Solutions to Quiz 1, Unit 2.5

Consider again the differential equation for the temperature T of an object:

$$\frac{dT}{dt} = 0.2(20 - T) \,, \tag{1}$$

with the an initial temperature of 5: $T(0) = 5$. For the two questions below apply Euler's method to this equation using $\Delta t = 1$. ($\Delta t = 2$ was used in the video.)

1. What does Euler's method give for $T(1)$, the temperature at time $t = 1$?

   A. 5

   B. 6

   C. 8

   D. 11

   Solution: The answer is **C**. At $t = 0$ the temperature T is 5 so the rate of change of the temperature is:

   $$\frac{dT}{dt} = 0.2(20 - 5) = 0.2(15) = 3\,\text{C/min} \,. \tag{2}$$

   So, if the temperature starts at 5, after one minute the temperature is:

   $$T(1) = 5\,\text{C} + (3\,\text{C/min} \times 1\,\text{min}) = 8\,\text{C} \,. \tag{3}$$

2. Using your answer to question 1, what does Euler's method give for $T(2)$, the temperature at time $t = 2$ minutes?

   A. 10.4

   B. 11.0

   C. 11.6

   D. 14.6

   Solution: The answer is **A**. At $t = 1$ the temperature T is 8 so the rate of change of the temperature is:

   $$\frac{dT}{dt} = 0.2(20 - 8) = 0.2(12) = 2.4\,\text{C/min} \,. \tag{4}$$

   The temperature at $t = 1$ is 8. So one minute later the temperature is:

   $$T(2) = 8\,\text{C} + (2.4\,\text{C/min} \times 1\,\text{min}) = 10.4\,\text{C} \,. \tag{5}$$