Differential Equations: Some Notes on Terminology

We have looked at equations of the form

$$\frac{dX}{dt} = f(X)$$

. There are other types of differential equations.

- If the right-hand side of the equation does *not* depend on time, the equation is **autonomous**.
- If the right-hand side of the equation does depend on time, then the equation is **non-autonomous**.
- We will study autonomous equations in this course.

Differential Equations: Some Notes on Terminology

$$\frac{dX}{dt} = f(X) \ .$$

- The **order** of a differential equation is the highest derivative that appears in the equation.
- For example, if there are only first and second derivatives in an equation it is second order.
- We will study only first-order equations in this course.
- Higher-order equations can be converted into systems of first-order equations.

Differential Equations: Some Notes on Terminology

$$\frac{dX}{dt} = f(X) \ .$$

- A differential equation is **ordinary** if it only contains ordinary (full) derivatives.
- "Ordinary differential equations" is often written ODE.
- A **partial differential equation** involves partial derivatives. For example:

$$\frac{\partial^2 u}{\partial t^2} \ = \ k\frac{\partial^2 u}{\partial x^2} \ .$$

- We will study only ordinary differential equations in this course.

Differential Equations: Existence and Uniqueness

Consider the differential equation

$$\frac{dX}{dt} = f(X) \ , \tag{1}$$

with an initial condition $X(0) = x_0$. If the function $f(X)$ is continuous and smooth (differentiable), then the solution to Eq. (1) **exists** and is **unique**.

- These conditions are met in most applications.
- So there is one and only one solution.
- The “rule” specified by the equation is unambiguous.