Introduction to Exponential Functions Summary

KEY TERMS

- power
- base
- exponent
- horizontal asymptote
- extracting square roots

LESSON 1

It's a Generational Thing

An expression used to represent the product of a repeated multiplication is a *power*. A **power** has a *base* and an *exponent*. The **base** of a power is the expression that is used as a factor in the repeated multiplication. The **exponent** of a power is the number of times that the base is used as a factor in the repeated multiplication.

You can write a power as a product by writing out the repeated multiplication.

$$2^7 = (2)(2)(2)(2)(2)(2)(2)$$

The power 2^7 can be read as:

- "two to the seventh power."
- "the seventh power of two."
- "two raised to the seventh power."

Parentheses can change the value of expressions containing exponents. When the negative sign is not in parentheses, it's not part of the base. For example, $-1^2 = -1$, but $(-1)^2 = 1$.

Properties of Powers	Words	Rule
Product Rule of Powers	To multiply powers with the same base, keep the base and add the exponents.	$a^m \cdot a^n = a^{m+n}$
Power to a Power Rule	To simplify a power to a power, keep the base and multiply the exponents.	$(a^m)^n = a^{mn}$
Quotient Rule of Powers	To divide powers with the same base, keep the base and subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$, if $a \neq 0$
Zero Power	The zero power of any number expect for 0 is 1.	$a^0 = 1$, if $a \neq 0$
Negative Exponents in the Numerator	An expression with a negative exponent in the numerator and a 1 in the denominator equals 1 divided by the power with its opposite exponent placed in the denominator.	$a^{-m} = \frac{1}{a^m}$, if $a \neq 0$ and $m > 0$
Negative Exponents in the Denominator	An expression with a negative exponent in the denominator and a 1 in the numerator equals the power with its opposite exponent.	$\frac{1}{a^{-m}} = a^m$, if $a \neq 0$ and $m > 0$

LESSON 2

Show What You Know

The properties of powers can be used to simplify numeric expressions.

For example, you can simplify the expression $\left(\frac{2^5}{2^4}\right)^3$.

$\left(\frac{2^5}{2^4}\right)^3 = (2^1)^3$ Quotient Rule of Powers

$= 2^3$ Power to a Power Rule

LESSON 3

A Constant Ratio

An exponential function is a function of the form $f(x) = ab^x$, where a and b are real numbers and b is greater than 0 but not equal to 1.

Geometric sequences with positive common ratios belong in the exponential function family. The common ratio of a geometric sequence is the base of an exponential function.

If a geometric sequence represents an exponential function, you can use the Product Rule of Powers and the definition of negative exponents to rewrite the explicit formula for the sequence as an exponential function.

For example, to represent $g_n = 45 \cdot 2^{n-1}$ using function notation, first rewrite it as $f(n) = 45 \cdot 2^{n-1}$. Next, rewrite the expression $45 \cdot 2^{n-1}$.

$f(n) = 45 \cdot 2^n \cdot 2^{-1}$	Product Rule of Powers
$f(n) = 45 \cdot 2^{-1} \cdot 2^n$	Commutative Property
$f(n) = 45 \cdot \frac{1}{2} \cdot 2^n$	Definition of negative exponent
$f(n) = \frac{45}{2} \cdot 2^n$	Multiply

So, $g_n = 45 \cdot 2^{n-1}$ written in function notation is $f(n) = \frac{45}{2} \cdot 2^n$.

The variable a in $f(x) = a \cdot b^x$ is the y-intercept, and b is the constant ratio.

LESSON 4

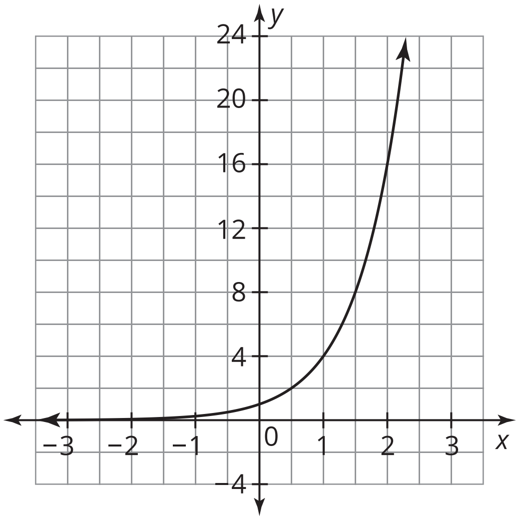
The Power Within

An exponential function is continuous, meaning that there is a value $f(x)$ for every real number value x. If the difference in the input values is the same, an exponential function shows a constant ratio between output values, no matter how large or how small the gap between input values. A constant ratio can be used to determine output values for integer and for non-integer inputs.

An exponential function has a **horizontal asymptote**, which is a horizontal line that a function gets closer and closer to but never intersects.

Consider the table and graph represented by the function $f(x) = 4^x$.

x	$f(x)$
−2	$\frac{1}{16}$
−1	$\frac{1}{4}$
0	1
1	4
2	16



There are no x-intercepts. The y-intercept is at (0, 1). The horizontal asymptote is $y = 0$. The domain is all real numbers, and the graph increases over the entire domain. The range is $y > 0$.

A rational exponent is an exponent that is a rational number. You can write each nth root using a rational exponent. If n is an integer greater than 1, then $\sqrt[n]{a} = a^{\frac{1}{n}}$.

For example, $\sqrt[4]{b} = b^{\frac{1}{4}}$ and $6^{\frac{1}{5}} = \sqrt[5]{6}$.

Write expressions with rational exponents in radical form using the known properties of integer exponents. Write the power as a product using a unit fraction. Use the Power to a Power Rule rule and the definition of a rational exponent to write the power as a radical.

For example, consider the expressions $8^{\frac{2}{3}}$ and $(\sqrt[7]{c})^3$.

$$8^{\frac{2}{3}} = 8^{\left(\frac{1}{3}\right)(2)} = (\sqrt[3]{8})^2$$

$$(\sqrt[7]{c})^3 = c^{\left(\frac{1}{7}\right)(3)} = c^{\frac{3}{7}}$$

Rewrite radical expressions with an index of 2 by **extracting square roots**. This is the process of removing perfect squares from under the radical symbol.

For example, consider the irrational number represented by the radical expression $\sqrt{40}$.

$$\begin{aligned}\sqrt{40} &= \sqrt{(4 \cdot 10)} \\ &= \sqrt{4} \cdot \sqrt{10} \\ &= 2\sqrt{10}\end{aligned}$$

The product $2\sqrt{10}$ is an irrational number. The product of a nonzero rational number and an irrational number is always an irrational number, but the product of two irrational numbers can be irrational or rational.