Quantities and Relationships Summary

KEY TERMS

- dependent quantity
- independent quantity
- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute maximum
- absolute minimum
- quadratic functions
- linear absolute value functions
- *x*-intercept
- *y*-intercept

LESSON 1

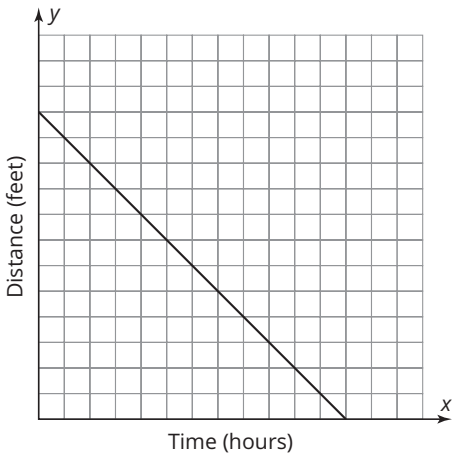
A Picture Is Worth a Thousand Words

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity.** The quantity that the dependent quantity depends upon is called the **independent quantity.**

Graphs relay information about data in a visual way. Connecting points on a coordinate plane with a line or smooth curve is a way to model or represent relationships. The independent quantity is graphed on the horizontal, or *x*-axis, while the dependent quantity is graphed on the vertical, or *y*-axis. Graphs can be straight lines or curves, and can increase or decrease from left to right.

For example, consider the graph which models the situation where Pedro is hiking in a canyon. At the start of his hike, he was at 3500 feet. During the first 20 minutes of the hike, he descended 500 feet at a constant rate. Then he rested for half an hour before continuing the hike at the same rate.

Time is the independent quantity and distance is the dependent quantity.



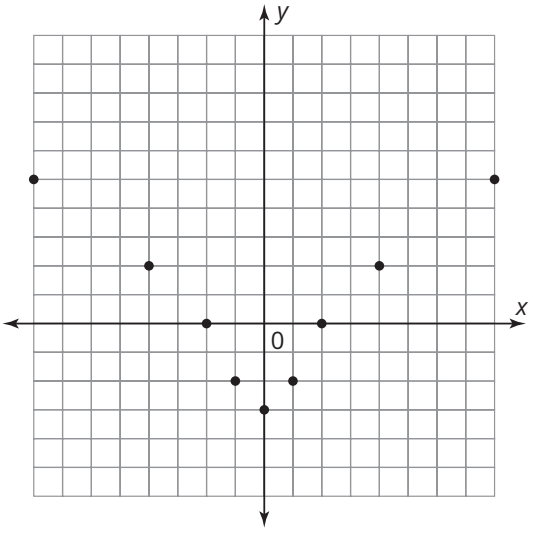
LESSON
2

A Sort of Sorts

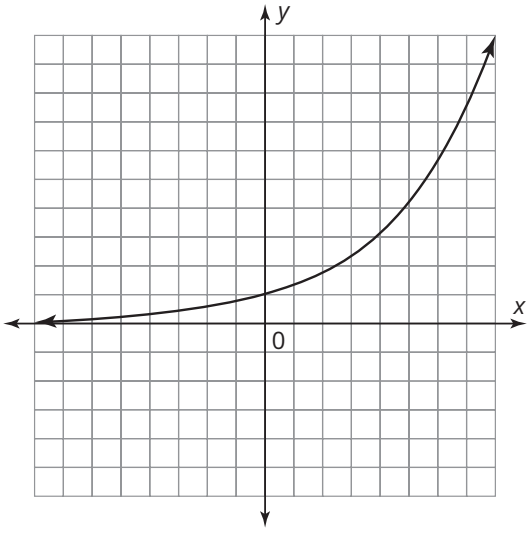
Looking for patterns can help when sorting and comparing graphs. Some graphs show vertical symmetry (if a vertical line were drawn through the middle of the graph, the image is the same on both sides). Other possible patterns to look for include: only goes through two quadrants, always increasing from left to right, always decreasing from left to right, straight lines, smooth curves, the graph goes through the origin, the graph forms a U shape, the graph forms a V shape.

For example, Graph A has vertical symmetry. Graph B is a smooth curve that increases from left to right.

Graph A



Graph B



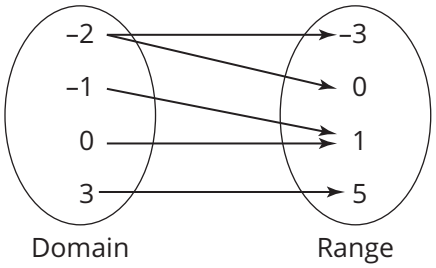
LESSON
3

F of X

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

A **function** is a relation between a given set of elements, such that for each element in the domain there exists exactly one element in the range. If each value in the domain has one and only one range value, then the relation is a function. If any value in the domain has more than one range value, then the relation is not a function.

The value −2 in the domain has more than one range value. The mapping does not represent a function.



Each element in the domain has exactly one element in the range. The table represents a function.

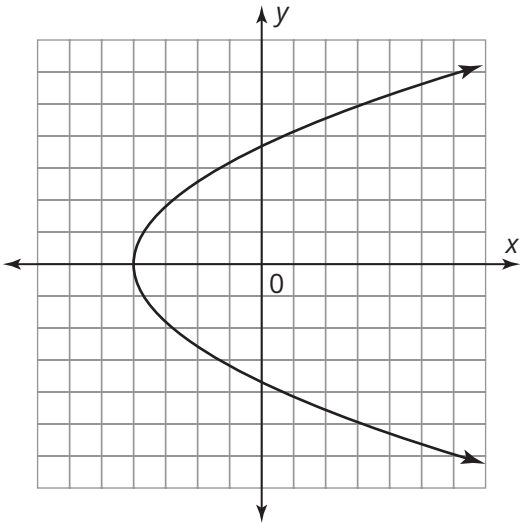
Domain	Range
2	1
6	3
10	5
14	7

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function $f(x)$ is read as "f of x" and indicates that x is the independent variable.

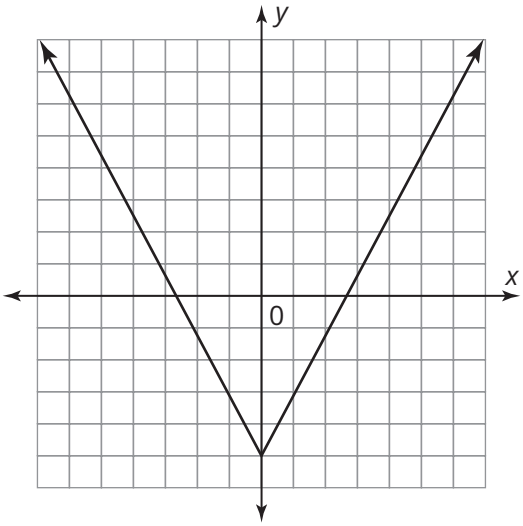
For example, consider the situation in which U.S. Shirts charges $8 per shirt plus a one-time charge of $15 to set up a T-shirt design. The equation that models the situation, $y = 8x + 15$, where x represents the number of shirts ordered and y represents the total cost of the order, can be written in function notation as $f(x) = 8x + 15$. The cost, defined by f, is a function of x, defined as the number of shirts ordered.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function. The Vertical Line Test applies to both discrete and continuous graphs. A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve with no breaks in the graph.

A line drawn vertically through the graph touches more than one point. The graph does not represent a function.



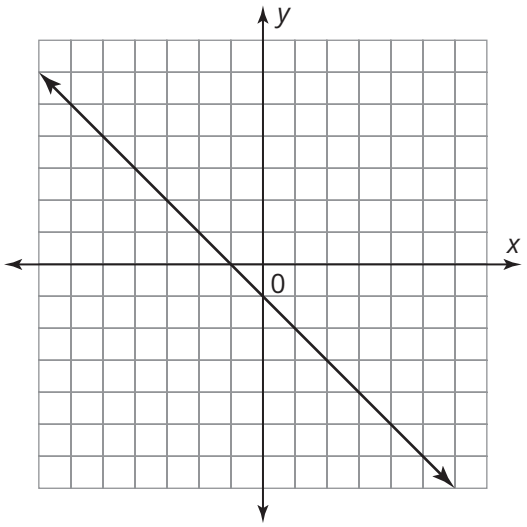
A line drawn vertically through the graph only touches one point. The graph represents a function.



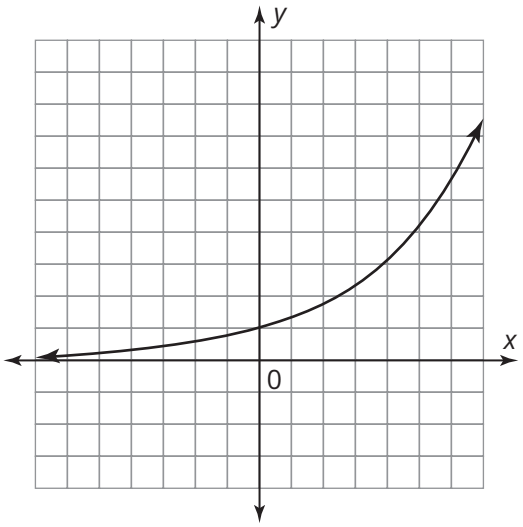
A function is described as increasing when both the independent and dependent variables are increasing. If a function increases across the entire domain, then the function is called an **increasing function**. A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.

A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

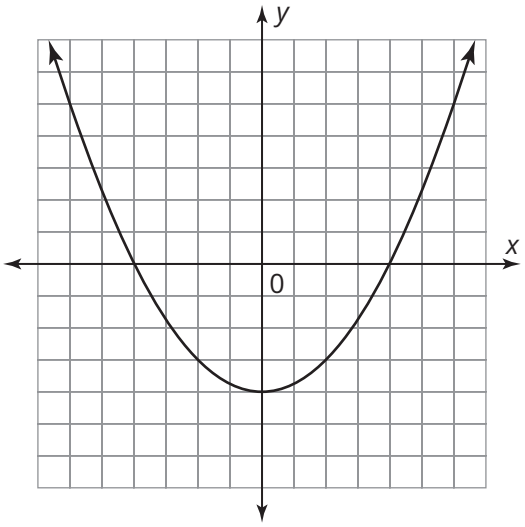


The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a, b, and c are real numbers, and b is greater than 0, but not equal to 1.

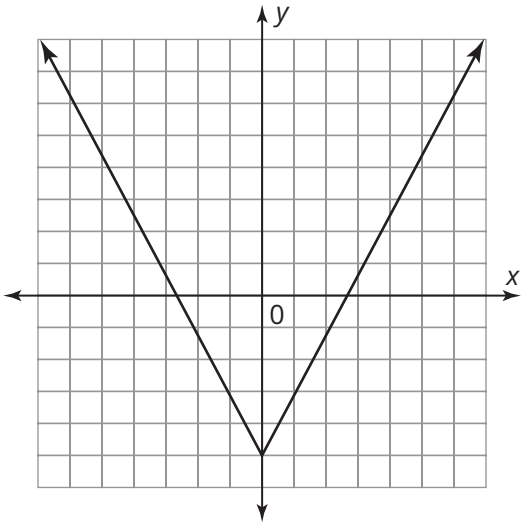


Quadratic and linear absolute value functions have an absolute maximum or an absolute minimum. An **absolute maximum** is a point on the graph of the function that has a y-coordinate that is greater than the y-coordinate of every other point on the graph. An **absolute minimum** is a point on the graph of the function that has a y-coordinate that is less than the y-coordinate of every other point on the graph.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.



The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.



LESSON
4

Function Families for 2000, Alex

Function families have key characteristics that are common among all functions in the family. Knowing these key characteristics is useful when sketching a graph of the function. For example, to sketch a graph of a continuous quadratic function with a maximum, the graph should be a smooth, U-shaped curve that increases to point and then decreases again.