

Solutions to Quiz 3, Unit 1.5

1. For the function $f(x) = 2x - 4$, which of the following is a fixed point?

- A. 0
- B. 2
- C. 4
- D. 8

Solution: The answer is **C**. One way to see this is to try out the four possible answer and see if $f(x) = x$. For example, to test $x = 2$,

$$f(2) = 2? \tag{1}$$

$$2 \times 2 - 4 = 2? \tag{2}$$

$$4 - 4 = 2? \tag{3}$$

$$0 = 2? \tag{4}$$

$$\tag{5}$$

Since 0 does not equal 2, we conclude that 2 is not a fixed point of $f(x)$. Performing a similar analysis with the other three possible answers will show that 4 is a fixed point.

Alternatively, one could use this fixed-point equation to solve for the fixed point. (This was covered in the optional lecture. It is not required that you know how to do this.)

$$f(x) = x, \tag{6}$$

$$2x - 4 = x, \tag{7}$$

$$-4 = -x, \tag{8}$$

$$4 = x. \tag{9}$$

So $x = 4$ is a fixed point. (To go from Eq. (7) to Eq. (8) I subtracted $2x$ from both sides of the equation.)

2. For the function $g(x) = x + 2$, which of the following statements is true?

- A. -2 is a fixed point
- B. 0 is a fixed point
- C. 2 is a fixed point
- D. $g(x)$ has no fixed points

Solution: The answer is **D**; $g(x)$ has no fixed points. There is no number that stays the same when 2 is added to it. If you try to solve the fixed point equation, one will see that it has no solution:

$$g(x) = x \tag{10}$$

$$x + 2 = x \tag{11}$$

$$2 = 0. \tag{12}$$

Since 2 does not equal 0, we conclude that the fixed point equation does not have a solution. There is no x that makes the equation $g(x) = x$ true.

3. Which of the following numbers are fixed points for the square-root function, $f(x) = \sqrt{x}$?
- A. 0
 - B. 1
 - C. 0 and 1
 - D. 0, 1, and 2

Solution: The answer is **C**; both 0 and 1 are fixed points. These numbers do not change when they are square-rooted:

$$\sqrt{1} = 1, \tag{13}$$

and

$$\sqrt{0} = 0, \tag{14}$$