LESSON 24

Frequency Distributions

Key Concept

This lesson introduces discrete and continuous random variables. Then students create a relative frequency histogram. This leads to a more general examination of probability distributions and frequency distributions, followed by an introduction to the key properties of normal distributions.

Key Question: Example 1

Can a discrete random variable take on values that are not whole numbers? If so, give an example. If not, why not?

Yes; for example, the shoe size of a randomly selected person at a bowling alley is a discrete random variable and the variable can take on the value $9\frac{1}{2}$.

Activity

- **Materials** number cubes
- **Goal** Students roll two number cubes 50 times and find the relative frequency of each sum. Then they use the results to make and analyze a relative frequency histogram.
- **Teaching Strategy** Connect this activity to what students have already learned about probability. In particular, help students see that the relative frequencies that they calculate in Step 2 are simply the experimental probabilities of each sum.
- **Key Discovery** The shape of a relative frequency histogram tells you about the underlying distribution. In this case, the majority of students' histograms will show that 7 is the most likely sum, with sums of 6 and 8 the next most likely, and so on.

Key Question: Example 2

How is this probability distribution histogram related to the relative frequency histogram from the preceding activity?

As you add more and more trials to the experiment in the activity, the relative frequency histogram approaches the theoretical results in the probability distribution histogram.

Key Question: Example 3

What is the median height of adult women in the United States? How is your answer related to the shape of the normal curve?

64 inches; because the normal curve is symmetric, half the values lie below the mean and half the values lie above the mean. Therefore, the mean of 64 inches is also the median.

Key Question: Example 4

What can you conclude about Justin's and Ann's heights based on their *z*-scores? Explain.

Ann's *z*-score has a smaller absolute value than Justin's, therefore her height is closer to the mean than Justin's. You can conclude that Justin's height is less common for an adult man than Ann's height is for an adult woman.

Avoiding Common Errors

Example 4 When calculating the *z*-score for a value that is less than the mean, students might subtract the value from the mean. Tell students they should always subtract the mean from the value, even if this results in a negative *z*-score. Point out that the sign of the *z*-score tells which side of the mean the value lies on.

Closing the Lesson

Have students answer the following question:
What are some things you can say about data that are normally distributed with a mean of m and a standard deviation of s?

68% of the data are between $m - s$ and $m + s$; 95% of the data are between $m - 2s$ and $m + 2s$; 99.7% of the data are between $m - 3s$ and $m + 3s$; half the data are less than m; half the data are greater than m; etc.

Homework Help

Example 1: Exs. 1–4

Activity: Exs. 13–18

Example 2: Exs. 5, 6

Example 3: Exs. 19–21, 23–28

Example 4: Exs. 22, 29–33

Enrichment: Exs. 7–12, 34–40

Homework Check

To quickly check student understanding of key concepts, go over the following exercises: 2, 5, 6, 19, 20, 31.

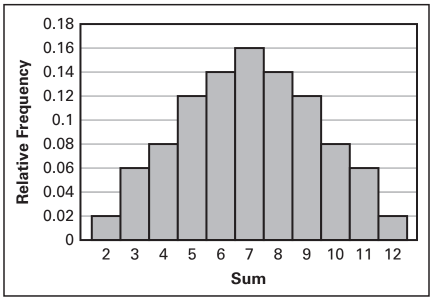
ANSWERS

Activity Answers

1 and 2. *Sample answer:*

Sum	Frequency	Relative Frequency
2	1	0.02
3	3	0.06
4	4	0.08
5	6	0.12
6	7	0.14
7	8	0.16
8	7	0.14
9	6	0.12
10	4	0.08
11	3	0.06
12	1	0.02

3. *Sample answer:*



4. *Sample answer:* Although the relative frequencies of specific sums may differ from histogram to histogram, the sum of all the relative frequencies in every histogram is 1. Most of the relative frequency histograms will be mound-shaped.

Check Answers

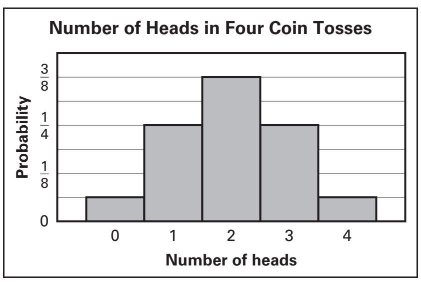
1. continuous

2. discrete

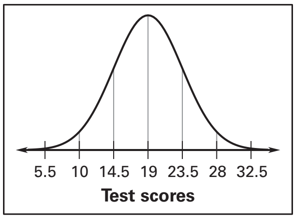
3.

Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

4.



5.



6. 14.5 and 23.5

7. 95%

8. $1.\overline{3}$

Exercise Answers

1. discrete

2. continuous

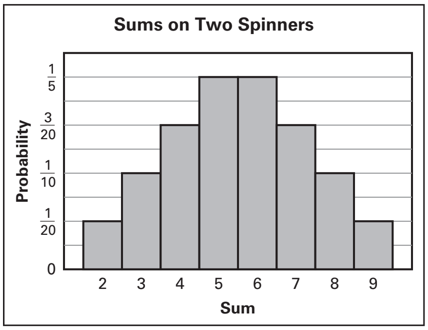
3. discrete

4. continuous

5.

Sum	2	3	4	5	6	7	8	9
Probability	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{1}{20}$

6.



7. skewed

8. mound-shaped

9. mound-shaped

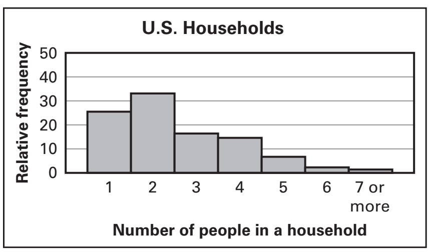
10. uniform

11. bimodal

12. skewed

13. 1.4%

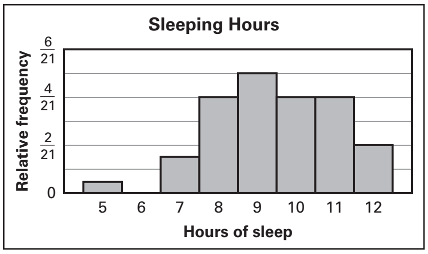
14.



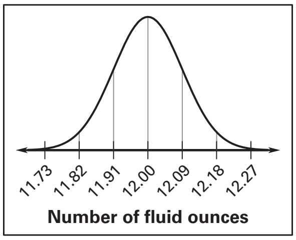
15. skewed

16.

Hours	5	6	7	8	9	10	11	12
Freq.	1	0	3	8	10	8	8	4
Relative Freq.	$\frac{1}{42}$	0	$\frac{1}{14}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{4}{21}$	$\frac{2}{21}$

17. 

18. mound-shaped

19. 

20. 11.82 fl oz and 12.18 fl oz

21. 50%

22. $-3.\overline{3}$

23. 68%

24. 84%

25. 99.7%

26. 34%

27. 16%

28. 0.3%

29. 8 points

30. 100 points

31. about 1.9

32. $1.\overline{1}$

33. Laura; her score was about 1.9 standard deviations above the mean, while Max's was $1.\overline{1}$ standard deviations above the mean.

34. **a.** 2000 hours; the normal curve is symmetric about the mean, so the mean is also the median.

b. 2000 hours; the peak of the normal curve occurs at the mean, so the mean is also the mode.

35. 0.3%

36. The *z*-score is less than −3 or greater than 3.

37. 2.75; −3.25; the bolt that is 3.32 cm long is an outlier since its *z*-score is less than −3.

38. Driving Test: mound-shaped; New School: skewed right; College: skewed left

39. Driving Test: mean, median, mode: 16; range: 6; New School: mean: approx. 14.8; median, mode: 14; range: 6; College: mean: approx. 17.2; median: 17.5; mode: 18; range: 6

40. Driving Test: the three measures coincide at the center of the distribution; New School: mean is to the right of the median and mode; College: mean is to the left of the median, which is to the left of the mode.