

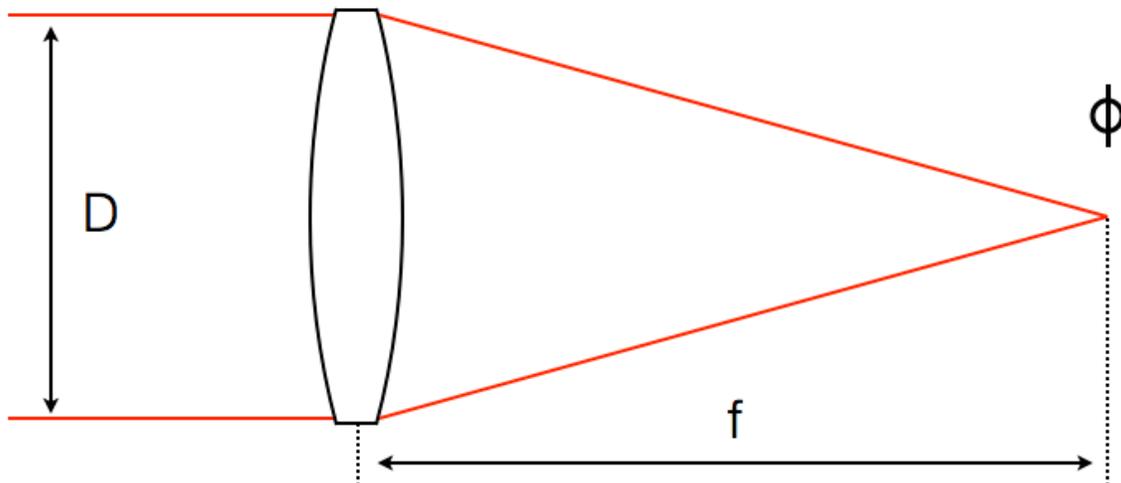
Aperture and F-number

The Math Behind the Number

Electromagnetic radiation is made up of photons, elementary particles that exhibit properties of waves and particles. Acting as a wave, a photon travels through space at the speed of light with a specific frequency that defines its energy and, therefore, its classification along the electromagnetic spectrum. Our eyes can only detect a very specific frequency range which makes up the visible spectrum (remember Roy G. Biv?). On the other hand, a photon acts as a particle such that the number of photons available determines the amount of light, and therefore the brightness, of a particular object emitting or reflecting the photons. Twice as many photons will appear twice as bright, or in photographic terms, “one stop” brighter.

Shutter speed is somewhat easy to understand in these terms. Double the shutter speed (say, from 1/2 second to 1 second) to double the amount of time that the sensor is exposed and consequently double the number of photons reaching the film or sensor. The F-number (also known as f-stop or aperture value), related to the size of the aperture, is not as simple.

A camera lens contains a diaphragm that restricts the amount of light reaching the film plane in a similar way to the iris of the human eye. Whereas the hole that lets in light in the middle of the eyes’ iris is the pupil, the hole in the middle of a diaphragm of a lens is the aperture.



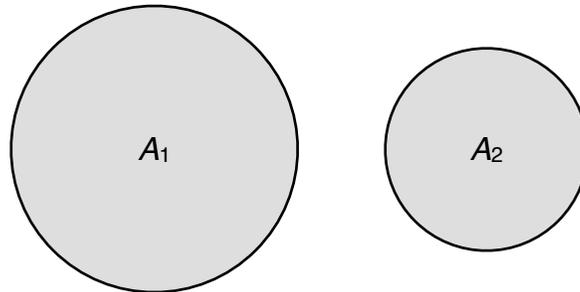
$$F\text{-number} = \frac{f}{D}$$

If the area of the aperture is doubled, the number of photons that can reach the sensor through the lens is also doubled. The F-number of a lens is the ratio of its focal length divided by the diameter of the aperture. Since the F-number is a ratio involving the diameter, and not the area, we lose the ability to nicely double or halve a number to calculate a stop.

Notice that, counterintuitively, higher F-numbers indicate that less light travels through the lens. To convince yourself of this, imagine what would happen to the F-number if the diameter decreases while the focal length remains the same or while the focal length increases while the diameter is the same. Also realize that the diagram above is an oversimplification of lenses. Almost all modern lenses are made up of more than one glass element, and it is not necessarily the very front element that contains the smallest diameter. An element inside of the lens may be a little bit smaller, for example, but this does not affect the focal length.

Back on topic. Doubling or halving the area of the aperture respectively doubles or halves the amount of light entering the lens. It should be clear that the F-number is related to the diameter of the lens and not the area. The question therefore arises: which F-numbers constitute a one stop difference?

To answer this question, let's assume we have two separate lenses that have areas A_1 and A_2 , shown below.



A_1 is twice the area of A_2 , and we will assume that the diameter (d_1) and focal length (f) of A_1 is 1, so the F-number would be 1. Let us further assume that the focal length of the lenses are the same. To figure out the F-number of A_2 , we need to determine how much smaller the diameter is compared to that of A_1 . Let's do the math!

$$\text{Area of a circle} = \pi \cdot r^2 = \pi \cdot \left(\frac{d}{2}\right)^2 \quad r = \frac{\text{diameter}}{2}$$

$$A_1 = \pi \left(\frac{d_1}{2}\right)^2 = \pi \left(\frac{(d_1)^2}{4}\right) = \frac{\pi \cdot (d_1)^2}{4}$$

$$A_2 = \pi \left(\frac{d_2}{2}\right)^2 = \pi \left(\frac{(d_2)^2}{4}\right) = \frac{\pi \cdot (d_2)^2}{4}$$

$$A_1 = 2 \cdot A_2$$

$$\frac{\pi \cdot (d_1)^2}{4} = 2 \cdot \frac{\pi \cdot (d_2)^2}{4}$$

$$\frac{\pi \cdot (d_1)^2}{4} = \frac{2 \cdot \pi \cdot (d_2)^2}{4}$$

$$\left(\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)$$

$$(d_1)^2 = 2 \cdot (d_2)^2$$

$$d_2 = \sqrt{(d_1)^2 \cdot \frac{1}{2}} = (d_1) \cdot \frac{\sqrt{1}}{\sqrt{2}} = d_1 \cdot \frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = d_1 \cdot \frac{\sqrt{2}}{2}$$

This means, for for A_2 to be half the area of A_1 , the diameter d_2 must be $\frac{\sqrt{2}}{2}$ times d_1 . We know that the diameter d_1 is 1, so the diameter d_2 is $\frac{\sqrt{2}}{2}$ which is approximately equal to 0.70. Let's continue with the calculations.

$$F\text{-number} (A_2) = \frac{f}{D} = \frac{f}{d_2} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} = 1.4$$

The F-number for A_2 , then, is 1.4. Note that even though the area A_2 is half the area of A_1 , the diameter d_2 is about 0.7 times as much as d_1 , so the F-number is affected similarly.

As an exercise, try doing the same thing with A_2 and a third circle with area A_3 which is equal to half of the area A_2 . If you keep applying this math for halved areas, you get a list of F-numbers that are one stop apart:

F-numbers: f/1.0, f/1.4, f/2.0, f/2.8, f/4.0, f/5.6, f/8, f/11, f/16, f/22, f/32, f/44, f/64, etc.

These numbers may be difficult to memorize, so a trick is to remember the first two F-numbers (1.0 and 1.4) and double each; these are the next two F-numbers. Like so:

┌── x2 ─┐ ┌── x2 ─┐ ┌── x2 ─┐
F-numbers: f/1.0, f/1.4, f/2.0, f/2.8, f/4.0, f/5.6, f/8, f/11, etc.
 └── x2 ─┘ └── x2 ─┘ └── x2 ─┘

Diameter of the Pinhole

It is possible to calculate the diameter of an aperture, including that of a pinhole, using the F-number ratio. In order to calculate the diameter two values must be known: the F-number and the focal length.

The focal length is easy to calculate. Measure the distance perpendicular from the focal plane to the end of the pinhole. The location of the film plane on an SLR is indicated by the focal plane icon printed or embossed on the body.

In the case of the pinhole used in class, the focal length is 50mm.

Next, the F-number of the pinhole must be calculated. This can be calculated by determining the number of stops less light that enters the camera. Set up the camera on a tripod with a lens with a known aperture value and take a photo at ISO 100 at a shutter speed the camera recommends. Maintaining the same lighting conditions, take photos of various lengths with the pinhole until one is found that matches the exposure of that taken with the lens.

For this calculation, the lens had an F-number of $f/2.0$ and the camera reported a shutter speed of $1/60$ second. After many attempts, the pinhole exposure at ISO 100 was measured at 70 seconds. Dividing 70 by $1/60$ shows that the pinhole image required a shutter speed of 4,200 times longer than that of the lens! Stops are measured in doubling or halving of light so we must figure out how many times 2 is multiplied with itself to reach 4,200:

$$\begin{aligned}2^x &= 4200 \\ \log(2^x) &= \log(4200) \\ x \log(2) &= \log(4200) \\ x &= \frac{\log(4200)}{\log(2)} = \log_2 4200 = 12.04\end{aligned}$$

The light was halved about 12 times. From this, we can determine the F-number of the pinhole by counting 12 stops away from $f/2.0$:

F-numbers: $f/2.0$, $f/2.8$, $f/4.0$, $f/5.6$, $f/8$, $f/11$, $f/16$, $f/22$, $f/32$, $f/44$, $f/64$, $f/88$, **$f/128$**

Now for the final calculation!

$$\begin{aligned}F\text{-number} &= \frac{f}{D} \\ 128 &= \frac{50\text{mm}}{d} \\ d &= \frac{50\text{mm}}{128} = 0.4\text{mm}\end{aligned}$$