How to support your student as they learn about

Inverting Functions

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module your student will deepen their understanding of exponential and logarithmic functions and all other function types covered so far. There are 3 topics in this module: *Exponential and Logarithmic Functions, Exponential and Logarithmic Equations*, and *Applications of Exponential Functions*. Your student will use what they already know about exponential functions, their properties and transformations, in this module.

Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Describe
Definition	• To represent or give an account of in words. • Describing communicates mathematical ideas to others.
Questions to Ask Your Student	• How should I organize my thoughts? • Is my explanation logical? • Did I consider the context of the situation? • Does my reasoning make sense?
Related Phrases	• Demonstrate • Label • Display • Determine • Define • What are the advantages? • What are the disadvantages? • What is similar? • What is different?

TABLE OF CONTENTS

Explain how to change the equation $100 = \log_2(x + 3)$ to exponential form. **What are the advantages** of changing the form?

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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Problem Types You Will See: ***Who's Correct***

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

3. Pippa and Kate disagree about the solution to the logarithmic equation $\log_5 x^2 - \log_5 4 = 2$.

$$\log_5 x^2 - \log_5 4 = 2$$
$$\log_5 \left(\frac{x^2}{4}\right) = 2$$
$$5^2 = \frac{x^2}{4}$$
$$25 = \frac{x^2}{4}$$
$$100 = x^2$$
$$x = 10, -10$$

Kate says the solutions are $x = 10$, $x = -10$. Pippa says that the solution $x = -10$ should be rejected, because the argument of a logarithm must be greater than zero.

Who is correct? Explain your reasoning.

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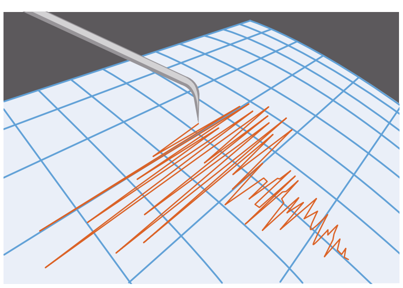
TEXAS MATH SOLUTION

Module Overview

TOPIC 1	TOPIC 2	TOPIC 3
Exponentials and Logarithmic Functions	**Exponential and Logarithmic Equations**	**Applications of Exponential Functions**
20 Days	15 Days	10 Days
Your student will expand on what they already know about exponential functions and their inverses, logarithmic functions.	**Your student will learn about the different properties and rules of logarithms needed in order to solve equations with multiple logarithms.**	**Your student will explore different uses for the different function families that have been learned throughout the course.**
Did you know? The Richter Scale is used to calculate the energy released by earthquakes. The Richter scale is logarithmic, meaning that whole-number jumps in the rating indicate a tenfold increase in the wave amplitude of the earthquake. For example, the wave amplitude in a Level 5 earthquake is ten times greater than the amplitude of a Level 4 earthquake. 	Solving the Mystery Crime investigators use logarithmic equations to estimate time of death based on two temperature readings of the body. Specifically, investigators use what's known as Newton's Law of Cooling, which states that an object cools down at a rate that is proportional to the temperature difference between the object and the environment. 	Part of Nature Fractals may seem a bit strange, but they are actually a naturally occurring event. They are seen all the time in things like ice formations, plants, and even sea life. Can you think of any specific examples where you have seen fractals before? 

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Topic 1: Exponential and Logarithmic Functions

Key Terms	
• half-life • natural base *e* • logarithm	• logarithmic function • common logarithm • natural logarithm
The **natural base e** is an irrational number equal to approximately 2.71828. $e^2 \approx 2.7183^2 \approx 7.3892$	A **common logarithm** is a **logarithm** with a base of 10. Common logarithms are usually written without a base. log (10*x*) or log *x* are examples of a common logarithm.
Follow the link to access the Mathematics Glossary: https://www.carnegielearning.com/texas-help/students-caregivers/	

In this topic, students analyze, graph, and transform exponential functions and their inverses, **logarithmic functions**.

Half-Life and Continuous Compounding

Calculating the half-life of a substance is a classic example of exponential decay. Half-life is used in carbon dating, radioactive decay, and to measure the effectiveness of medical drugs over time. Similar to exponential decay, there can also be growth, and how the growth happens depends on the situation.

Money can be compounded at different points of time by a bank while some things like population and cells compound continuously, which means that they never stop compounding. The irrational number *e* represents the continuous compounding of things like populations.

The table shows the amount of caffeine in a person's system over time, starting with 80 milligrams of caffeine. The caffeine has a half-life of 5 hours.

Time Elapsed (hours)	0	5	10	15	20	25
Caffeine in System (mg)	80	40	20	10	5	2.5
Number of Half-Life Cycles	0	1	2	3	4	5

An equation to represent the half-life of the caffeine would be, $g(n) = 80\left(\frac{1}{2}\right)^{\left(\frac{n}{5}\right)}$, where n is the number of hours.

Inverses and Changing Forms

Like other function families, exponential functions have inverses which are called logarithms. The base, exponent, and argument of an exponential function get rearranged into a logarithmic equation. Any number can be the base of a logarithm, but the three most common numbers used are 2, 10, and *e*. If 10 is used it is generally not written, or if *e* is used it is called the natural log and will be written as "ln" instead of $\log_e$.

exponent argument

$$3 = \log_2 8$$

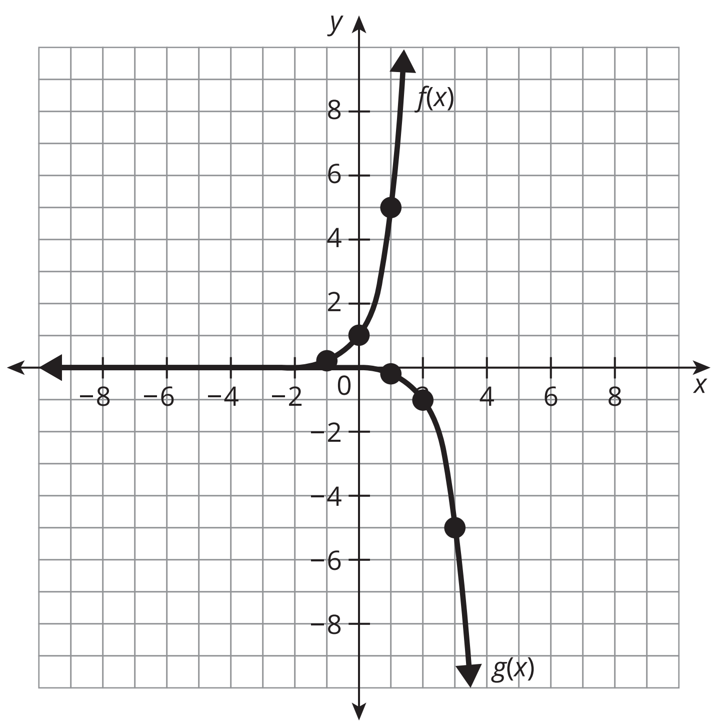
base

Transformations and Inverse Transformations

Transformations of exponential functions and logarithmic functions are both covered much like other function families have been. Because the two are inverses of one another, their relationship with regard to transformations is no different than with polynomials and radical functions.

To create $g(x)$, the graph of $f(x)$ is horizontally translated right 2 units and reflected across the x-axis.

$g(x) = -f(x - 2)$



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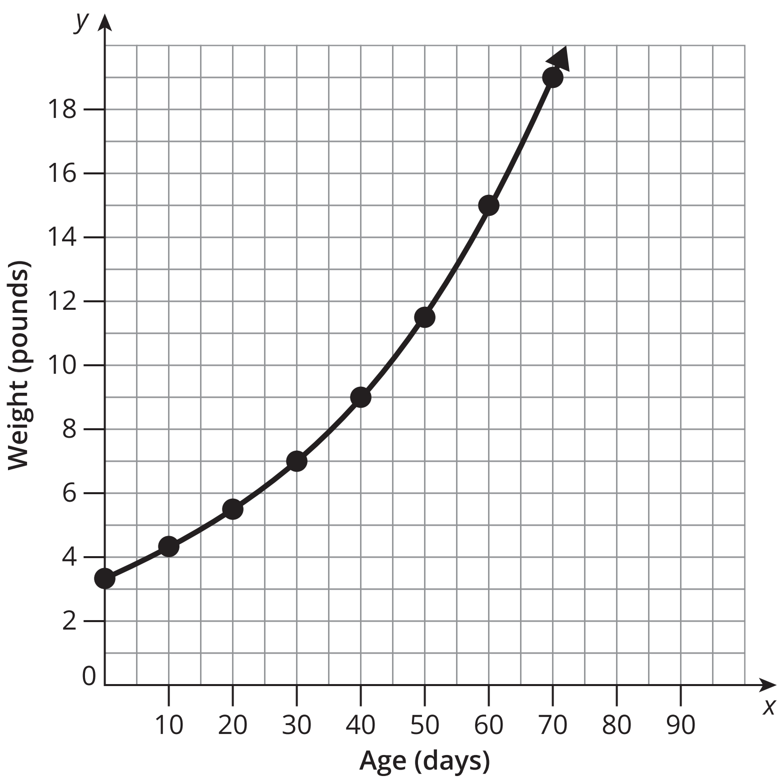
Regression Models

Exponential regression was covered in a previous course and should be familiar and similar to the regression modeling that was used with polynomial and linear functions. Students will use technology to generate regression equations that can be used to make predictions.

The exponential regression equation for this scenario is $f(x) = 3.31(1.025)^x$. The correlation coefficient, r, is 0.9999.



Age (days)	Weight (pounds)
0	3.25
10	4.25
20	5.5
30	7
40	9
50	11.5
60	15
70	19

Topic 2: Exponential and Logarithmic Equations

Key Terms

- logarithmic equation
- Zero Property of Logarithms
- logarithm with same base and argument
- Product Rule of Logarithms
- Quotient Rule of Logarithms
- Power Rule of Logarithms
- Change of Base Formula

A **logarithmic equation** is an equation that contains a logarithm.

The equation $\log_2(x) = 4$ is a logarithmic equation.

The **Change of Base Formula** allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base. It is especially helpful when using a calculator.

The Change of Base Formula states:

$$\log_b(c) = \frac{\log_a(c)}{\log_a(b)}, \text{ where } a, b, c > 0 \text{ and } a, b \neq 1.$$

$$\log_4(50) = \frac{\log 50}{\log 4} \approx 2.821928095$$

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In this topic students begin by building an understanding of exponential and logarithmic expressions, including converting between the two, estimating the values of logarithms, and evaluating logarithmic expressions for given values. They then use the properties of exponents to derive the properties of logarithms.

Changing Forms to Solve

Exponential and logarithmic equations can be rewritten to solve for the unknown piece or variable. For any equation that does not have a whole number solution, logarithms can also be estimated with a decimal value.

Argument Is Unknown	Exponent Is Unknown	Base Is Unknown
$\log_4 y = 3$	$\log_4 64 = x$	$\log_b 64 = 3$
$4^3 = y$	$4^x = 64$	$b^3 = 64$
$64 = y$	$4^x = 4^3$	$b^3 = 4^3$
	$x = 3$	$b = 4$

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Rules of Logarithms

There are several properties and rules of logarithms to help with simplifying and solving logarithms. The Change of Base Formula is very important because it allows logarithms of any base and argument to be changed to a base of 10. This allows them to be easily divided using a calculator.

For example, consider the expression $\log_5 30$.

$$\log_5 30 = \frac{\log 30}{\log 5} \approx 2.11$$

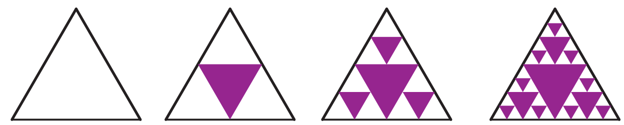
Solving Exponential Equations

If a logarithmic equation involves multiple logarithms, you can use the properties of logarithms to rewrite the equation in a form you already know how to solve. See the example below of an equation with multiple logarithms, simplified using properties.

$$\begin{aligned}
\log 5 + \log x &= 2 \\
\log (5x) &= 2 \\
10^2 &= 5x \\
100 &= 5x \\
x &= 20
\end{aligned}$$

MATH PROCESS STANDARDS **How do the activities in *Exponential and Logarithmic Equations* promote student expertise in the math process standards?**	
NOTE: This is an example of the math process standard: *Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.* • I can apply mathematical ideas to solve problems. Have your student refer to page 2 for more "I can" statements.	$2\log 6 = \log x - \log 2$ Solve the equation and justify each step with rules or properties.

Topic 3: Applications of Exponential Functions

Key Terms	
• geometric series • fractal	• self-similar • iterative process
A **geometric series** is the sum of the terms of a geometric sequence. The geometric series corresponding to the geometric sequence 2, 4, 8, 16 is 2 + 4 + 8 + 16, or 30.	A **fractal** is a complex geometric shape that is constructed by a mathematical pattern. Fractals are infinite and **self-similar**.  Stage 0 Stage 1 Stage 2 Stage 3
Follow the link to access the Mathematics Glossary: https://www.carnegielearning.com/texas-help/students-caregivers/	

In this topic, students begin by deriving formulas for geometric series. They then solve real-world problems, including payoff amounts for credit cards. Next, students draw and interpret graphics, paying attention to the domain restrictions required by the image. Finally, they explore objects and use functions to model their growth.

Geometric Series

Previously, geometric sequences were used to build exponential functions. The difference with a series is the numbers in the sequence are added together. Series are useful for things like calculating payments of loans or credit cards to determine total payments.

For example, consider the geometric series 2 + 6 + 18 + 54 + 162.

$$g_1 = 2,\ r = 3,\ n = 5$$

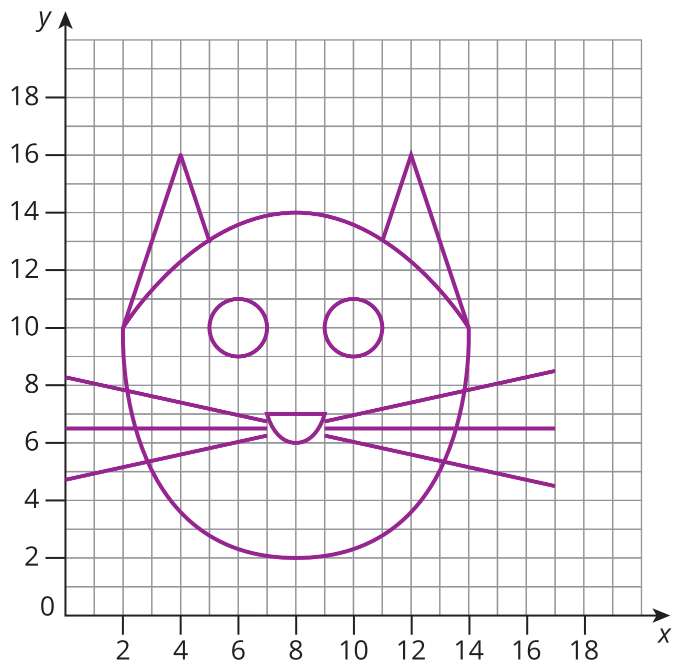
$$S_n = g_1\frac{(r^n - 1)}{r - 1}$$

$$S_5 = \frac{2(3^5 - 1)}{3 - 1}$$

$$= 242$$

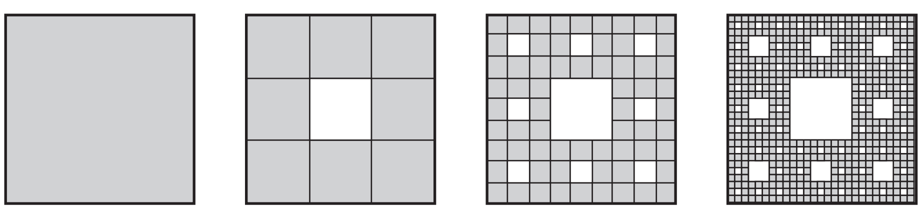
Applying Transformations to All Functions

Transformations of all the different function families covered so far will be used to create graphical images. Domains will be restricted so only pieces of graphs are visible at a time.



Fractals

Students will analyze and follow the patterns of fractals and use what they have learned so far to create expressions to determine the next stage or stages of the patterns.



Stage (n)	0	1	2	3	n
Shaded squares	2^0, or 1	2^3, or 8	2^6, or 64	2^9, or 512	2^{3n}

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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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