How to support your student as they learn about

Extending Beyond Polynomials

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module your student will deepen their understanding of the rational and radical function families as they relate to polynomials. There are 2 topics in this module: *Rational Functions* and *Radical Functions*. Your student will use what they already know about polynomial functions and inverses in this module.

Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Estimate
Definition	• To make an educated guess based on the analysis of given data. • Estimating first helps inform reasoning.
Questions to Ask Your Student	• Does my answer make sense? • Is my solution close to my estimation?
Related Phrases	• Predict • Approximate • Expect • About how much?

It takes Maureen 90 minutes to water the garden. When Maureen and Sandra Jane work together, they can complete the job in 40 minutes. **About how long** should it take Sandra Jane to water it on her own?

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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Analyze mathematical relationships to connect and communicate mathematical ideas.

Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.
- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Problem Types You Will See:
Thumbs Up Thumbs Down

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connection between steps.

Ask Yourself

- Why is this method correct?
- Have I used this method before?

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself

- Where is the error?
- Why is it an error?
- How can I correct it?

Marissa

$$\frac{2x+2}{x+1} + \frac{1}{x} = \frac{2\cancel{(x+1)}}{\cancel{(x+1)}} + \frac{1}{x}$$
$$= 2 + \frac{1}{x}$$
$$= \frac{2(x)}{(x)} + \frac{1}{x}$$
$$= \frac{2x+1}{x}$$

2. $\frac{2}{3} + \frac{4}{x} = 1$

$$3x\left(\frac{2}{3} + \frac{4}{x}\right) = 1$$
$$2x + 12 = 1$$
$$2x = -11$$
$$x = -\frac{11}{2}$$

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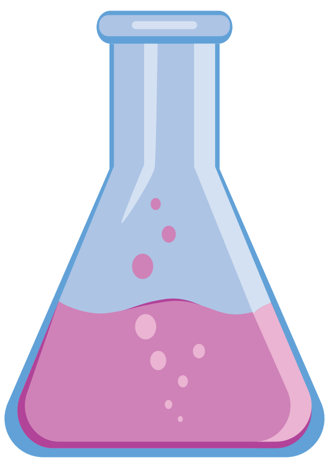
Module Overview

TOPIC 1	TOPIC 2
Rational Functions	Radical Functions
16 Days	17 Days
Your student will learn about creating rational functions, their key attributes, applying transformations, and how these functions are used in various types of problem situations.	**Your student will expand what they know about inverses from Module 1 to develop radical functions.**
Mixing it up! Chemistry is an exact science that doesn't always have the perfect ingredients. Diluting and concentrating ingredients is important to get correct and safe reactions. Rational equations are an important part of the pre-work! 	DId you know? Predicting tsunamis is not an exact science, but scientists use a variety of radical functions to quickly estimate the arrival time of any potential dangers so people can head for safety. 

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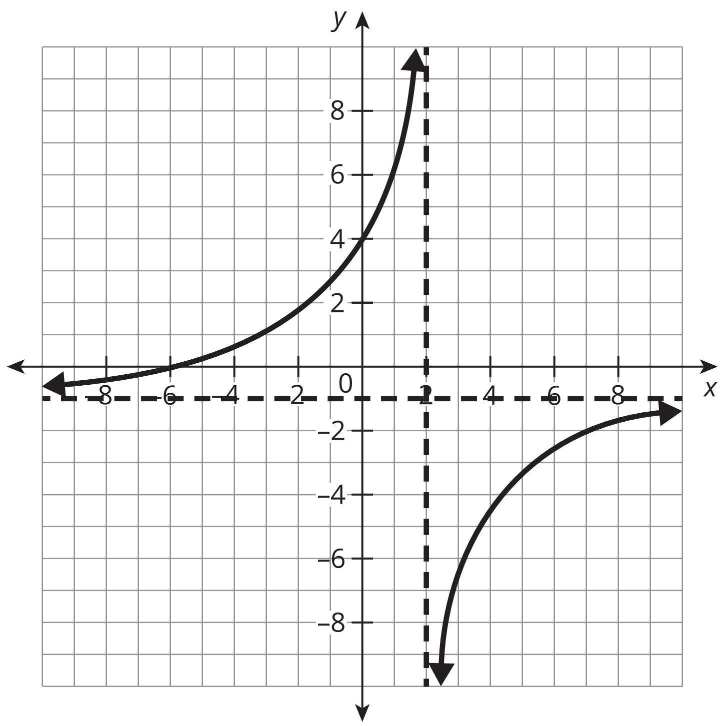
Topic 1: Rational Functions

Key Terms		
• rational function	• vertical asymptote	• rational equation

A **vertical asymptote** is a vertical line that a function gets closer and closer to, but never intersects. The asymptote does not represent points on the graph of the function. It represents the value that the graph approaches.

The graph has two asymptotes: a vertical asymptote $x = 2$ and a horizontal asymptote $y = -1$.



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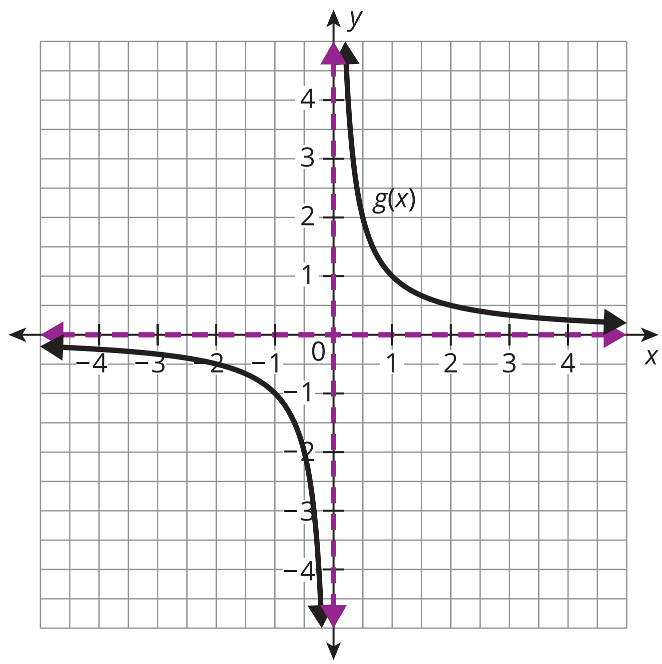
Rational Functions

A rational function is any function that can be written as the ratio of two polynomial functions. A rational function can be written in the form $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions and $Q(x) \neq 0$. These are similar to exponential functions in that they can have horizontal asymptotes, but they can also have vertical asymptotes, which will have an effect on the range.



Rational Equations

A rational equation is an equation that contains one or more rational expressions. Students will use different methods to solve equations depending on the operations used in the problem. Generally students will need previous skills learned, such as factoring polynomials, finding common denominators, and using proportional reasoning, to solve the equations they encounter.

Sidonie

$$\frac{6}{x^2 - 4x} + \frac{4}{x} = \frac{2}{x - 4}$$

$$\frac{6}{x(x - 4)} + \frac{4}{x} = \frac{2}{x - 4}$$

$x \neq 0, 4$

$$(x(x - 4)) \cdot \left[\frac{6}{x^2 - 4x} + \frac{4}{x} = \frac{2}{x - 4}\right]$$

$$6 + 4(x - 4) = 2x$$

$$6 + 4x - 16 = 2x$$

$$4x - 10 = 2x$$

$$2x = 10$$

$$x = 5$$

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TEXAS MATH SOLUTION

Applications of Rational Equations

Rational equations are used in many types of problems related to work, mixture, cost, and distance. Students will build tables to assist in building the equations that represent different situations. Below is an example of a mixture problem set up with a table.

Manuel is taking a college chemistry course, and some of his time is spent in the chemistry lab.

He is conducting an experiment for which he needs a 2% salt solution. However, all he can find in the lab is 120 milliliters (mL) of 10% salt solution. Calculate the amount of water Manuel would need to add to the 120 mL of 10% salt solution to make a 2% salt solution.

	Liquid	% Salt	Amount of Salt
Water	x	0	0
10% Solution	120	10	$0.10(120) = 12$
2% Solution	$120 + x$	2	$0.02(120 + x)$

MATH PROCESS STANDARDS

How do the activities in *Rational Functions* promote student expertise in the math process standards?

NOTE: This is an example of the math process standard:

Create and use representations to organize, record, and communicate mathematical ideas.

- I can create an understandable representation of a problem situation.

Have your student refer to page 2 for more "I can" statements.

	Portion of the Rink Completed	Time Spent Working	Rate of Work
	Rinks	Hours	$\frac{\text{Rinks}}{\text{Hour}}$
Anita's Team		x	
Martin's Team		x	
Entire Job, or 1 Rink		x	

Why is the variable x the same for Anita's Team and Martin's team?

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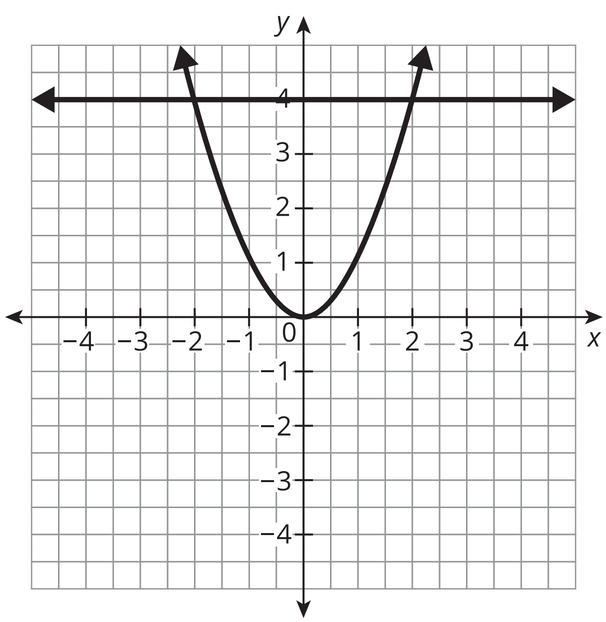
Topic 2: Radical Functions

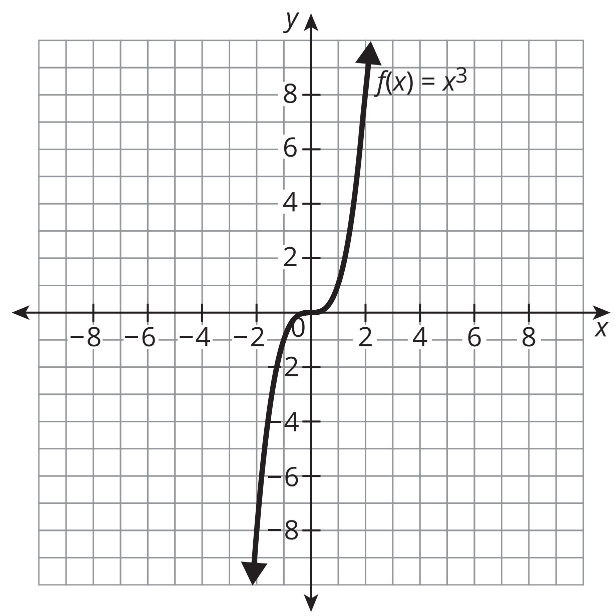
Key Terms	
• inverse of a function • invertible function • horizontal line test • square root function	• cube root function • radical function • composition of functions

The **Horizontal Line Test** is a test to determine if a function is one-to-one. To use the test, imagine drawing every possible horizontal line on the coordinate plane. If no horizontal line intersects the graph of a function at more than one point, then the function is one to one.

The function $y = x^2$ does not pass the Horizontal Line Test because a horizontal line can be drawn that intersects the graph at more than one point. So, the function is not one to one.



An **invertible function** is a function whose inverse exists. It is one-to-one and passes the Horizontal Line Test, so its inverse will also be a function.



The graph of $f(x) = x^3$ is an invertible function because it is one-to-one and passes the Horizontal Line Test. Therefore, its inverse will also be a function.

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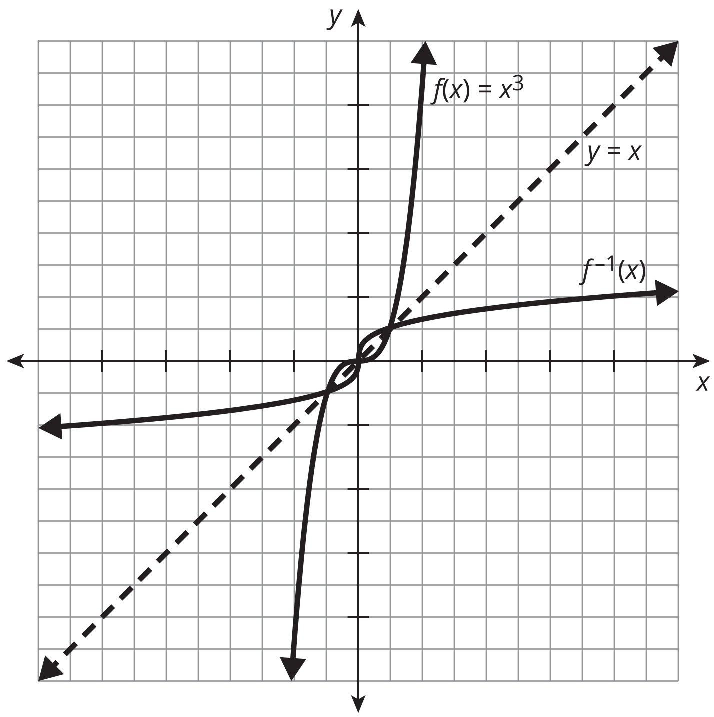
Inverse Functions

Building on polynomial functions, students will explore the functions that their inverses create. They have seen what inverses look like previously in Module 1, but they are expanded upon now that polynomial functions have been covered. The functions covered in this topic are radical functions, mostly square root and cube root functions.



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TEXAS MATH SOLUTION

Transformations and Inverses

Like all other functions, transformations are applied to radical functions in the same way as they have been for other function families. Because of how inverses relate to their parent functions in that they reflect across the line $y = x$, there is a pattern relating transformations to the inverse function and its transformations.

Transformation of Quadratic Function, $f(x)$	Transformation of Inverse Function, $f^{-1}(x)$
translation up D units	translation right D units
translation down D units	translation left D units
translation right C units	translation up C units
translation left C units	translation down C units

MATH PROCESS STANDARDS

How do the activities in *Radical Functions* promote student expertise in the math process standards?

NOTE: This is an example of the math process standard:

Analyze mathematical relationships to connect and communicate mathematical ideas.

- I can look closely to identify patterns or structure.

Have your student refer to page 2 for more "I can" statements.

Given: $f(x) = \sqrt{x + 4} + 7$ and $g(x) = (x - 7)^2 - 4$

How can you use transformations to prove that these two functions are inverses or not inverses of each other?

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Solving Radical Equations

Radical equations are solved with the same reverse operations that other equations have used. Similar to rational equations, extraneous solutions need to be accounted for by checking that any potential solutions are valid.

$$\sqrt{x+2}+10=x$$
$$\sqrt{x+2}=x-10$$
$$(\sqrt{x+2})^2=(x-10)^2$$
$$x+2=x^2-20x+100$$
$$0=x^2-21x+98$$
$$0=(x-14)(x-7)$$
$$x=14 \text{ or } x=7$$

There is one solution, $x = 14$.

Check:

$$\sqrt{14+2}+10 \stackrel{?}{=} 14$$
$$\sqrt{16}+10 \stackrel{?}{=} 14$$
$$14 = 14 \checkmark$$

Check:

$$\sqrt{7+2}+10 \stackrel{?}{=} 14$$
$$\sqrt{9}+10 \stackrel{?}{=} 14$$
$$13 \neq 14$$

Extraneous solution

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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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