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DEVELOPMENT AND APPLICATION OF THE ELASTICA SHAPE FOR PRESSURIZED BALLOONS[1]

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Abstract

It is now widely accepted that the optimum shape for a pressurized balloon is the elastica shape, also known as the natural or pumpkin shape. This shape is the result of mathematical and experimental explorations of the problems associated mostly with balloons and parachutes and has been known since the first part of the 20th Century. In this shape, the pressure loads are carried almost entirely by longitudinal stresses in the shell and circumferential stresses in the film are very small. The tops of natural shaped balloons use the upper half of the elastica shape with the bottom half being closer to a cone shape. The shape of a deployed flat circular parachute also follows the form of the top half of the elastica. The purpose of this paper is to review the experiences of various projects that have either studied or used elastica balloons and to relate their results with current knowledge of these balloons.

1. Background of the Elastica Shape

The elastica shape is developed by placing a flaccid structure under a uniform normal load. This development is analogous to the catenary shape which is developed by placing a flaccid body under a constant gravity load. This comparison is demonstrated in Figure 1.

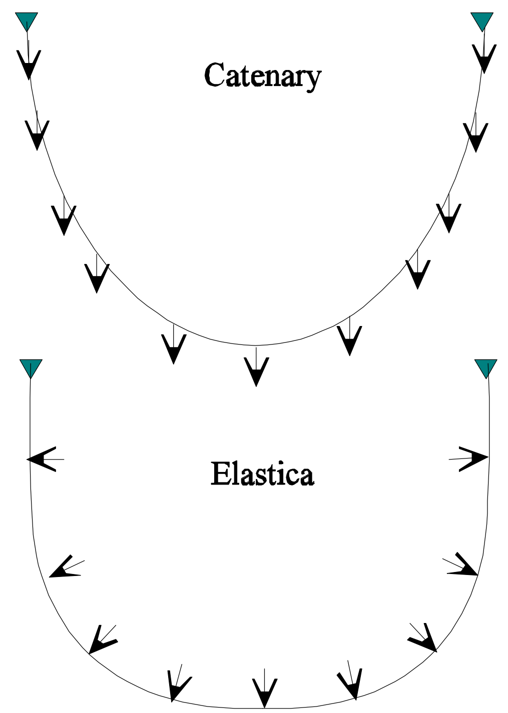


Figure 1 - Catenary vs. Elastica Analogy

If the elastica shape in Figure 1 is rotated around the two anchor points, the natural shape superpressure balloon profile, in Figure 2, is generated. The ratio of height to diameter is 0.6. The ratio of diameter to gore length is 0.76.

In 1919, Sir Geoffery Taylor [1] applied the elastica in the calculation of the shapes of descending parachutes. His analysis showed that parachutes with sufficient excess material would assume the elastica

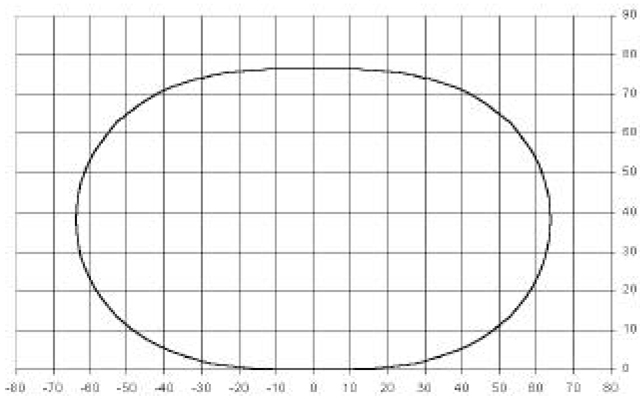


Figure 2 - Elastica Balloon Profile

shape while inflated. To validate his calculations, he reinforced a rubber weather balloon with longitudinal strings. The resulting shape matched the calculations. Although the photograph is crude, the shape shown in Figure 3 is significant, especially when one realizes that it was taken in the year 1919.

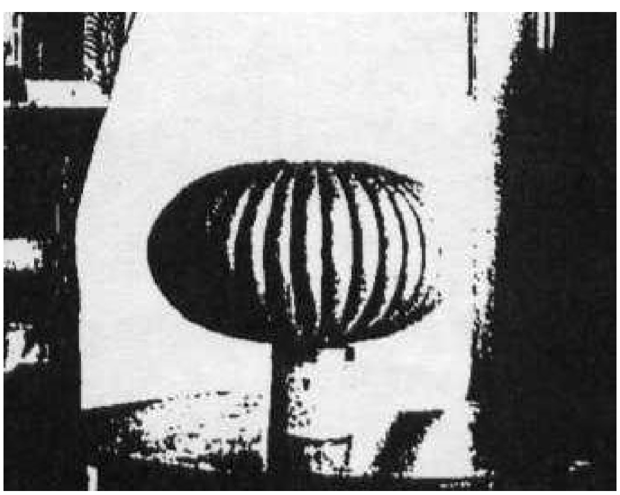


Figure 3 - Taylor Shape Validation Model CIRCA 1919 [1]

2. Early Balloon Experiments

In the 1950's and 1960's some experimentation was done with highly stressed film balloons which were built with straight gores and allowed to assume the elastica shape naturally. Others were built with tailored gores and used longitudinal load tapes to bear the longitudinal loads generated by the shape. All of these studies showed that the strength of available materials for the load tapes. A secondary consideration was the difficulty in retaining the material in the end fittings. The work done by Smalley [2] in the late 1960's reiterated that the elastica shape theoretically produces no circumferential stresses in the shell and that small bulges in the balloon material between the load tapes could be achieved by stretch in the material. He also suggested that extra material could be added to the widths of the gores to facilitate the bulging.

3. The Endeavour Experience

In 1984, Julian Nott used an elastica shape balloon with built in bulges in the gores for a balloon circumnavigation attempt. The small prototype balloon had 60 gores. As shown in Figure 4, the balloon deployed properly.

Figure 4 - The Endeavor Prototype

The first inflation test, the balloon had 64 gores. It showed the first encounter with a new phenomenon in balloon design. As shown in Figure 5, the amount of extra material built into the balloon shell did not deploy evenly. It buckled in the center. In order to cause the balloon to deploy properly, four gores were removed from the shell.

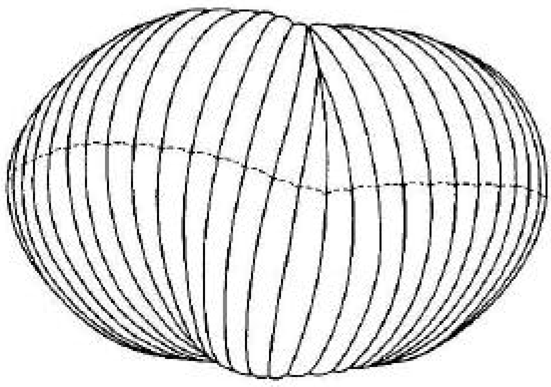


Figure 5 - Buckling Due to Excess Material

It was later shown by Calladine [3], that there is a limit to the amount of built-in lobing that can be used on an elastica balloon. The limit is a function of the number of lobes or gores in the balloon. As the number of gores increases, the maximum amount of built-in lobing decreases. The relationship, given in Equation 1 was developed using a combination of theory and experimental results.

$$n < 47/á^{2.5} \qquad \textbf{(1)}$$

In Equation 1, á is the half angle of the lobed gore and *n* is the number of gores. The geometry of a single lobe is described in Figure 6. In the figure, c is the distance between tendons. It is also the gore width that would be used if the balloon were to be constructed as a smooth surface of revolution without lobing. The symbol s represents the manufactured gore width.

By rearranging Equation 1 and using simple geometric relationships, a maximum width ratio (s/c_{max}) can be developed as a function of the number of gores. This is shown in equations 2 and 3 in unreduced form for clarity. Angle $á_{max}$ is in radians.

$$á_{\mathrm{max}} = (47/n)^{0.4} \qquad \textbf{(2)}$$

$$s/c_{max} = á_{\mathrm{max}}/\sin á_{max} \qquad \textbf{(3)}$$

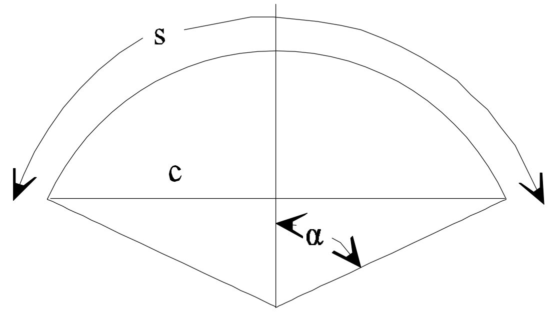


Figure 6 - Lobed Gore Geometry

4. Experiments for the ULDB Project

4.1 Shapes Observed in Test Flights

The relationships described in the previous section were compared to results of test flights and model tests for NASA's Ultra Long Duration Balloon (ULDB) project. The motivation for this work was the result of a shape anomaly observed during the test flight of a full scale ULDB from Australia in March of 2001. During that flight, the fully inflated balloon was observed in a telescope and found to have a shape that was very similar to the one in Figure 5. The photo in Figure 7 clearly shows the buckled shape with undeployed material.

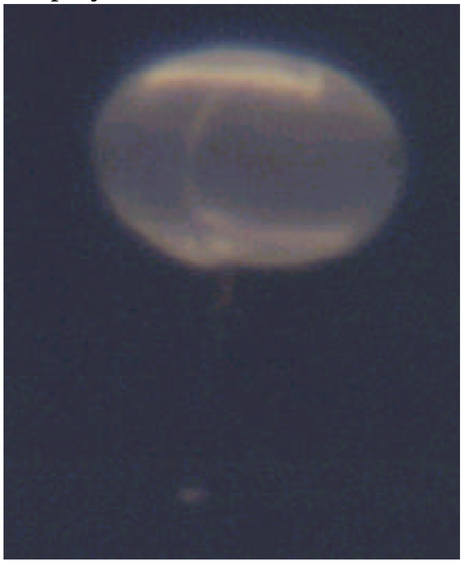


Figure 7 - Full-Scale ULDB with Undeployed Material

The amount of built-in lobing in this balloon was 13%. With 290 gores, the Calladine relationship gives a s/c_{max} of 1.035.

Two previous flight test balloons were flown without any lack of deployment. The first balloon was flown in Phase II of the ULDB Project. It had 134 gores and a s/c_{max} of 1.05. The Calladine limit for 134 gores is 1.076. The balloon flown in Phase III had 150 gores and a s/c of 1.08. The Calladine limit of 1.07 for this balloon suggests that some adjustment to the constant in Equation 2 may in order.

4.2 Model Testing

A series of 4 m diameter, 48 gore model balloons were constructed with various amounts of built-in lobing. The first two models were built with over 50% extra gore width. The typical deployment for these is shown in Figure 8.

Figure 8 - 48 Gore Model with s/c of 1.6

A third model was constructed with s/c of 1.2 which was equal to the Calladine limit. The balloon had a cleft similar to the one in Figure 8 up until the last moment of inflation when the balloon began to pressurize. As the pressure began to increase, the balloon deployed fully as shown in Figure 9.

Figure 9 - 48 Gore Model with s/c of 1.2

This model had a constant s/c down its length. The balloon then had a constant lobe angle á down its length. To test the limit of the gore width from Calladine, the balloon was further pressurized to cause the gore material to strain beyond the 20% limit. The balloon began to buckle as the gore material stretched in the circumferential direction.

A fourth model was constructed with a s/c of 1.4 at the equator, but a decreasing s/c near the poles of the balloon. As described by Schur [4], this allowed the balloon to fully deploy with a constant lobe radius instead of a constant lobe angle as on the other models. The deployed balloon is shown in Figure 10.

Figure 10 - Variable s/c with Constant Lobe Radius

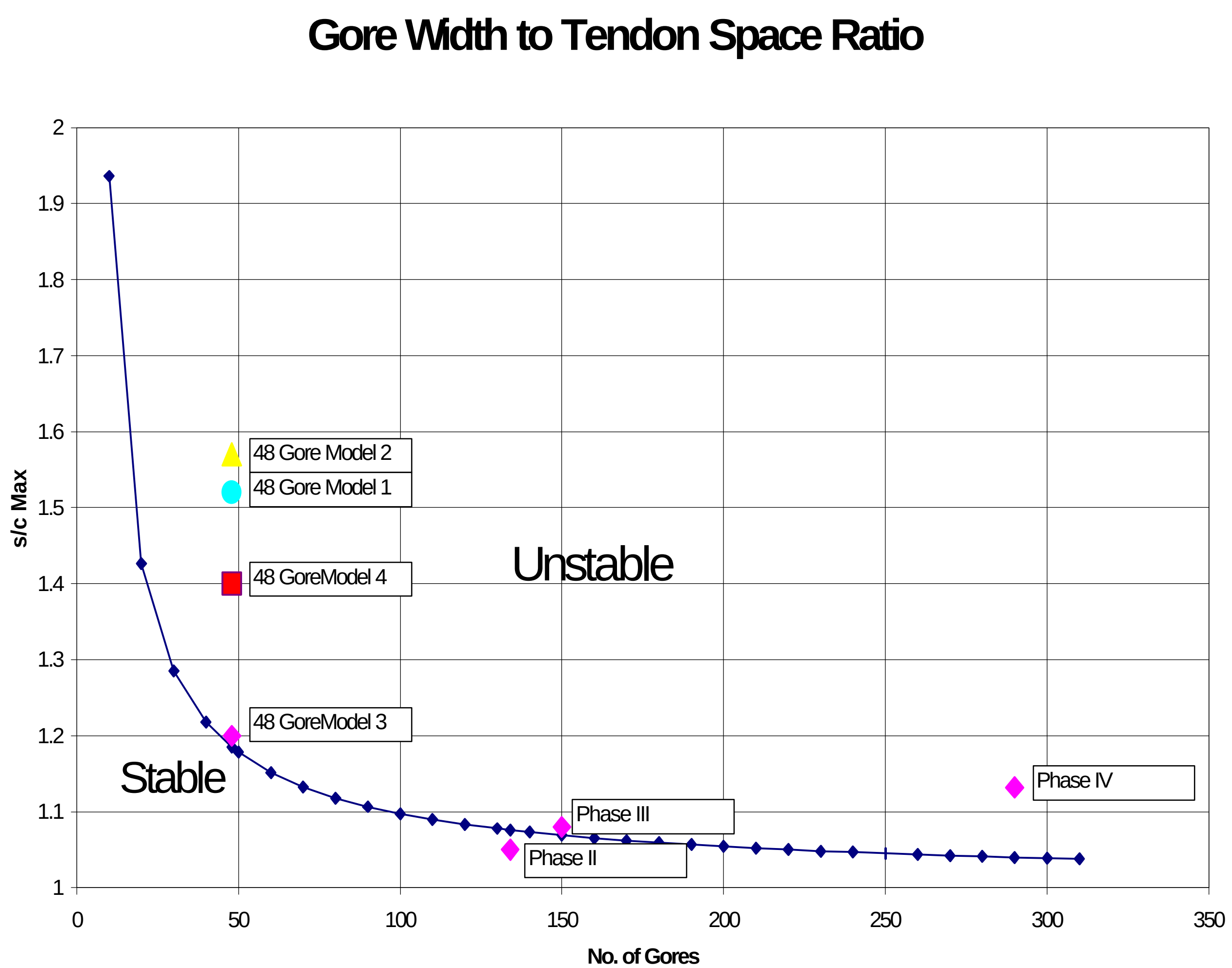
A summary of the flight test and model balloons is shown in Figure 11. The balloons with deployment problems are clearly above the Calladine limit line. The only exception to this is Model 4 which had the variable s/c ratio. This variable s/c design was chosen by NASA for the next full-scale balloon test flight in June of 2002.

5. Recommendations for Future Development Plans

The Model 3 balloon was different from all previous models tested. It is the only one with a 1:1 ratio of film gore length to tendon length. The lobing in the longitudinal direction was achieved by material strain only. Smalley [2] proposed to achieve the lobing by material strain in the longitudinal direction and adding material in the circumferential direction. This approach had two main advantages: greatly reduced complexity in fabrication and reduced weight.

A summary of the various types of approaches to achieving the lobed gore is presented in Figure 12. The proposed method, which was used in Model 3, needed 3% strain in the meridional direction to achieve the desired lobing. After achieving the desired shape, the meridional strain did not increase as pressure increased. Only the transverse strain increased as the pressure increased.

The model balloon had only 48 gores. In a large balloon such as the full scale ULDB, the number of gores would be 290. The meridional strain needed to achieve the desired gore shape would only be 1%.



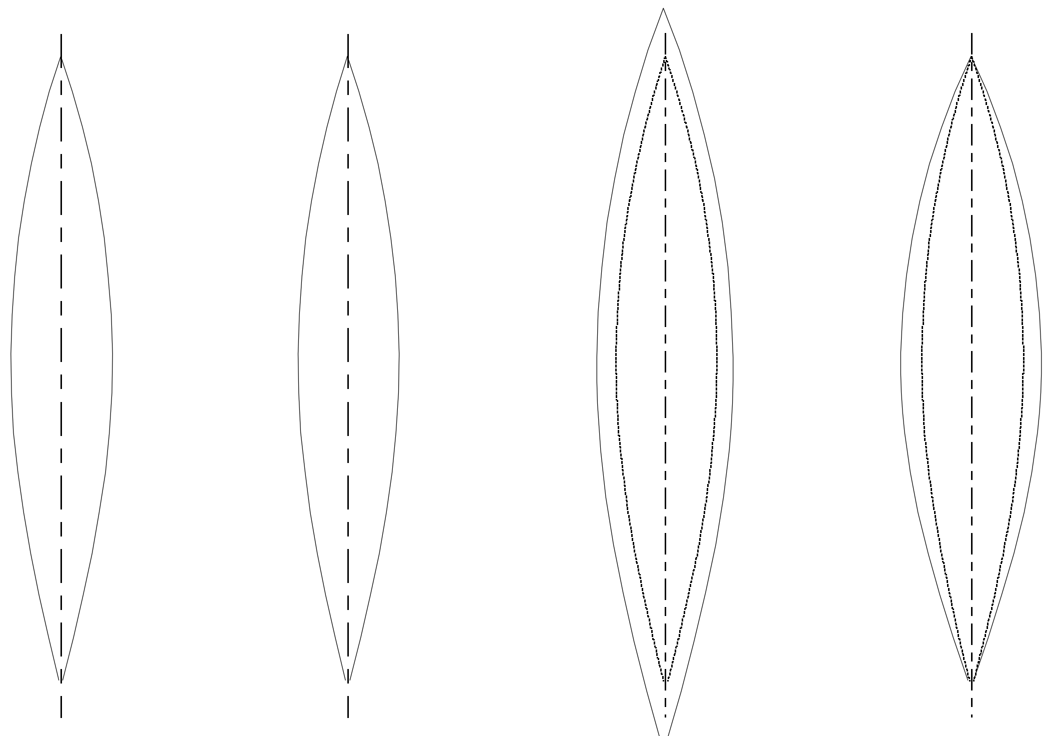


Figure 12 - Methods for Achieving Lobed Gores

6. Conclusions

It has been shown that there is a limit to the amount of built-in lobing that can be added to an elastica balloon shape. There is a possible solution to this limitation by varying the amount of extra material to force a smaller lobe radius at the equator where it is needed the most.

A new approach to achieving the lobed gore has been proposed. This approach would greatly facilitate fabrication and would require only a small amount of material strain in the meridional direction to achieve the desired amount of lobing.

7. Acknowledgment

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