Developing and Assessing Addition Fact Fluency

Gina Kling and Jennifer Bay-Williams
NCTM Annual Meeting
April 17, 2015
Our Session

• Overview of basic fact fluency
• Games to support basic fact fluency
• Assessment overview
• Assessment strategies for basic fact fluency
• Results of a strategy-based approach to learning basic facts
What do we mean by fluency with basic facts?
CCSS-M Descriptions

Grade 1 (1.0A.C.6):
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.0A.B.2):
Fluently add and subtract within 20 using mental strategies (reference to 1.0A.C.6). By end of Grade 2, know from memory all sums of two one-digit numbers.
CCSS-M Descriptions

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Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.0A.B.2):
Fluently add and subtract within 20 using mental strategies (reference to 1.0A.C.6). By end of Grade 2, know from memory all sums of two one-digit numbers.
This would suggest that fluency is different from automatic retrieval. Research heavily supports this…

So, what does fluency really mean?
The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO, 2010, p. 6).
Procedural Fluency

Knowing from Memory ≠ Memorization
Developing Addition Fact Fluency
Mastering Basic Facts

Phase 1: Counting
(counts with objects or mentally)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery
(efficient production of answers)

Adapted from Baroody, 2006
Mastering Basic Facts

Phase 1 Example: Solving $5 + 7$ by counting on from 5.

Phase 2 Example: Solving $5 + 7$ by starting from $5 + 5 = 10$ and then adding 2 more to get 12.

Phase 3 Example: Answering $5 + 7 = 12$ with little/no hesitation.
Reasoning Strategies for Addition Facts
K-2 Trajectory

Foundational Fact Strategies
- Sums within 5  K
- +/- 1 or 2  K
- Doubles  K 1
  (2 + 2, 6 + 6, etc.)
- Combinations of Ten
  (3 + 7, 8 + 2)  K 1

Derived Fact Strategies
- Near Doubles  1 2
  (6 + 7, 8 + 7)
- Making Ten  1 2
  (8 + 3, 9 + 5)
## Basic Fact Fluency: A 3-yr. Progression

<table>
<thead>
<tr>
<th>K</th>
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<th>2</th>
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<tr>
<td>Sums within 5</td>
<td>Doubles</td>
<td>Near Doubles</td>
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<tr>
<td>+/- 1</td>
<td>Combo’s of 10</td>
<td>Making 10</td>
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<td>+/- 2</td>
<td>Sums within 10</td>
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Meaningful Activities for Learning Basic Facts

- Solve Number Stories
- Use Quick Looks with dot patterns and ten frames
- Discuss/write about strategy use
- Play basic facts games with a focus on reasoning strategies

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Story Telling

(a) Join

Initial → Change → Result

(b) Separate

Initial → Change → Result

(c) Part-part-whole

Part + Part = Whole

(d) Compare

Large set → Difference → Small set

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Phase 1 → Phase 2

Quick Images

PreK:
- ✓ number recognition
- ✓ representation

Kindergarten:
- ✓ decomposing and recomposing numbers
- ✓ subitizing

Grade K- 2:
- ✓ recognizing and developing strategies for basic +/− combinations
Developing Fact Strategies through Quick Looks

What arrangements of dots on double ten frames could encourage strategies such as near doubles and making ten?
Developing Fact Strategies through Quick Looks

Teachers need to help children transition from developing strategies with Quick Looks to symbolic notation for strategies…
“Practice that follows substantial initial experiences that support understanding and emphasize ‘thinking strategies’ has been shown to improve student achievement with single-digit calculations.” (NRC, 2001).
Games Across Phases

- Close to 20
- Double It
- Name that Number
- Roll and Total
- Tens Go Fish
- Top-It
Games as Meaningful Practice

As you explore each game, consider the following:

• What is the mathematical content of the each game?

• What makes these games examples of “meaningful practice”?

• What questions do you have about these games?

• Which games might work best with your students?
Where does each game fit in?

Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Meaningful Practice

Games:

• Are engaging.
• Provide opportunities for strategy discussion and assessment.
• Should be sequenced developmentally (for example, playing combinations of ten games before exploring making ten strategies).
• Can be targeted practice or general practice.
• Lend to differentiation.
November 2014 article discusses these games.
NCTM NEW Book with more Games

Mastering Basic Math Skills

Developing and Assessing Addition Fact Fluency

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Assessing Addition Fact Fluency
The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO, 2010, p. 6).
Assessing Basic Fact Fluency

What can we learn from this assessment related to:
- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use
Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

Timed Tests

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<td>10 + 1 =</td>
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Timed Testing: Issues

The issues with timed testing include:

1) Limitations as an assessment tool
2) Can impede progress when mastering facts
3) Psychological effects
Timed Testing: Issues

1) Limitations with respect to the four components of fluency.

A child finishes a 20-fact timed test in 60 seconds.

• Did the child spend 3 seconds on each fact?

  *Or...*

• Did the child spend 1 second on 16 facts and 10 seconds each on 4 of the facts?
Timed Testing: Issues

2) Can impede progress in mastering facts

A study of nearly 300 first graders found that children who were more frequently exposed to timed testing demonstrated lower progress towards knowing facts from memory than their counterparts.

Henry & Brown, 2008
Timed Testing: Issues

2) Can impede progress in mastering facts

A study of 2\textsuperscript{nd} and 4\textsuperscript{th} graders showed that children in experimental classrooms with a focus on strategy development vastly outperformed those in the control classrooms, even on traditional timed assessments.

Thornton, 1978
Timed Testing: Issues

3) Can have negative psychological effects

- The stress that children experience with timed testing is not experienced when they complete the same tasks in untimed conditions.
- “Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range.”

Boaler, 2014
3) Can have negative psychological effects

Anxiety over timed testing is often not related to achievement. Even high-achieving children share concerns such as “I feel nervous. I know my facts, but this just scares me.”

Boaler, 2012
Timed Testing: Issues

3) Can have negative psychological effects

Children experience math anxiety as early as first grade and this anxiety is not correlated with reading achievement. This suggests that the children’s anxiety is specific to mathematics, not general academic work.

Ramirez et al. 2013
Timed Testing: Issues

3) Can have negative psychological effects

Children who tended to use more sophisticated mathematical strategies experienced the most negative impact on achievement due to math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively impacted by timed testing.

Ramirez et al. 2013
Timed Tests: Alternatives

- Observation
- Interviews
- Writing prompts
- Strategy quizzes
- Self-assessment
Salute!
Observation Checklist

Addition Facts Fluency Chart

<table>
<thead>
<tr>
<th>Student</th>
<th>Models and counts all</th>
<th>Counts on Derived Fact</th>
<th>Recall (double or combo of 10)</th>
<th>Recall</th>
<th>Comments</th>
</tr>
</thead>
</table>

Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use
<table>
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<th>Flexibility</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve $6 + 7$ using one strategy. Now try solving it using a different strategy.</td>
<td>What is the answer to $7 + 8$? How do you know it is correct (how might you check it)?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>Appropriate Strategy Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>For which facts did you <strong>just know</strong>?</td>
<td>Emily solved $6 + 8$ by changing it in her mind to $4 + 10$. What did she do? Is this a good strategy?</td>
</tr>
<tr>
<td>For which facts did you <strong>use a strategy</strong>?</td>
<td>Tell why or why not.</td>
</tr>
</tbody>
</table>
Timed Tests: Alternatives

Interviews: “Riley” at the end of Grade 1

At first, Riley incorrectly answers that $8 + 7 = 17$.

- Which components of fluency were you able to assess through the follow-up questions?

- How does this assessment experience compare to traditional facts assessments?
Timed Tests: Alternatives

Interviews: “Riley” at the end of Grade 2

Riley solves $5 + 9$ in one second.

- Which components of fluency were you able to assess through the follow-up questions?

- How does this assessment experience compare to traditional facts assessments?
Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

Journal Writing

If your friend didn’t know the answer to $4 + 5$, how would you tell him to figure it out?
Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

Journal Writing

Review the four student responses:

What might you infer about each child’s level of fluency?
Timed Tests: Alternatives
Journal Writing Samples

May 10, 2012
I would tell my friend
to take 5 and
count 4 in your hand
I would tell my friend to start with 5 then add 2 then one more 2 and then you have 9.
I would tell my friend to pose a double plus 1. \(4+4=8\), so count 1 up. Now you get your answer.
Timed Tests: Alternatives
Journal Writing Samples

I would tell my friend to take away one number from ten. And that is nine. I know that five plus five equals ten.
Addition Fact Fluency Quiz

Solve these problems and tell how you solved it.

4 + 5 = ______  Check one:  ____ I used this strategy: ______________
____ I just knew.

10 + 6 = ______  Check one:  ____ I used this strategy: ______________
____ I just knew.

6 + 2 = ______  Check one:  ____ I used this strategy: ______________
____ I just knew.
Assessing Basic Fact Fluency

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children’s fact fluency.

By Gina Kling and Jennifer M. Bay-Williams
What Works with Addition Fact Fluency
Promising Results

Kling (2013) followed 30 children from 2 different schools, 4 different classrooms, in Kalamazoo, MI.; 21 had no exposure to timed testing or drill in the classroom in either 1st or 2nd grade. By the end of 2nd grade those 21 children demonstrated:

- automaticity with addition facts (solved within 3 seconds) 95% of the time.
- Strategy use (e.g., making ten) so quickly that it was impossible to distinguish between strategy use and “knowing from memory.”
… and RETENTION

18 of the children were interviewed once more in the first week of 3rd grade prior to any fact strategy review.

- Children demonstrated automaticity 91% of the time.
- Were accurate and used strategies (not counting) 99.99% of the time.
Conclusions

Basic facts instruction, practice, and assessment must truly encompass all four components of fluency:

- **Flexibility**
- **Accuracy**
- **Efficiency**
- **Appropriate strategy use**

Traditional approaches to teaching and assessing basic facts do not support these goals. But when *fluency* is the focus, children can achieve meaningful mastery of basic facts.
Assessing Your Fact [teaching] Fluency
Whip Around

At your table, share your answer to one of these prompts:

• An activity I will use is…
• An assessment strategy I will use is…
• Something surprising I heard is…
• I am going to/not going to…
Bibliography


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