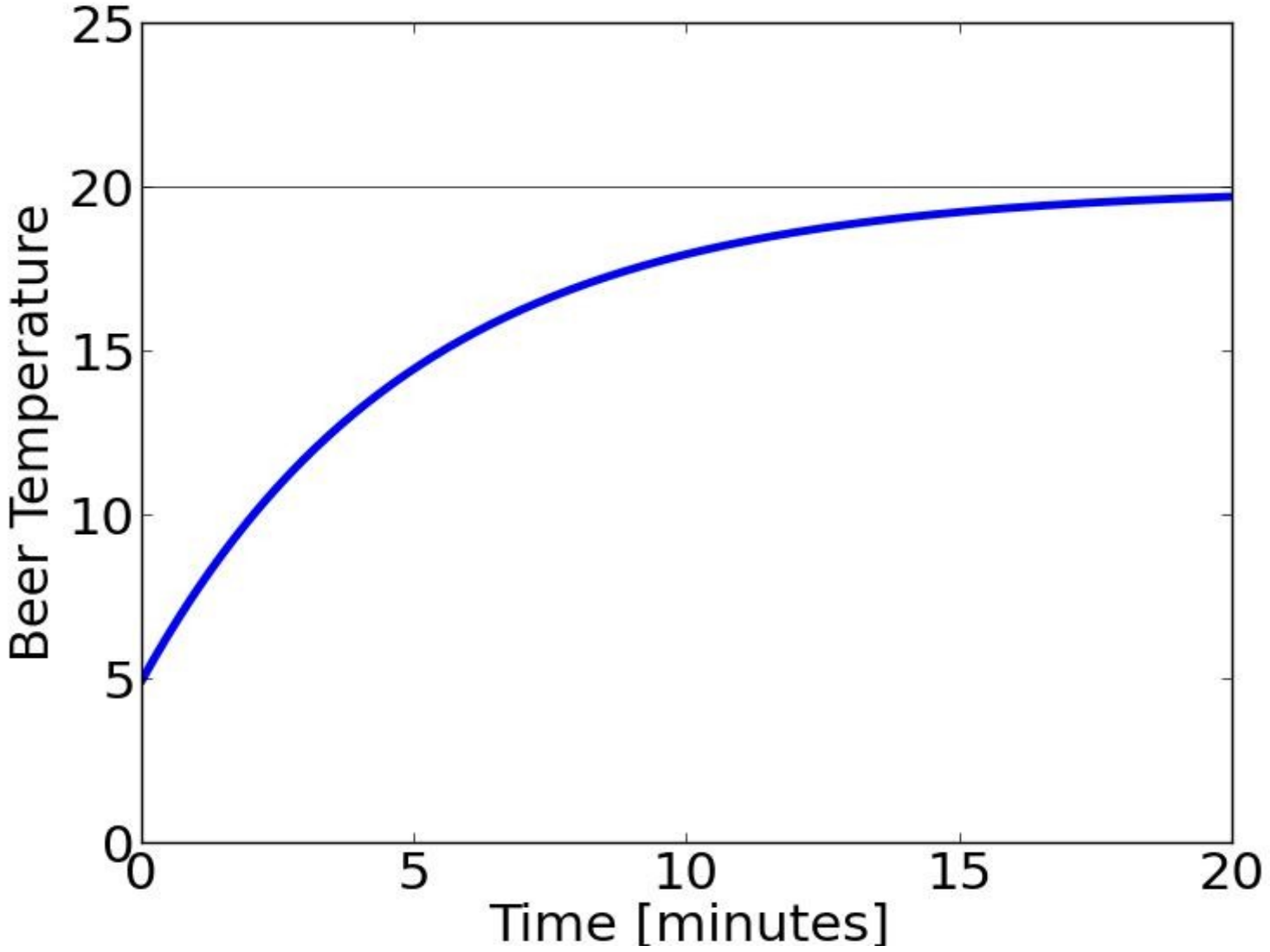
An Initial (Non-Butterfly Effect) Example

- Newton's Law of Cooling describes the temperature of a glass of beer as it warms to room temperature
- $$\frac{dT}{dt} = 0.2(20 - T)$$

- This is a differential equation. It describes how temperature T changes with time t.
- The solution to this equation is T(t), the temperature as a function of time.

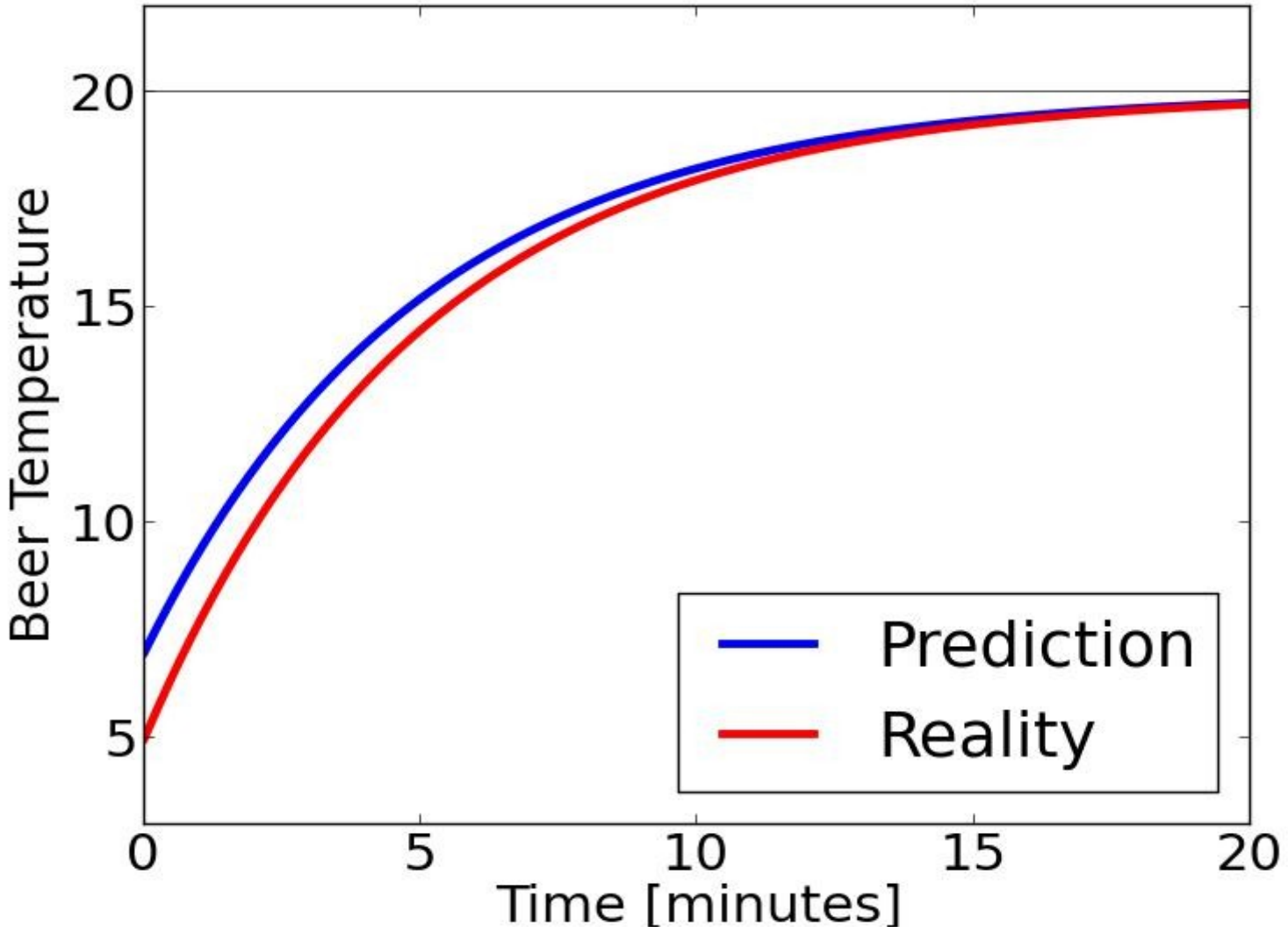
The Beer Warms Up



Predicting Beer Temperature

- We can use the equation to predict the temperature of the beer at a later time.
- We might be slightly wrong in our initial temperature measurement, but it won't matter much.

Predicting Beer Temperature



Laplace's Demon

An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

(Pierre-Simon Laplace, 1814)

Another Example: The Lorenz Equations

- Here is another equation to study
- $$\frac{dx}{dt} = 10(y - x) \ , \quad \frac{dy}{dt} = x(28 - z) - y \ , \quad \frac{dz}{dt} = xy - \frac{8}{3}z$$

- Three equations, one each for x, y, and z.
- A very simple model of atmospheric convection
- Three coupled differential equations. (We'll study these in more detail later.)
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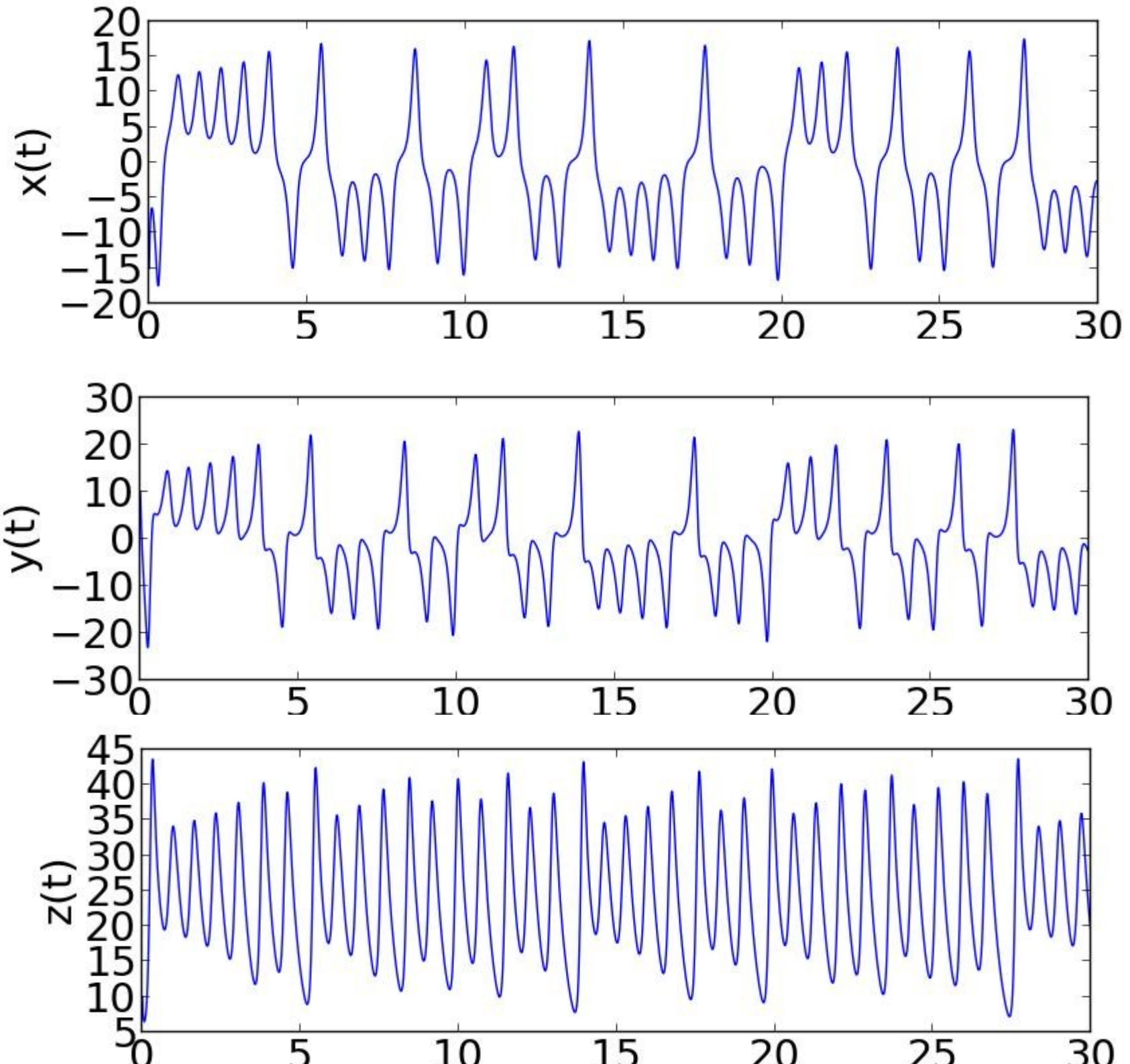
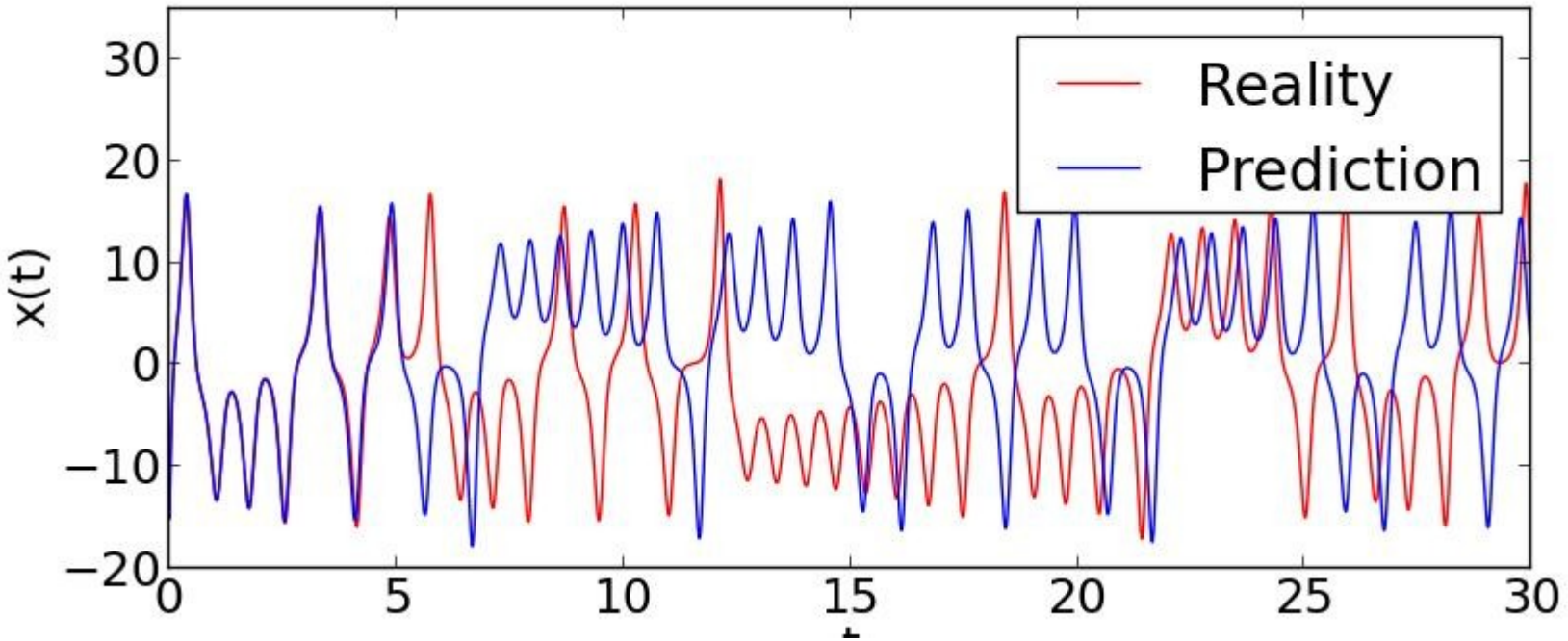

x(t)
y(t)
z(t)

Prediction with the Lorenz Equations

The initial measurement is slightly off:

Model: x=15.1. Reality: x=15.0

We can predict until around t=5

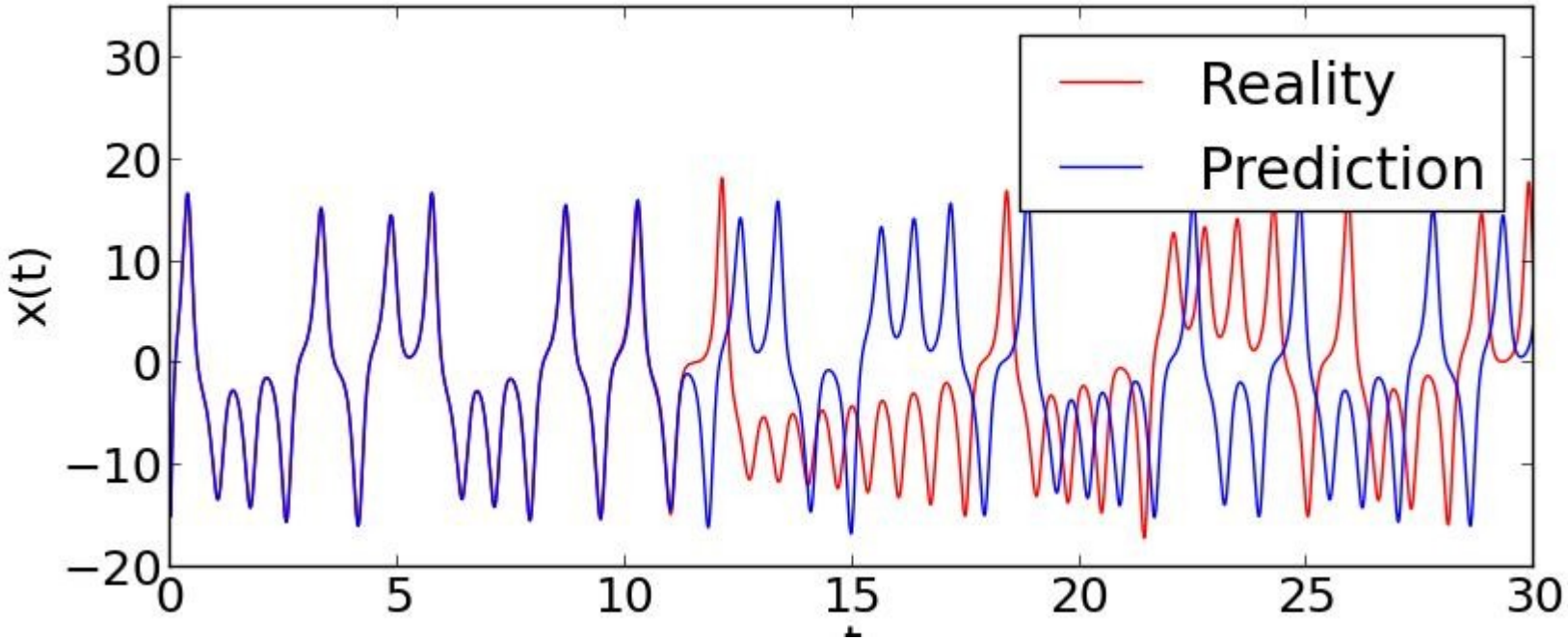


Prediction with the Lorenz Equations

The initial measurement is **much** better:

Model: x=15.0001. Reality: x=15.0

We can now predict until around t=11

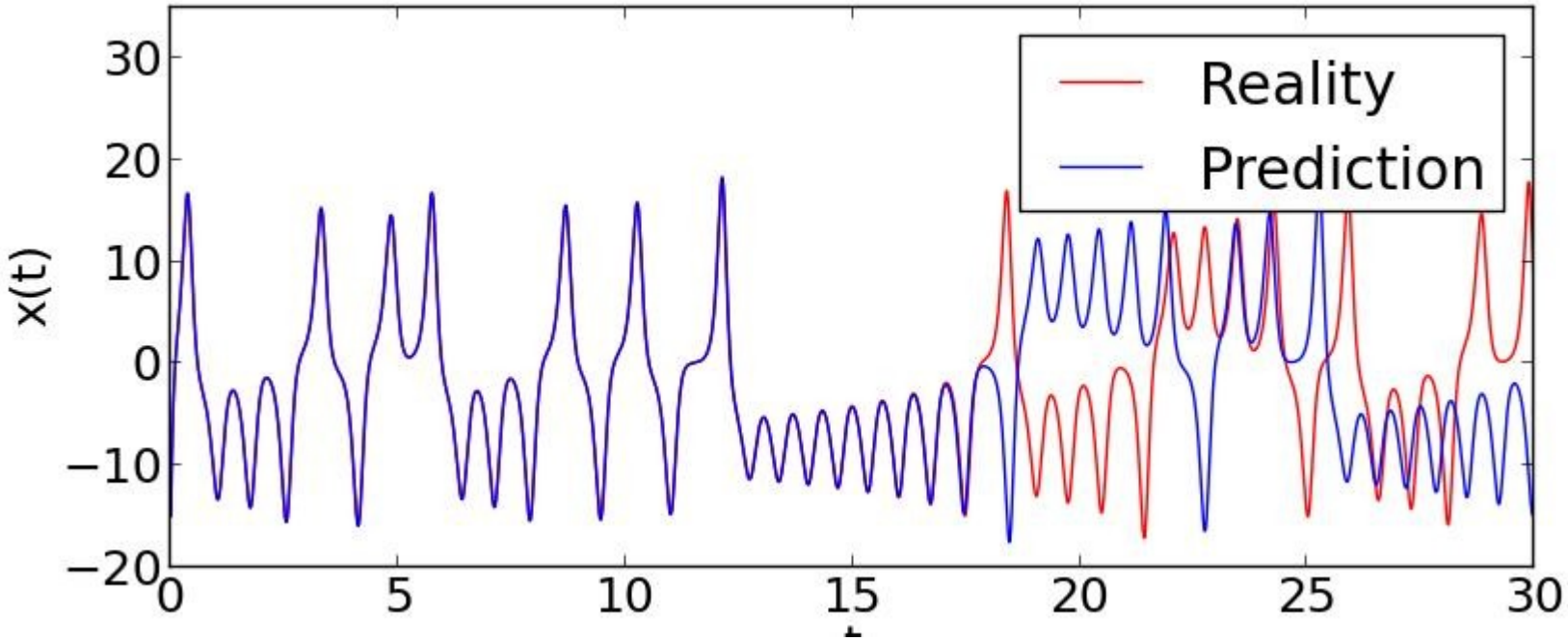


Prediction with the Lorenz Equations

The initial measurement is **amazing**:

Model: x=15.00000001. Reality: x=15.0

We can now predict until around t=18.



Butterfly Effect

- This is the Butterfly Effect
- A very small error in the initial condition grows extremely rapidly
- Long-term prediction is impossible
- A deterministic (rule-based) system can behave unpredictably
- Predictability causes unpredictability