TEXAS MATH SOLUTION

How to support your student as they learn about

Developing Structural Similarities

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module your student will deepen their understanding of the structure of polynomials and how to restructure them based on their needs. There are 2 topics in this module: *Relating Factors and Zeros* and *Polynomial Models*. Your student will use what they already know about factored polynomials, zeros, and combinations of objects in this module.

Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Represent
Definition	• To display information in various ways. • Representing mathematics can be done using words, tables, graphs, or symbols.
Questions to Ask Your Student	• How can you organize your thoughts? • How can you use a model to show an idea? • What does this representation mean to you? • Is your representation accurate?
Related Phrases	• Show • Sketch • Draw • Create • Plot • Graph • Write an equation • Complete the table

Create a factor tree to show the prime factorization of each number.

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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Problem Types You Will See: ***Worked Examples***

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connection between steps.

Worked Example

Factor $x^4 - 29x^2 + 100$ using quadratic form.

$x^4 - 29x^2 + 100$	Determine whether you can factor the given trinomial into 2 factors.
$(x^2 - 4)(x^2 - 25)$	Determine whether you can continue to factor each binomial.
$(x - 2)(x + 2)(x - 5)(x + 5)$	

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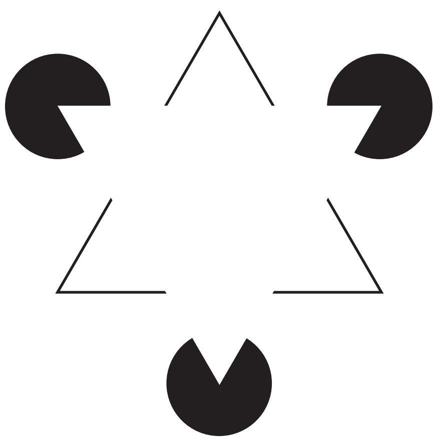
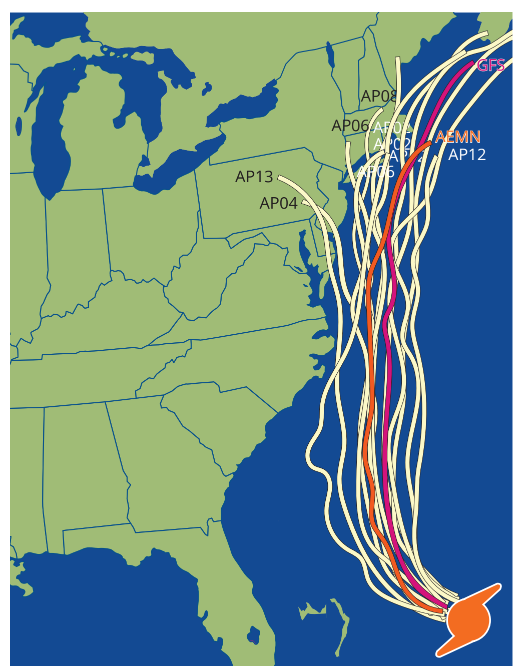
Module Overview

TOPIC 1	TOPIC 2
Relating Factors and Zeros	**Polynomial Models**
10 Days	8 Days
Your student will expand their ability to factor polynomials, find factors using polynomial division, and determine closure for polynomials.	**Your student will expand binomials using identities, Pascal's Triangle, and the Binomial Theorem. Then they will expand their use of regression to polynomials and determine the best model for sets of data.**
Did you know... The word closure can mean many things depending on the context. Closure may also refer to humans' ability to perceive objects as wholes, even when some of the parts are missing. Your brain fills in the missing parts. For example, you perceive a white triangle below, even though it is not drawn there at all. 	Hurricane prediction models Meteorologists use algorithms and data about the weather. Combined with supercomputers, they can make predictions about where deadly storms may travel in an effort to warn people days in advance, so they can prepare or leave if necessary. 

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Topic 1: Relating Factors and Zeros

Key Terms	
• Factor Theorem • polynomial long division • Remainder Theorem	• synthetic division • closed under an operation

The **Factor Theorem** states that a polynomial is divisible by $(x - r)$ if the value of the polynomial at r is zero.

The polynomial $x^3 - 2x^2 + 2x - 1$ is divisible by $x - 1$ because $(1)^3 - 2(1)^2 + 2(1) - 1 = 0$.

The **Remainder Theorem** states that the remainder when dividing a polynomial by $(x - r)$ is the value of the polynomial at r.

The value of the polynomial $x^2 + 5x + 2$ at 1 is $(1)^2 + 5(1) + 2 = 8$. So, the remainder when $x^2 + 5x + 2$ is divided by $x - 1$ is 8.

$$\begin{array}{r} x+6 \\ x-1\overline{)\,x^2+5x+2} \\ \underline{x^2-x} \\ 6x+2 \\ \underline{6x-6} \\ 8 \end{array}$$

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Factoring

Being able to factor polynomials is a skill that is used across many different levels of mathematics. Many different methods may be used to factor polynomials, and it is important to know each of them as some may be more efficient than others or may be the only possibility.

- Factoring out a GCF
- Chunking
- Special cases
- Factoring by grouping
- Factoring by using quadratic form

TEXAS MATH SOLUTION

MATH PROCESS STANDARDS

How do the activities in *Relating Factors and Zeros* promote student expertise in the math process standards?

<table>
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NOTE: This is an example of the math process standard:

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

- I can use a variety of different tools that I have to solve problems.

Have your student refer to page 2 for more “I can” statements.

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Given the polynomial $9x^2 + 21x + 10$, can you use more than one method to factor the polynomial into two binomials?

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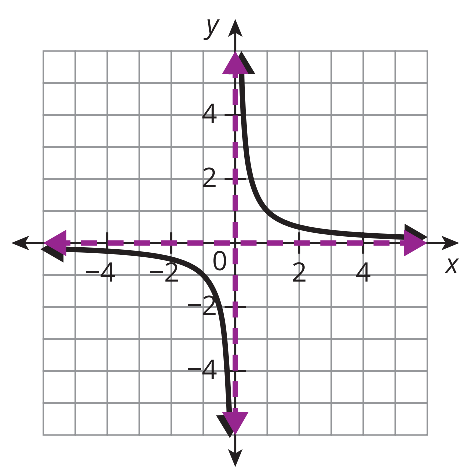
Polynomial Division

Just as polynomials are able to be added, subtracted, and multiplied together, they can be divided as well. Two methods used are long division and **synthetic division**. Long division is very similar to the method students used in elementary school to divide larger numbers. Synthetic division only works for linear divisors of the form $(x - r)$.

Long Division	Synthetic Division
$\begin{array}{r} x^2+0x+3 \\ x+1\overline{)\,x^3+x^2+3x+3} \\ \underline{-(x^3+x^2)} \\ 0x^2+3x \\ \underline{-(0x^2+0x)} \\ 3x+3 \\ \underline{-(3x+3)} \\ 0 \end{array}$	$r = -1$ $\begin{array}{r\|rrrr} -1 & 1 & 1 & 3 & 3 \\ & & -1 & 0 & -3 \\ \hline & 1 & 0 & 3 & \boxed{0} \end{array}$ multiply by r, add
$(x^3+x^2+3x+3) \div (x+1) = (x^2+3)$	

Closure

A set is **closed under an operation** if the operation is performed on any of the numbers in the set and the result is a number that is also in the same set. An example that students should already know is whole numbers are closed under addition because any two whole numbers added together produce another whole number. For example, you cannot add any two whole numbers to produce the number 8.5 or $1\frac{1}{3}$. This idea will be applied to polynomials where it should be discovered that polynomials are not closed under division and will use a graph to help justify.



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Topic 2: Polynomial Models

Key Terms	
• Euclid's Formula • Binomial Theorem	• regression equation • coefficient of determination
Euclid's Formula is a formula used to generate Pythagorean triples given any two positive integers. Given positive integers r and s, where $r > s$, Euclid's Formula is $(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$. Let $r = 3$ and $s = 1$. $(3^2 + 1^2)^2 = (3^2 - 1^2)^2 + (2 \cdot 3 \cdot 1)^2$ $10^2 = 8^2 + 6^2$ So, one Pythagorean triple is 6, 8, 10.	The **Binomial Theorem** states that it is possible to extend any power of $(a + b)$ into a sum of the form shown. $(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n-1} a^1 b^{n-1} + \binom{n}{n} a^0 b^n$ Use the Binomial Theorem to find the third term of $(x + y)^{20}$. $(x+y)^{20} = \binom{20}{2} x^{20-2} y^2 = \frac{20!}{18!2!} x^{18} y^2$ $= \frac{20 \cdot 19}{2 \cdot 1} x^{18} y^2 = 190 x^{18} y^2$
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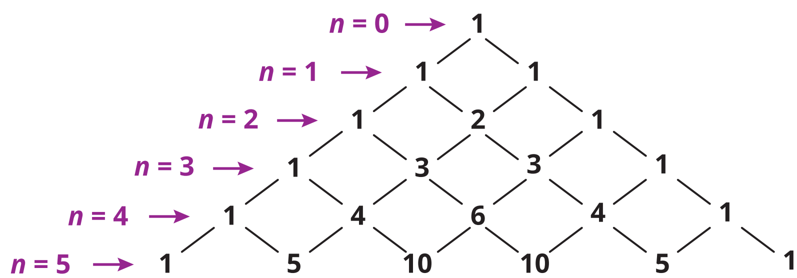
Special Cases

There are a set of special cases of differences and sums of both squares and cubes that factor the same way each time. Students should learn to recognize these special cases and their patterns for how to factor efficiently.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2)$$
$$(a-b)^3 = (a-b)(a^2 - 2ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

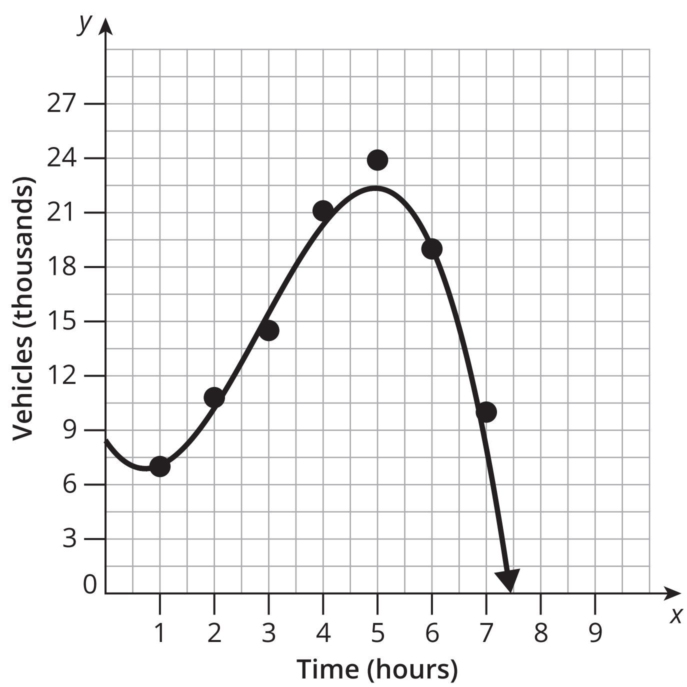
Pascal's Triangle

Pascal's Triangle is a mathematical pattern that can help with binomial expansion. The row number would be the exponent and the numbers within that row would represent the coefficients of each term of the expanded binomial. Building the triangle down 10 or 20 more rows would be very time consuming, so the pattern has been turned into an algorithm called the Binomial Theorem.



Modeling With Polynomials

Expanding on linear regression and quadratic regression, students can now model situations with polynomials of any degree and determine which is the best fit for a situation. Once they have chosen a model, they will then use the model to make predictions for the situation.



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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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