

Introduction to Dynamical Systems and Chaos

Homework for Unit 1: Iterated Functions

Santa Fe Institute.

<http://www.complexityexplorer.org>

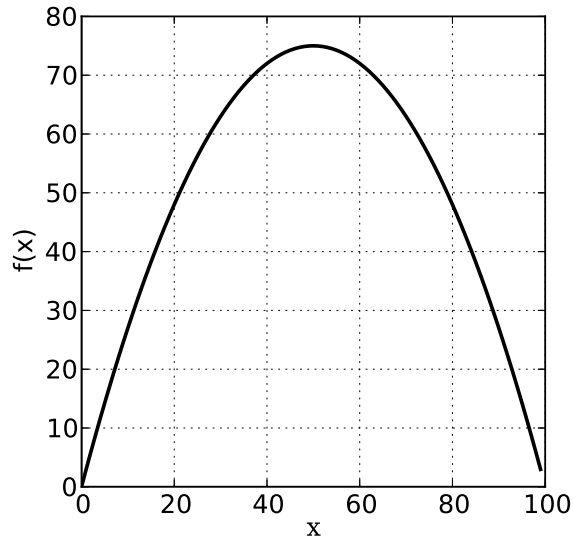


Figure 1: A graph of $f(x)$.

Beginner

1. These problems refer to the function $f(x)$ shown in Fig. 1.
 - (a) What is $f(40)$?
 - (b) What is $f(20)$?
 - (c) Find approximate values for the first three iterates, x_1 , x_2 , and x_3 , using the seed $x_0 = 10$.
2. Consider the function $g(x) = 4x - 12$.
 - (a) What are the first three iterates of the seed $x_0 = 0$?
 - (b) What are the first three iterates of the seed $x_0 = 5$?
3. A function has an attracting fixed point at 1 and a repelling fixed point at 5. Sketch this function's phase line.
4. Consider the function $h(x) = \frac{1}{2}x + 10$. Calculate the first six iterates of h for the initial condition $x_0 = 4$, and then sketch this itinerary in a time series plot.

5. There are three fixed points for the function whose phase line is shown in Fig. 2. What is the stability of each fixed point?
6. For the function whose phase line is shown in Fig. 2,
 - (a) What is the long-term behavior of the seed $x_0 = 0$?
 - (b) What is the long-term behavior of the seed $x_0 = 1$?
 - (c) What is the long-term behavior of the seed $x_0 = 2$?
 - (d) What is the long-term behavior of the seed $x_0 = 5$?

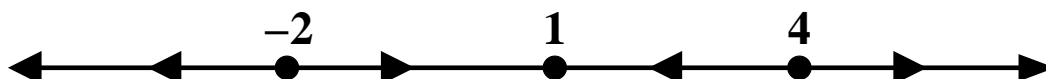


Figure 2: A phase line.

Intermediate

1. Consider the function $f(x) = x^3$.
 - (a) Find all the fixed points of $f(x)$.
 - (b) What is the phase line for the function $f(x)$? Consider both positive and negative seeds.
 - (c) Determine the stability of all of the fixed points of $f(x)$.
2. Consider the function $g(x) = x^2 - 3$.
 - (a) Determine the first few iterates for the seed $x_0 = 1$.
 - (b) Make a time series plot of the orbit. How would you describe this behavior?
 - (c) Is this behavior stable? Try iterating a seed close to x_0 and see what happens.
3. Find all fixed points for the function $f(x) = x + 1$.

Advanced

1. Determine all fixed points for the function $f(x) = \sqrt{2x}$. Which fixed point(s) are stable and which are unstable?
2. Find all fixed points for the function $f(x) = 2^x$.
3. Write a program or make a spreadsheet that will calculate orbits for the function $f(x) = 2.5x(1 - x)$. What is the long-term behavior of seeds between 0 and 1 for this function?