

Solutions to Quiz 4, Unit 1.5

1. The function $f(x) = 2x - 4$ has a fixed point at $x = 4$. Is this fixed point stable or unstable? Answer this by choosing initial conditions near the fixed point iterating (using a calculator if you wish) and seeing what happens to the orbits.

- A. The fixed point is unstable
- B. The fixed point is stable

Solution: The answer is **A**. If you choose an initial condition near 4 you will find that its orbit moves away from 4. For example, the orbit of 3 is: $3 \rightarrow 2 \rightarrow 0 \rightarrow -4 \rightarrow -12 \dots$. The orbit moves away from the fixed point. Hence, the fixed point is unstable.

2. Consider the square root function $f(x) = \sqrt{x}$. What is its phase line (for positive x)? Choose from the four phase lines shown below. (To answer this question you might want to choose a few different initial conditions and use a calculator to determine their orbits.)

Solution: When a number larger than 1 is square rooted, it gets smaller. For example, $\sqrt{49} = 7$. And when a number between 0 and 1 is square rooted it gets larger. For example,

$$\sqrt{\frac{1}{4}} = \frac{1}{2}, \tag{1}$$

and $1/2$ is larger than $1/4$. Thus, the correct phase line is the one in the **bottom** of the figure, as it shows numbers between 0 and 1 increasing and approaching 1, and all numbers larger than 1 get smaller and approach 1.

